

Improving Crew Scheduling By Incorporating Key Maintenance Routing Decisions

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Crew Scheduling

- A *crew pairing* is a sequence of flights that can be flown by a single crew:
 - beginning and ending at a crew's home airport
 - spanning one or more days
 - satisfying FAA rules and collective bargaining agreements, such as:
 - maximum flying time in a day
 - minimum rest requirements
 - minimum connection time between two flights

Crew Pairing (CP)

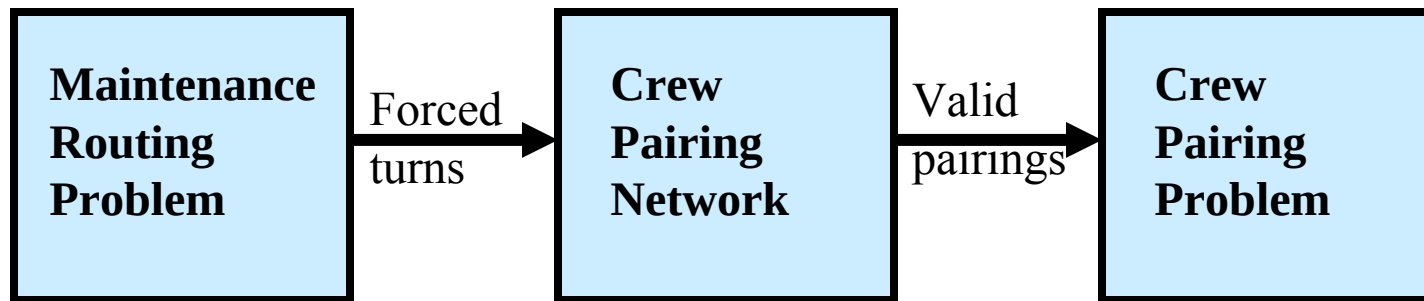
- Crew pairing problem: Find a minimum cost set of feasible pairings such that each flight is included in exactly one pairing
- Set partitioning problem with an exponential number of pairings:

$$\begin{aligned} & \min \sum_p c_p y_p \\ & st \\ & \sum_p \delta_{fp} y_p = 1 & \forall f \\ & y_p \in \{0,1\} & \forall p \end{aligned}$$

Link Between MR and CP

- Maintenance routing problem (MR) assigns aircraft to flights to ensure adequate maintenance opportunities
- Crew needs enough time between two sequential flights to travel through the terminal -- *minimum connect time*
- If both flights are covered by the same aircraft, this travel time is not needed -- tighter connections can be permitted
- A *forced turn* is a connection that is only crew-feasible if both flights are covered by the same aircraft

Sequential Approach



An Example

- Flights:
A B C D E F G H
- Forced turns:
A-B A-D D-G
- MR solution (x_1) uses
A-D and D-G
- MR solution (x_2) uses
A-B
- Pairings:
 - E-F-G-H (y_1) - \$1
 - B-C-E-F (y_2) - \$2
 - A-D-G-H (y_3) - \$5
 - A-B-C-D (y_4) - \$4
- Solutions:
 - $x_1 \Rightarrow$ pairings 2, 3 -- \$7
 - $x_2 \Rightarrow$ pairings 1, 4 -- \$5

Klabjan, Johnson, and Nemhauser

- Solve the crew pairing problem first, including all forced turns in the crew pairing network
- Given the crew solution, require all forced turns included in it to be part of the maintenance solution, which is solved second
- For “good” instances, this yields the optimal solution to the integrated problem (and many problems are “good”)
- For “bad” instances, this leads to maintenance infeasibility

Cordeau, Stojković, Soumis, Desrosiers

- Directly integrate string-based models
- Basic maintenance routing and crew pairing variables and constraints, plus linking constraints
- Benders decomposition approach using a heuristic branching strategy
- For non-hub-and-spoke networks, positive computational results

