

A Network Design Problem in Freight Transportation with Non-Linear, Cross-Arc Costs

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Project Background

- “Third party logistics provider subsidiary of one of the major U.S. package carriers”
- Looking at moving freight between locations in the U.S. and Europe with purchased transportation resources
- Freight moves by ground from origin to US hub, then by air to European hub, then by ground to destination
- How to minimize air costs?

Arc Costs As a Function of System Flow

- Airlines do not offer per-unit cost on each flight
- Instead, cost is negotiated as a function of *total volume* in the system
- We consider specifically the case where the airline offers a base list per-unit price on each arc, then a system-wide discount based on volume
- Tradeoff: Extra ground costs and base air costs in exchange for consolidation to fewer airlines (and thus fewer flights, hubs) to get higher discounts

Cost Structure of Air Freight

	Airline 1	Airline 2	...	Airline A
Range 1 $l_1 - u_1$	$p_{11} \%$	$p_{12} \%$	...	$p_{1 A } \%$
Range 2 $l_2 (u_1) - u_2$	$p_{21} \%$	$p_{22} \%$	...	$p_{2 A } \%$
...	...	...	...	...
Range R $l_{ R } (u_{ R -1}) - u_{ R }$	$p_{ R 1} \%$	$p_{ R 2} \%$	...	$p_{ R A } \%$

Problem Statement

Given a set of commodities, with origins in the US and destinations in Europe, determine how to route these commodities from origin to US hub to European hub to destination so as to minimize overall transportation costs subject to airline-specific volume discounts.

Literature Review

- Network flows

- ◆ Ahuja, Magnanti, and Orlin (1989)
- ◆ Goldberg, Tardos, and Tarjan (1990)

- Non-linear, general cost flow problems

- ◆ Dembo, Mulvey, and Zenios (1989)

- Multi-commodity flows

- ◆ Jones, et al (1993)
- ◆ Hane, et al (1995)
- ◆ Schneur and Orlin (1995)

Literature Review, cont.

■ Network design

- ◆ Magnanti and Wong (1984)
- ◆ Minoux (1989)
- ◆ Holmberg and Hellstrand (1998)

■ Applications

- ◆ Armacost, Barnhart, and Ware (2002)
- ◆ Budenbender, Grunert, and Sebastian (2000)
- ◆ Kim, et al (1999)

■ Little (if any) on cross-arc costs

Modeling Approach

- Goal is to linearize the objective function
- Multi-commodity flow problem with side constraints
- Instead of one arc per flight, one arc per flight/range combination
- Binary variables, constraints enforcing only one range per airline

Model – Notation

■ Sets:

- ◆ O = set of origins
- ◆ D = set of destinations
- ◆ K = set of commodities (origin and destination)
- ◆ $k^O(o)$ = set of commodities from origin o
- ◆ $k^D(d)$ = set of commodities to destination d
- ◆ U = set of US hubs
- ◆ E = set of European hubs
- ◆ A = set of airlines
- ◆ F = set of flights (US hub, European hub, and airline)
- ◆ $F^O(u)$ = set of flights originating at US hub u
- ◆ $F^D(e)$ = set of flights terminating at European hub e
- ◆ $F(a)$ = set of flights on airline a
- ◆ R = set of pricing ranges

Model – Notation

■ Data:

- ◆ d_k = demand of commodity k
- ◆ c_{ou} = cost per unit of flow from origin o to US hub u
- ◆ c_{ed} = cost per unit of flow from European hub e to destination d
- ◆ c_f = base cost per unit of flow on flight f
- ◆ $a(f)$ = airline associated with flight f
- ◆ l_r = lower bound on flow for discount range r
- ◆ u_r = upper bound on flow for discount range r
- ◆ p_{ar} = percent discount for airline a in range r

Model – Notation

■ Variables:

- ◆ o_{ku} = amount of outbound flow of commodity k using US hub u (continuous, non-negative)
- ◆ i_{ke} = amount of inbound flow of commodity k using European hub e (continuous, non-negative)
- ◆ a_{fr} = amount of flow on flight f in range r (continuous, non-negative)
- ◆ y_{ar} = 1 if total flow on airline a is in range r , else 0 (binary)

Model – Objective

Min

$$\sum_{o \in O} \sum_{k \in K^O(o)} \sum_{u \in U} c_{ou} o_{ku} +$$

(origin ground cost)

$$\sum_{f \in F} \sum_{r \in R} (1 - p_{a(f)r}) c_f a_{fr} +$$

(air costs, weighted by discount factor)

$$\sum_{d \in D} \sum_{k \in K^D(d)} \sum_{e \in E} c_{ed} i_{ke}$$

(destination ground cost)

Model – Balance Constraints

$$\sum_{u \in U} o_{ku} = d_k \quad \forall k \in K$$

(all commodities have to flow to an origin hub)

$$\sum_{e \in E} o_{ke} = d_k \quad \forall k \in K$$

(all commodities have to flow from a destination hub)

$$\sum_{k \in K} o_{ku} = \sum_{f \in F}^O(u) \sum_{r \in R} a_{fr} \quad \forall u \in U$$

$$\sum_{k \in K} i_{ke} = \sum_{f \in F}^D(e) \sum_{r \in R} a_{fr} \quad \forall e \in E$$

(for each hub, total flow in equals total flow out –
NOT commodity specific!)

Model – Linearizing Constraints

$$\sum_{r \in R} y_{ar} = 1 \quad \forall a \in A$$

(each airline falls into one range)

$$\sum_{f \in F(a)} a_{fr} \geq l_r y_{ar} \quad \forall a \in A, r \in R$$

$$\sum_{f \in F(a)} a_{fr} \leq u_r y_{ar} \quad \forall a \in A, r \in R$$

(flow on an arc is zero or else falls within the range)

Potential Challenge

- Very weak LP relaxation!
- Example:
 - ◆ Single commodity
 - ◆ Single airline
 - ◆ Single US, European hub
 - ◆ Only “decision” is which range to put freight in (i.e. use optimization model to compute a non-linear cost)

Potential Challenge, cont.

- Ranges:

- ◆ $0 - 9$
- ◆ $10 - 19$
- ◆ ...
- ◆ $90 - 99$

- Flow = 5

- Optimal integer solution –
 $y_1 = 1, \quad x_1 = 5$

- Optimal LP solution

- ◆ $y_1 = 85/90, x_1 = 0$
- ◆ $y_{10} = 5/90, x_{10} = 5$

- Branch on $y_{10} = 0$

- ◆ $y_1 = 75/80, x_1 = 0$
- ◆ $y_9 = 5/80, x_9 = 5$

- ...

- Have to eliminate all ranges

Challenge Resolved

- In theory, flow in an LP solution will always be assigned to highest available range
- In practice, standard CPLEX preprocessing is sufficient!
- We're still looking for counter-examples...

Computational Experience

- Instances based on variations of actual data
 - ◆ ~1800 commodities
 - ◆ 8 US, 11 European hubs
 - ◆ 10 airlines
 - ◆ 236 flights
 - ◆ 4 ranges
- Problem size
 - ◆ ~35,000 continuous variables
 - ◆ 40 binary variables
 - ◆ ~3,700 constraints
- Solved using ILOG AMPL 8.100 on a Sun Ultra 80 running Solaris 8 with 2 gigabytes of RAM and 4 450MHz Ultra II Sparc processors
- All instances so far have solved in under 1 minute

Next Steps

- More testing – is performance network-specific?
- How do arc-based capacity constraints impact performance?
- How do no-splitting constraints impact performance?
- Integrate with hub location problem