

Improving Airline Crew Schedules By Expanding the Solution Space

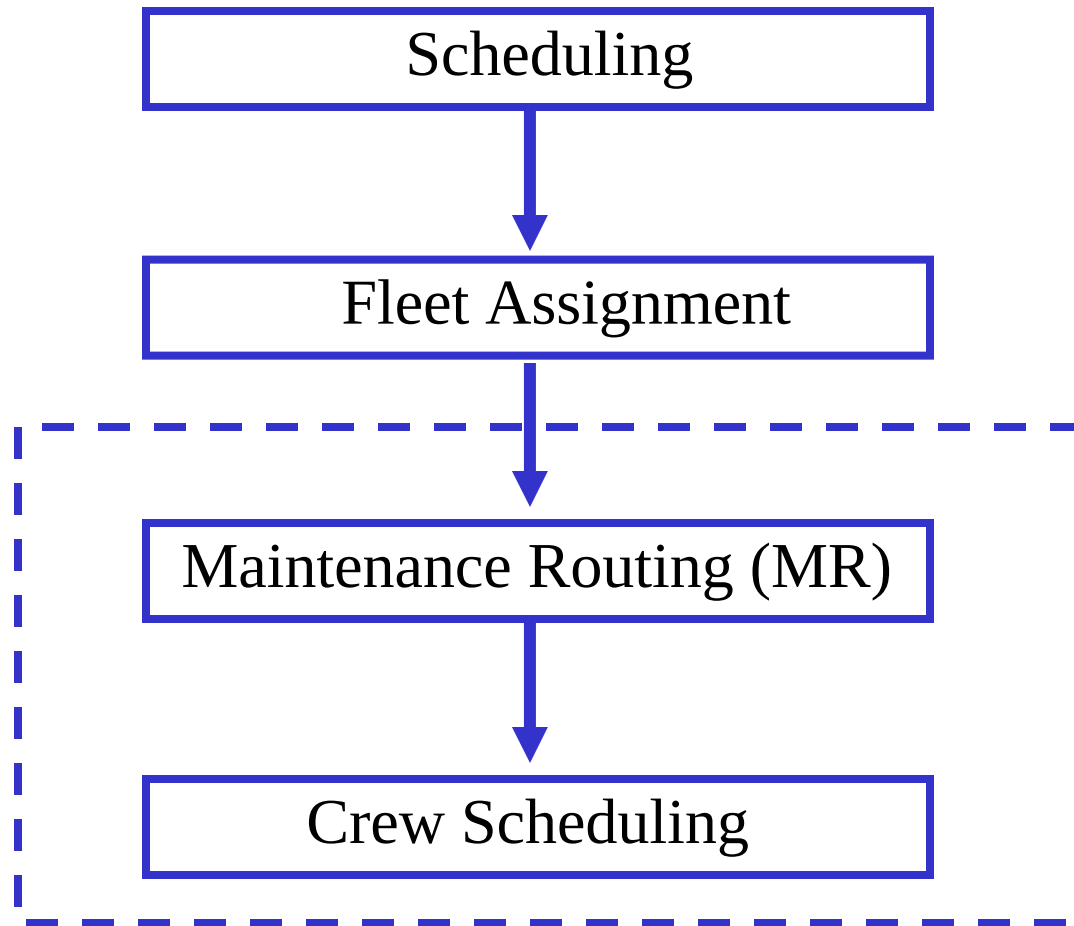
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Overview

- Crew scheduling is critical to the airline industry
 - Second largest operating expense
 - Small improvement in solution quality has significant financial impact
- For decades, researchers have worked on finding better crew scheduling algorithms
- Our approach is to instead improve solution quality by expanding the feasible set of solutions

The Airline Planning Problem



The Crew Pairing Problem

- A **pairing** is a sequence of flights which:
 - spans one or more days
 - begins and ends at a pilot's home airport
 - satisfies FAA rules and collective bargaining agreements, such as:
 - maximum flying time in a day
 - minimum rest requirements
 - minimum connection time between two flights

Crew Pairing (CP)

- Find a set of feasible pairings for a single fleet type such that:
 - Each flight is covered exactly once
 - Cost is minimized
- Set partitioning model:
 - Exponential number of pairings
 - Usually solved with branch-and-price

CP Formulation

$$\min \sum_p c_p y_p$$

st

$$\sum_p \delta_{fp} y_p = 1 \quad \forall f$$

$$y_p \in \{0,1\} \quad \forall p$$

Link Between MR and CP

- Min. connect time for crew can be relaxed to min. aircraft turn time if the same aircraft is used for both flights
 - Aircraft turn time -- 30 minutes
 - Crew connect time -- 45 minutes
- **Forced turn** -- connection of 30 - 45 minutes -- both flights must be assigned the same aircraft to be assigned the same crew

An Example

- Flights:
A B C D E F G H
- Forced turns:
A-B A-D D-G
- MR solution (x_1) uses
A-D and D-G
- Pairings:
 - Feasible connect time:
 - E-F-G-H (y_1)
 - B-C-E-F (y_2)
 - Maintenance feasible:
 - A-D-G-H (y_3)
 - Infeasible:
 - A-B-C-D (y_4)

Alternate Approaches

- CP first (Nemhauser, Johnson, and Klabjan)
 - Promising research in this area
 - Does not ensure a feasible MR solution
- Direct integration of basic models
 - Route strings and pairings
 - Tractability -- combining two large-scale IPs
 - Extendability -- if it works, can you add anything else?

