

Problem

Test-and-
Prune
Algorithm

Automotive
Stamping

Conclusion

Test-and-Prune Approach to High-Level and Low-Level Resource Planning

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General Problem: Description

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- **Resource planning problems**
 - High-level decisions: *Resource Allocation*
 - Allocation of resources drives **cost**
 - Low-level decisions: *Resource Utilization*
 - Tasks must be completed to satisfy demand
 - Utilization of resources affects **feasibility**
- **Decisions need to be integrated to have effective plans**
 - How are the resources distributed? (*resource allocation*)
 - What happens during the horizon? (*resource utilization*)

General Problem: Examples

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■ Telemarketing

- Resource: Labor
- Allocation: Determine staffing during the horizon
- Utilization: Conduct all required customer surveys

■ Surgical Scheduling

- Resource: Operating Rooms
- Allocation: Assign capabilities to rooms
- Utilization: Schedule pending surgeries

■ Manufacturing

- Resource: Machines
- Allocation: Determine availability of machines
- Utilization: Sequence parts to meet demand

General Problem: Formulation

■ Formulation

- y is the resource allocation decision vector.
- x is the resource utilization decision vector.
- $\mathbf{X}$ is the set of integrality and bounds on x .
- $\mathbf{Y}$ is the set of bounds on y .

$$\begin{aligned} \min \quad & f(y) \\ \text{s.t.} \quad & \\ & g^i x - y \leq 0 \quad \forall i \in I \\ & Hx \leq 0 \\ & x \in \mathbf{X} \\ & y \in \mathbf{Y} \end{aligned}$$

General Problem: Challenges

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■ Challenges

- What is $\mathbf{Y}$?
 - Possibilities: non-linear, discrete, or non-convex
- What is the structure of $f(y)$?
 - Possibilities: non-linear, non-convex, not closed-form
- Weak LP relaxation
 - Minimum cost often has fractional resources

General Problem: Formulation

- **Key assumptions motivate our approach**

- Fixing the resource allocation leaves an easier feasibility problem
 - Reduces complexity

$$\begin{array}{ll}\min & f(y) \\ \text{s.t.} & \\ & g^i x - y \leq 0 \quad \forall i \in I \\ & Hx \leq 0 \\ & x \in \mathbf{X} \\ & y \in \mathbf{Y}\end{array}$$

General Problem: Formulation

- **Key assumptions motivate our approach**

- Fixing the resource allocation leaves an easier feasibility problem
 - Reduces complexity

$$\min \quad 0$$

s.t.

$$g^i x \leq y' \quad \forall i \in I$$

$$Hx \leq 0$$

$$x \in \mathbf{X}$$

where $f(y')$ is a constant

General Problem: Formulation

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- **Key assumptions motivate our approach**
 - Resource allocation decisions are limited discrete and finite
 - Bounds can be created
 - Limited number of possible allocations to consider

General Problem: Formulation

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■ Key assumptions motivate our approach

- Information about resource allocation decisions can be used to eliminate other allocations
 - Few resources - more constrained problem
 - Many resources - less constrained problem
- Example - three resource types
 - If $\{3\ 2\ 3\}$ feasible
 - Ignore every y : $f(\{3\ 2\ 3\}) \leq f(y)$
 - Provides new upper bound on the solution

General Problem: Formulation

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■ Key assumptions motivate our approach

- Information about resource allocation decisions can be used to eliminate other allocation decisions
 - Few resources - more constrained problem
 - Many resources - less constrained problem
- Example - three resource types
 - If $\{3\ 2\ 3\}$ infeasible
 - $\{2\ 2\ 3\}$, $\{3\ 1\ 3\}$, $\{3\ 2\ 2\}$... is infeasible
 - May provide new lower bound on solution

Test-and-Prune Algorithm: Overview

■ Concept

■ *Build List*

- Enumerate all possible resource allocations

■ *Process List*

- For each resource allocation determine if it is feasible
- Pick the feasible allocation with the smallest cost

■ Enhancements

- *Pre-processing* - Reduce the initial list of allocations considered

- Remove clearly infeasible allocations

- *Pruning* - Eliminate allocations with information

- Prune by feasibility
- Prune by infeasibility

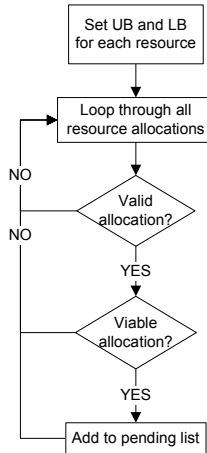
Test-and-Prune Algorithm: Build List

Problem

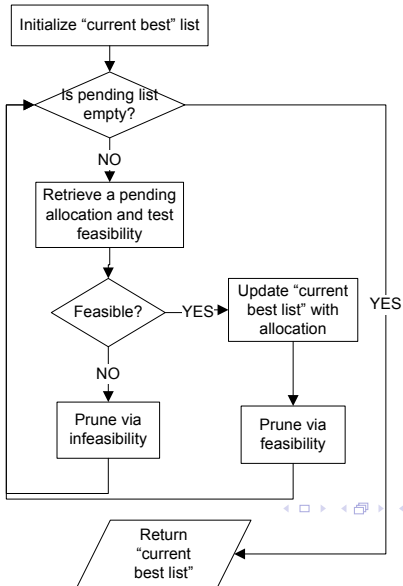
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Test-and-Prune Algorithm: Process List