Our Objectives

- Guarantee maintenance feasibility
- Allow the user flexibility to trade off between solution time and quality
- Leverage the fact that only a fraction of the maintenance routing decisions are relevant to the crew pairing problem

Philosophy

- Allow the crew scheduler to choose from a collection of MR solutions
- Problem: Don't want to solve one CP for each MR solution
- Solution: Extended crew pairing model (ECP)

The Extended Crew Pairing Model (ECP)

- Start with basic CP model (set partitioning problem)
- Add one variable for each MR solution
- Add one linking constraint for each forced turn
- Add one convexity constraint to choose the MR solution

Matrix Representation

	x_1	x_2	y_1	y_2	y_3	y_4		rhs	
Flights:	0	0	0	0	1	1	=	1	A
	0	0	0	1	0	1	=	1	B
	0	0	0	1	0	1	=	1	C
	0	0	0	0	1	1	=	1	D
	0	0	1	1	0	0	=	1	E
	0	0	1	1	0	0	=	1	F
	0	0	1	0	1	0	=	1	G
	0	0	1	0	1	0	=	1	H
Forced turns:	0	1	0	0	0	-1	$\geq$	0	A-B
	1	0	0	0	-1	0	$\geq$	0	A-D
	1	0	0	0	-1	0	$\geq$	0	D-G
Convexity:	1	1	0	0	0	0	=	1	Conv.

General Formulation

$$\min \sum_p c_p y_p$$

st

Flights: $\sum_p \delta_{fp} y_p = 1 \quad \forall f$

Forced turns: $\sum_s \delta_{ts} x_s - \sum_p \delta_{tp} y_p \geq 0 \quad \forall t$

Convexity: $\sum_s x_s = 1$

$$x_s, y_p \in \{0,1\} \quad \forall s, p$$

Does ECP Meet Our Goals?

- Maintenance feasibility: ✓
- User flexibility: ✓
 - One column for the original MR solution -- sequential approach
 - One column for each MR solution -- optimal approach
 - Any number of columns in between -- a priori, via column generation, or both
- Leveraging key MR decisions: ✓
 - Only forced turn information appears in ECP

ECP Enhancements

- Dramatically reduce the *number* of MR columns:
 - *Uniqueness*: multiple MR solutions use the same set of forced turns
 - *Maximal independence*: $\{1, 2, 3\}$ dominates $\{1, 2\}$
 - Theoretical bounds and computational observation
 - Example: 61 flights $\Rightarrow \gg 25,000$ solutions $\Rightarrow 4$ required columns
- Relax the *integrality* of MR columns:
 - Same number of binary variables as original CP

Generating MR Solutions

- Algorithm to generate only UMI columns -- can generate as many columns in advance as time permits
- Can generate k UMI columns in at most the time needed to solve k MR problems
- Can also generate columns using delayed column generation to take advantage of dual information
 - Pricing problem still yields UMI columns
 - Crew pricing problem doesn't change

LP Solvability

- Comparing CP and ECP
- ~200,000 columns
- 243 flights
- 187 FT's
- CP LP relaxation: 243 rows, 153.59 seconds
- ECP LP relaxation: 431 rows, 366.17 seconds

* HP9000 Model D370 running HPUX 10.20 and CPLEX 6.0

Heuristic Results

- Problem A:

16 MR solutions: 31,396.10

Lower bound: 31,396.10

Optimality gap: 0%

- Problem B:

20 MR solutions: 25,498.60

Lower bound: 25,076.60

Optimality gap: 1.7%

Benefits of Our Model

- Ensures maintenance feasibility
- Can be solved heuristically or to optimality, allowing user to trade off solution time and quality
- Leverages the fact that the only maintenance decisions that matter to crew pairing are forced turns

Benefits of Our Model II

- Has the same number of binary variables as the crew pairing model alone
- Flexible
 - can take advantage of advances in maintenance and crew solvers
 - can incorporate new maintenance constraints
 - well-suited to problems with multiple sub-fleets
- Tighter LP relaxation than basic integrated model