

Simplified Bidding And Solution Structures For
Combinatorial Procurement Auctions
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CMMI - Service Enterprise Engineering

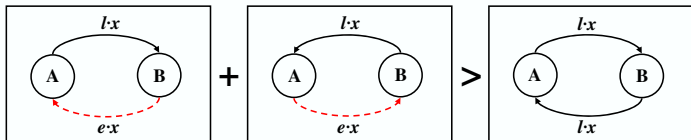
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Why Combinatorial Auctions?

- Enable bidders to express economies of scale and scope via their bids and preferences
- May result in lower bid prices and therefore lower cost for the auctioneer
- Successfully used in practice for billions of dollars worth of transactions

Power of Expressive Bidding: Truckload Example



- Two loads to be transported. One load from $A \rightarrow B$ and one load from $B \rightarrow A$
- Distance between A and B is x
- Empty and loaded costs per mile are e and l respectively
- Cost to transport both loads separately is $2lx + 2ex$
- Cost to transport both loads in a single tour is $2lx$

Computational Difficulties and Observations

Computational Difficulties:

- Bid evaluation and communication.
 - Exponential number of bundles ($2^n - 1$).
 - 30 items $\rightarrow$ 1,073,741,823 combinations
 - Communicating an exponential number of bundles is not possible
- Solving an exponentially-sized winner determination problem (WDP)

Observations:

- Current approaches assume exogenously endowed bids
- Current approaches do not consider the exhaustive set of bundles
- No current approach gives you an optimal solution within a practical time frame

Implicit Bidding Approach

- Bidders construct, price, and submit bids for bundles of items (up to $2^n - 1$)
- Bidders communicate bid prices to the auctioneer, who then solves a WDP
- A bidder's bid price (p_u^k) for a bundle u is obtained by solving a bid generating function (BGF) f^k
- Instead of the traditional WDP formulation, we present a novel alternative:

T-WDP

$$\begin{aligned} \min \quad & \sum_{k \in \mathcal{K}} \sum_{u \in \mathcal{U}^k} p_u^k x_u^k \\ \text{s.t.} \quad & \sum_{k \in \mathcal{K}} \sum_{u \in \mathcal{U}^k: l \in u} x_u^k = 1 \quad \forall l \in \mathcal{B} \\ & \sum_{u \in \mathcal{U}^k} x_u^k \leq 1 \quad \forall k \in \mathcal{K} \\ & x_u^k \in \{0, 1\} \quad \forall k \in \mathcal{K}, u \in \mathcal{U}^k \end{aligned}$$

I-WDP

$$\begin{aligned} \min \quad & \sum_{k \in \mathcal{K}} f^k(\mathbf{x}^k) \\ \text{s.t.} \quad & \sum_{k \in \mathcal{K}} x_l^k = 1 \quad \forall l \in \mathcal{B} \\ & x_l^k \in \{0, 1\} \quad \forall k \in \mathcal{K}, l \in \mathcal{B} \end{aligned}$$

Model Size Comparison

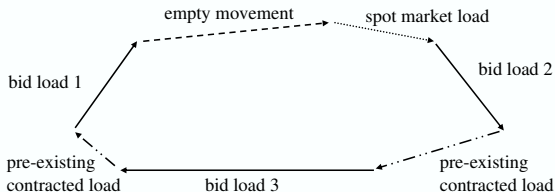
Let K represent the number of bidders and B represent the number of items in the auction.

	T-WDP	I-WDP
No. of binary variables	$(2^B - 1)K$	BK
No. of constraints	$B + K$	B

- Benefits: Exponential reduction in model size
- Weakness: Tractability of I-WDP depends on the structure of $BGF f^k$

What is BGF in the truckload problem?

- To minimize costs, and thus improve profitability, carriers must build efficient **continuous moves**. This can be accomplished not only by combining bid loads, but also by taking advantage of a carrier's pre-existing loads and spot market loads (*backhaul capacity*).