Example: Plant Overview

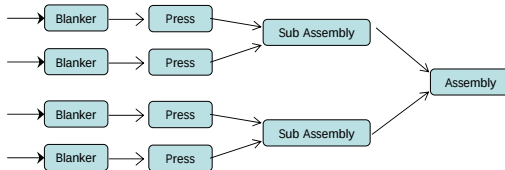
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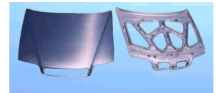
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Automotive Stamping Plant - Part of the automotive supply chain that produces parts that compose the frame of vehicles.



Example Plant Layout



Example Products

Example: Decisions, Goal, Rules

■ Decisions

- Allocate workforce and sequence jobs on the press lines

■ Goal

- Minimize the cost of the workforce

■ Rules

- Customer Constraints
 - Meet demand
- Production Constraints
 - Direct labor required
- Changeover Constraints
 - Indirect labor required
- Time Constraints
 - Ten day planning horizon

Example: In terms of Test-and-Prune

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- **Resource Allocation**
 - How many laborers in each shift?
- **Resource Utilization**
 - Determine the sequence jobs to meet demand with the allocated labor

■ Data Set

- 22 Press Lines
- Number of part types (across machines): 130
- Number of units (across machines): 1,024,650
- Maximum number of part types (on one machine): 13

■ Computational Results

- Implemented in C++/Concert Technology
- Solver CPLEX 10.2
- IBM x3455 running Redhat Enterprise Linux 4

Test-and-Prune Computational Results

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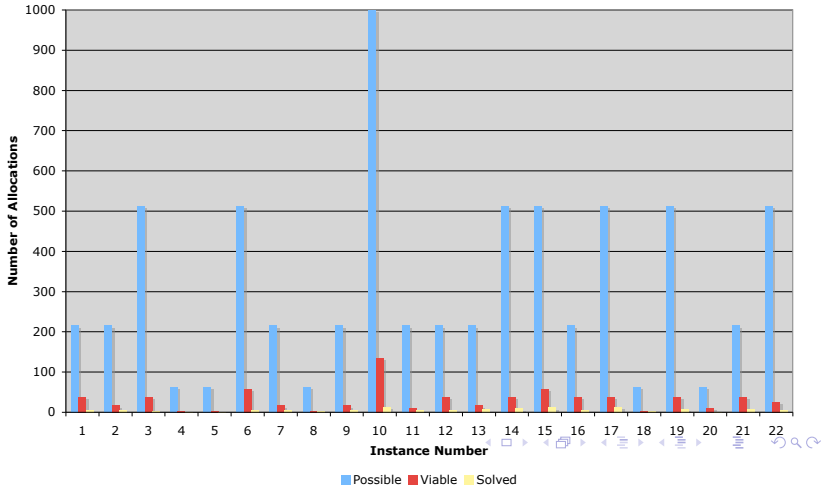
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- **Availability of direct and indirect workers**
- **Single Pressline - Single Machine**
 - How many laborers are available during each shift?
 - 22 machines - provably optimal solution for each machine in under 10 minutes
 - 19 machines - provably optimal solution in under 3 minutes

Test-and-Prune Computational Results

Test-and-Prune Results



Test-and-Prune Computational Results

■ Pressline Stage - Parallel Machine

- Is labor available during shift n ? (Y/N)
- Three Shift Types

Demand Level	Low	Medium	High
<i>#Possible</i>	64	64	64
<i>#Viable</i>	6	6	2
<i>#Feasible</i>	1	1	1
<i>#Infeasible</i>	1	2	0
<i>Run Time</i>	25 sec	1.28 min	8.48 min

Test-and-Prune Computational Results

■ Pressline Stage - Parallel Machine

- Is labor available during shift n ? (Y/N)
- Fifteen Shift Types

Demand Level	Low	Medium	High
<i>#Possible</i>	32768	32768	32768
<i>#Viable</i>	1525	1525	1525
<i>#Feasible</i>	6	6	5
<i>#Infeasible</i>	0	0	13
<i>Run Time</i>	3.13 min	6.65 min	5.41 hr

■ Resource Allocation and Utilization

■ *Test-and-Prune* Algorithm

- Cost function does not need to be linear or closed form
- Takes advantage of relationships between resource allocation decisions
- Provides provably optimal solution

■ Automotive Stamping Example

- *Test-and-Prune* used to solve workforce allocation and scheduling problems

- **Additional applications of Test-and-Prune**
 - Extensions to the stamping problem
 - Work begun to determine *number* of workers across presslines
 - Non-manufacturing case studies
- **Further generalize Test-and-Prune approach**
 - Multi-criteria optimization

Acknowledgements

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Questions

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