Our Approach

- Generate many different solutions to the maintenance routing problem
- Allow the crew pairing model to choose the maintenance routing solution with the most useful set of forced turns.

An Example

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A B C D E F G H
- Forced turns:
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- Pairings:
 - Feasible connect time:
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 - Infeasible:
 - A-B-C-D (y_4)

Matrix Representation

Flights:

y_1	y_2	y_3		rhs	
0	0	1	=	1	A
0	1	0	=	1	B
0	1	0	=	1	C
0	0	1	=	1	D
1	1	0	=	1	E
1	1	0	=	1	F
1	0	1	=	1	G
1	0	1	=	1	H

Matrix Representation

Flights:

y_1	y_2	y_3	y_4	=	rhs	
0	0	1	1	=	1	A
0	1	0	1	=	1	B
0	1	0	1	=	1	C
0	0	1	1	=	1	D
1	1	0	0	=	1	E
1	1	0	0	=	1	F
1	0	1	0	=	1	G
1	0	1	0	=	1	H

Matrix Representation

Flights:

x_1	y_1	y_2	y_3	y_4	rhs	
0	0	0	1	1	= 1	A
0	0	1	0	1	= 1	B
0	0	1	0	1	= 1	C
0	0	0	1	1	= 1	D
0	1	1	0	0	= 1	E
0	1	1	0	0	= 1	F
0	1	0	1	0	= 1	G
0	1	0	1	0	= 1	H

Matrix Representation

Flights:

x_1	y_1	y_2	y_3	y_4		rhs	
0	0	0	1	1	=	1	A
0	0	1	0	1	=	1	B
0	0	1	0	1	=	1	C
0	0	0	1	1	=	1	D
0	1	1	0	0	=	1	E
0	1	1	0	0	=	1	F
0	1	0	1	0	=	1	G
0	1	0	1	0	=	1	H

Convexity:

1	0	0	0	0	=	1
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Matrix Representation

	x_1	y_1	y_2	y_3	y_4		rhs	
Flights:	0	0	0	1	1	=	1	A
	0	0	1	0	1	=	1	B
	0	0	1	0	1	=	1	C
	0	0	0	1	1	=	1	D
	0	1	1	0	0	=	1	E
	0	1	1	0	0	=	1	F
	0	1	0	1	0	=	1	G
	0	1	0	1	0	=	1	H
Forced turns:	0	0	0	0	-1	$\geq$	0	A-B
	1	0	0	-1	0	$\geq$	0	A-D
	1	0	0	-1	0	$\geq$	0	D-G
Convexity:	1	0	0	0	0	=	1	Conv.

Matrix Representation

	x_1	x_2	y_1	y_2	y_3	y_4		rhs	
Flights:	0	0	0	0	1	1	=	1	A
	0	0	0	1	0	1	=	1	B
	0	0	0	1	0	1	=	1	C
	0	0	0	0	1	1	=	1	D
	0	0	1	1	0	0	=	1	E
	0	0	1	1	0	0	=	1	F
	0	0	1	0	1	0	=	1	G
	0	0	1	0	1	0	=	1	H
Forced turns:	0	1	0	0	0	-1	$\geq$	0	A-B
	1	0	0	0	-1	0	$\geq$	0	A-D
	1	0	0	0	-1	0	$\geq$	0	D-G
Convexity:	1	1	0	0	0	0	=	1	Conv.

Extended Formulation

$$\min \sum_p c_p y_p$$

st

Flights: $\sum_p \delta_{fp} y_p = 1 \quad \forall f$

Forced turns: $\sum_s \delta_{ts} x_s - \sum_p \delta_{tp} y_p \geq 0 \quad \forall t$

Convexity: $\sum_s x_s = 1$

$$x_s, y_p \in \{0,1\} \quad \forall s, p$$

MR Solution Column Set

- Heuristic methods:
 - Generate k columns in advance
 - Generate columns dynamically
- Optimal approach:
 - Include all feasible MR solution columns, either through column generation or in some cases through explicit enumeration
 - Many structural benefits to leverage

Two Key Results

- **Result:** We can **relax the integrality** of the maintenance routing solution columns
 - Only as many binary variables as the crew pairing model alone; roughly the same number of rows
- **Result:** The number of **relevant** maintenance columns is much smaller
 - For many instances, may be able to enumerate all relevant columns in advance

Number of Relevant Columns

- Only need solutions containing **unique** and **maximally independent** sets of forced turns
- Example:
61 flights => 13,477 route strings => >> 25,000 solutions => 4 required columns (bounded by 35)
- **Result:** Algorithm to generate k relevant columns with no more work than solving k maintenance routing problems

Additional Results

- **Result:** Sufficient condition for bypassing many iterations when generating maintenance columns dynamically
- **Result:** Modification of pricing problem to only generate unique and maximally independent columns
- **Result:** Tighter LP relaxation than basic integrated model

Conclusions

- Improve crew pairing solution quality while ensuring maintenance feasibility
- No more binary variables than the basic model
- The number of relevant columns is much smaller than the total number of maintenance routing solutions -- in some instances, we can enumerate all relevant columns to guarantee optimality
- The model therefore suggests the possibility of further extension.

Further Research

- Computational experiments
- Alternate cut-based model
- Link between variables and cuts for ensuring maintenance feasibility
- Further integration and extension of airline planning