Carrier's Bid-Generating Function

- Given a bundle of loads s , how does each carrier k determine the bid price to cover these loads?

$$f^k(\mathbf{x}^s) := \sum_{a \in \mathcal{A}^{\mathcal{B}}} d_a^k x_a^k + \min \left(\sum_{a \in \mathcal{A}^{\mathcal{E}}} e_a^k y_a^k + \sum_{i \in \mathcal{A}_k^{\mathcal{C}}} c_a^k z_a^k \right) \quad (1)$$

$$\begin{aligned} \text{subject to: } & \sum_{a \in \mathcal{A}^{\mathcal{B}}: \mathcal{O}(a)=i} x_a^k + \sum_{a \in \mathcal{A}^{\mathcal{E}}: \mathcal{O}(a)=i} y_a^k + \sum_{a \in \mathcal{A}_k^{\mathcal{C}}: \mathcal{O}(a)=i} z_a^k \\ & = \sum_{a' \in \mathcal{A}^{\mathcal{B}}: \mathcal{D}(a')=i} x_{a'}^k + \sum_{a' \in \mathcal{A}^{\mathcal{E}}: \mathcal{D}(a')=i} y_{a'}^k + \sum_{a' \in \mathcal{A}_k^{\mathcal{C}}: \mathcal{D}(a')=i} z_{a'}^k \quad \forall i \in \mathcal{N} \end{aligned} \quad (2)$$

$$z_a^k \leq u_a^k \quad \forall a \in \mathcal{A}_k^{\mathcal{C}} \quad (3)$$

$$y_a^k \in \mathbb{Z}^+ \quad \forall a \in \mathcal{A}^{\mathcal{E}} \quad (4)$$

$$z_a^k \in \mathbb{Z}^+ \quad \forall a \in \mathcal{A}_k^{\mathcal{C}} \quad (5)$$

- This is a classical min-cost flow problem
- In a traditional CPTA, carriers must price each bundle of interest individually by solving f^k

Implicit Winner Determination Problem (I-WDP)

$$\min \sum_{k \in \mathcal{K}} \sum_{a \in \mathcal{A}^{\mathcal{B}}} d_a^k x_a^k + \sum_{k \in \mathcal{K}} \sum_{a \in \mathcal{A}^{\mathcal{E}}} e_a^k y_a^k + \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{A}_k^{\mathcal{C}}} c_a^k z_a^k \quad (6)$$

$$\text{subject to: } \sum_{k \in \mathcal{K}} x_a^k = w_a \quad \forall a \in \mathcal{A}^{\mathcal{B}} \quad (7)$$

$$\begin{aligned} & \sum_{a \in \mathcal{A}^{\mathcal{B}}: \mathcal{O}(a)=i} x_a^k + \sum_{a \in \mathcal{A}^{\mathcal{E}}: \mathcal{O}(a)=i} y_a^k + \sum_{a \in \mathcal{A}_k^{\mathcal{C}}: \mathcal{O}(a)=i} z_a^k \\ = & \sum_{a' \in \mathcal{A}^{\mathcal{B}}: \mathcal{D}(a')=i} x_{a'}^k + \sum_{a' \in \mathcal{A}^{\mathcal{E}}: \mathcal{D}(a')=i} y_{a'}^k + \sum_{a' \in \mathcal{A}_k^{\mathcal{C}}: \mathcal{D}(a')=i} z_{a'}^k \quad \forall i \in \mathcal{N}, k \in \mathcal{K} \end{aligned} \quad (8)$$

$$z_a^k \leq u_a^k \quad \forall a \in \mathcal{A}_k^{\mathcal{C}}, k \in \mathcal{K} \quad (9)$$

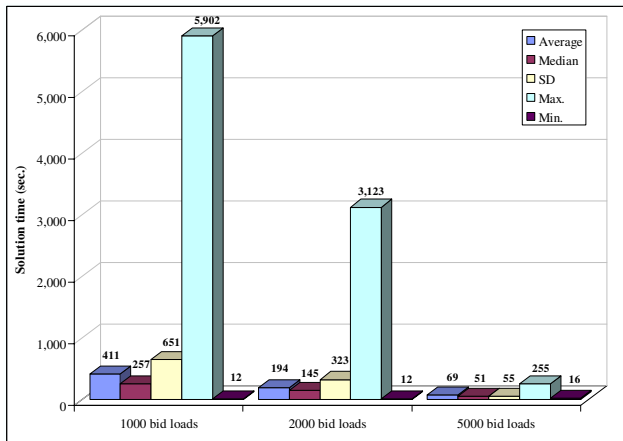
$$x_a^k \in \mathbb{Z}^+ \quad \forall a \in \mathcal{A}^{\mathcal{B}}, k \in \mathcal{K} \quad (10)$$

