

Labor Allocation in Automotive Planning: Leveraging a Flexible Workforce

Ada Barlatt Amy Cohn Mary Jo Luppino

University of Michigan

Oleg Gusikhin

Ford Motor Company

2007 INFORMS Meeting - Seattle ,WA

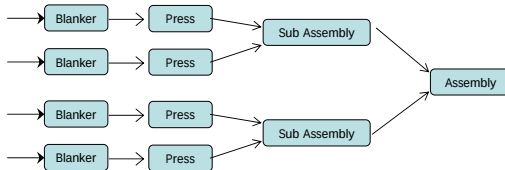
Example: Plant Overview

Automotive
Stamping

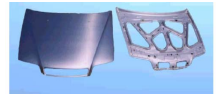
Test-and-
Prune

Conclusion

Automotive Stamping Plant - Part of the automotive supply chain that produces parts that compose the frame of vehicles.

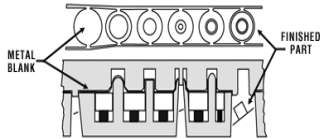


Example Plant Layout

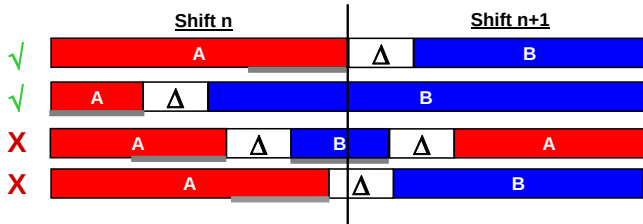
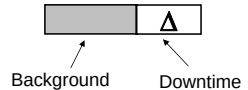


Example Products

Example: Stamping Press



Die Set Changeover



Example: Decisions, Goal, Rules

■ Decisions

- Allocate workforce and sequence jobs on the press lines

■ Goal

- Minimize the cost of the workforce

■ Rules

- Customer Constraints
- Production Constraints
 - Three shift types per day
 - Ten day planning horizon
 - Repeatable
- Changeover Constraints
- Labor Constraints
 - Direct labor requirements
 - Indirect labor requirements
 - Labor hired for sets of shifts

Problem: Challenges Motivate Composite Variable Modeling

Automotive
Stamping

Test-and-
Prune

Conclusion

Model Challenges

- Many complicated constraints
 - Order and duration of jobs
 - Changeovers
 - Shift interactions
- Large number of variables and constraints

Composite Variables

- Capture complexity in definition
- Successful applications
 - Airline Crew Scheduling
 - S. Lavoie, M. Minoux, & E. Odier (1988)
 - A. Cohn & C. Barnhart (2003)
 - Freight Transportation
 - A. Cohn, S. Root, & A. Wang (2005)
 - Manufacturing
 - Little if any application

Example: Composite Variable Definition

VARIABLE = SHIFT SCHEDULE

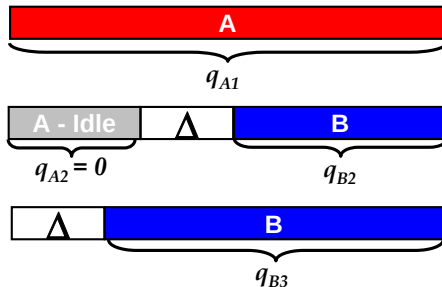
Shift Schedule: A set of *feasible* events in an eight hour shift.



Example: Composite Variable Definition

VARIABLE = SHIFT SCHEDULE

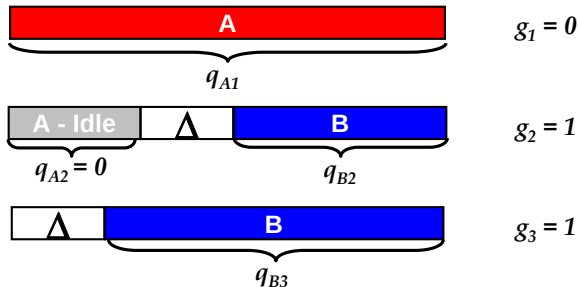
Shift Schedule: A set of *feasible* events in an eight hour shift.



Example: Composite Variable Definition

VARIABLE = SHIFT SCHEDULE

Shift Schedule: A set of *feasible* events in an eight hour shift.



Example: Composite Variable Definition

VARIABLE = SHIFT SCHEDULE

Shift Schedule: A set of *feasible* events in an eight hour shift.

$$f_{A1} = 1$$



$$l_{A1} = 1$$

$$f_{A2} = 1$$



$$l_{B2} = 1$$

$$f_{A3} = 1$$



$$l_{B3} = 1$$

Model: Variables

Automotive
Stamping

Test-and-
Prune

Conclusion

- x_{ns} - binary variables that specify if shift schedule s is assigned to shift n .
- y_r^d - integer variables that specify the number of direct laborers needed during labor shift type r .
- y_r^i - integer variables that specify the number of indirect laborers needed during labor shift type r .
- i_{tj} - continuous variables that specify the inventory of job j at the end of day t , (hours of production)
- w_{nj}^f - integer variables that specify the first job produced in shift n .
- w_{nj}^l - integer variables that specify the last job produced in shift n .

Model: Formulation

Automotive
Stamping

Test-and-
Prune

Conclusion

$$\min \sum_r c_r^i * y_r^i + \sum_r c_r^d * y_r^d$$

- Minimize labor cost

s.t.

$$\sum_s x_{ns} = 1 \quad \forall n \in N$$

- Select one shift schedule for each shift

$$\sum_s q_{js} * x_{(3t-3)s} + \sum_s q_{js} * x_{(3t-2)s} + \sum_s q_{js} * x_{(3t-1)s} + i_{(t-1)j} - d_{tj} = i_{tj} \quad \forall j \in J, t \in \{2 \dots 10\}$$

$$\sum_s q_{js} * x_{30s} + \sum_s q_{js} * x_{1s} + \sum_s q_{js} * x_{2s} + i_{10j} - d_{1j} = i_{1j} \quad \forall j \in J$$

- Calculate inventory for each day in the horizon

$$i_{0j} = i_{10j} \quad \forall j \in J$$

- Beginning inventory must equal ending inventory

Model: Formulation, continued

Automotive
Stamping

Test-and-
Prune

Conclusion

$$\sum_s f_{js} * x_{ns} = w_{nj}^f \quad \forall j \in J, n \in N$$

$$\sum_s l_{js} * x_{ns} = w_{nj}^l \quad \forall j \in J, n \in N$$

- Define the w variables

$$w_{nj}^l = w_{(n+1)j}^f \quad \forall j \in J, n \in \{1 \dots 29\}$$

- The last job on in a shift must be the first job in the following shift

$$w_{30j}^l = w_{1j}^f \quad \forall j \in J$$

- The last job in the horizon must equal the first job of the horizon

$$\sum_s a_s * x_{ns} \leq y_{r(n)}^d \quad \forall n \in N$$

$$\sum_s g_s * x_{ns} \leq y_{r(n)}^c \quad \forall n \in N$$

- Calculate direct and indirect labor

$$x_{ns} \in \{0, 1\} \quad \forall n \in N, s \in S$$

$$i_{tj} \geq 0 \quad \forall t \in T, j \in J$$

$$y_r^c, y_r^d \in \mathbb{Z}^+ \quad \forall r \in R$$

$$w_{nj} \in \{0, 1\} \quad \forall n \in N, j \in J$$

- Integrality and non-negativity constraints

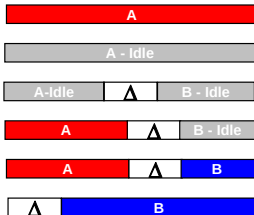
Composite Variable Definition: Variable Reduction

Automotive
Stamping

Test-and-
Prune

Conclusion

Result: $0 \leq x_{ns} \leq 1$ is the **proportion** of shift schedule s assigned to shift n



Jobs (Part Types)	Shift Schedules
5	90
10	380
15	870

- **Data Set**

- 22 Press Lines

- **Computational Results**

- Implemented in C++/Concert Technology
 - Solver CPLEX 10.2
 - IBM x3455 running Redhat Enterprise Linux 4

- **Optimization Computational Results**

- 7 of 22 machines - provable optimal solution in 2 hours
 - 15 of 22 machines - final gaps of 7.60 % to 49.53%

Results Motivate Test-and-Prune

Automotive
Stamping

Test-and-
Prune

Conclusion

- **High-level and Low-level Decisions**
 - High: Determine the number of workers hired for each shift type
 - Low: Determine the machine schedule, using the workers allocated

Results Motivate Test-and-Prune

Automotive
Stamping

Test-and-
Prune

Conclusion

■ Test-and-Prune

1 Build List

- Enumerate all possible high-level decision allocations
- *Pre-process* clearly infeasible allocations
- Three decision example: $\{0, 0, 0\}$

2 Process List

- While the list is not empty, select an allocation
- Solve corresponding feasibility problem
- *Prune* list based on result
- Three decision example: $\{3, 3, 3\}$ feasible
- Prune allocations with larger cost

Results Motivate Test-and-Prune

Automotive
Stamping

Test-and-
Prune

Conclusion

■ Test-and-Prune

1 Build List

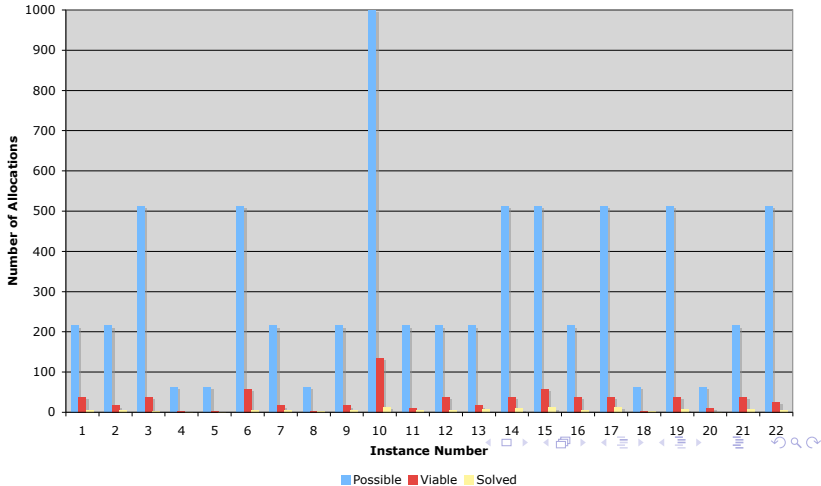
- Enumerate all possible high-level decision allocations
- *Pre-process* clearly infeasible allocations
- Three decision example: $\{0, 0, 0\}$

2 Process List

- While the list is not empty, select an allocation
- Solve corresponding feasibility problem
- *Prune* list based on result
- Three decision example: $\{3, 3, 3\}$ infeasible
- Prune dominated allocations $\{3, 2, 3\}, \{3, 2, 2\}, \dots$

Test-and-Prune Computational Results

Test-and-Prune Results



Computational Results: A Comparison

Automotive
Stamping

Test-and-
Prune

Conclusion

- **New Test-and-Prune Algorithm**

- All 22 machines - provably optimal solution in under 10 minutes
 - 19 machines - provably optimal solution in under 3 minutes

- **Traditional Optimization Approach**

- 7 of 22 machines - provable optimal solution in 2 hours
- 15 of 22 machines - final gaps of 7.60 % to 49.53%

■ Workforce Allocation and Scheduling

■ *Test-and-Prune* Algorithm

- Takes advantage of relationships between workforce decisions
- Proves provably optimal solutions quickly

■ Automotive Stamping Example

- Uses *Test-and-Prune* to solve real-world instances
- Formulation based on composite variables (shift schedules)

- **Additional applications of Test-and-Prune**
 - Extensions to the stamping problem
 - Non-manufacturing case studies
- **Further generalize Test-and-Prune approach**
 - Extend to general resource allocation and utilization
 - Multi-criteria optimization

Questions

Automotive
Stamping

Test-and-
Prune

Conclusion