$$y_a^k \in \mathbb{Z}^+ \quad \forall a \in \mathcal{A}^{\mathcal{E}}, k \in \mathcal{K} \quad (11)$$

$$z_a^k \in \mathbb{Z}^+ \quad \forall a \in \mathcal{A}_k^{\mathcal{C}}, k \in \mathcal{K} \quad (12)$$

- This can be re-formulated as a pure multi-commodity flow problem!

I-WDP Computational Results - Unconstrained



instances = 100/scenario # nodes(cities) = 100

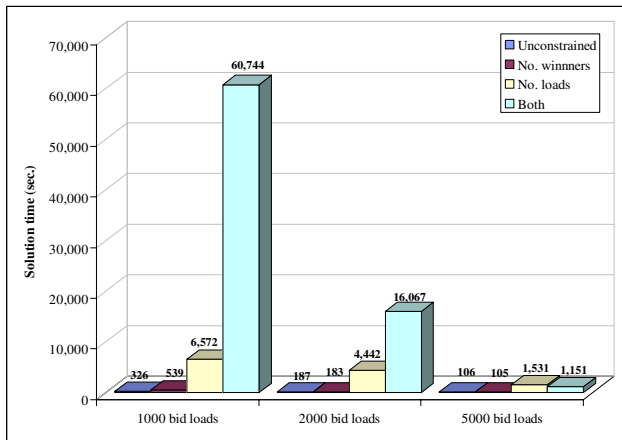
carriers = 50

backhaul capacity/carrier = 5%-15% of # of bid loads

Practical Considerations

- **Number of winners:** The shipper might want to award loads to no fewer than i_{min} carriers and to no more than i_{max} carriers.
- **Number of loads:** The shipper and carrier can specify the minimum and maximum numbers of loads that each carrier can be awarded.
- **Total mileage per carrier:** The shipper and carrier can specify a minimum, μ_{min}^k , and maximum, μ_{max}^k , miles that a carrier can travel.

I-WDP Computational Results - Constrained



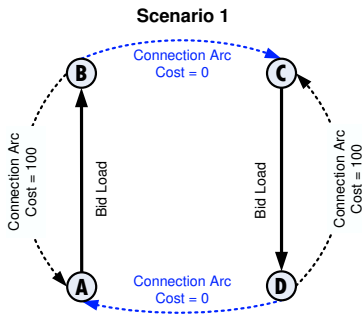
instances = 10/scenario # nodes(cities) = 100

carriers = 50

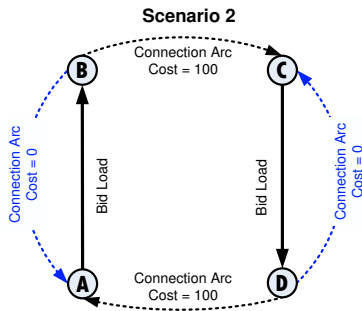
backhaul capacity/carrier = 5%-15% of # of bid loads

Stochastic Truckload Procurement Auction

- The connection costs are not known at the time of auction:



Optimal Routing: $A \rightarrow B \rightarrow C \rightarrow D$



Optimal Routing: $A \rightarrow B \rightarrow A$
And $C \rightarrow D \rightarrow C$

Stochastic Truckload Procurement Auction

- We assume a finite set of **scenarios** with respect to the cost of each connection arc
 - Each scenario $s \in \mathcal{S}$ has probability p^s
 - c_a^{ks} is the cost of connection arc a under scenario s for carrier k
- Carriers may **change** the routing after the information about their connection arc costs are revealed
- In this case we can minimize the **expected cost** over possible scenarios
- This is a **two-stage stochastic programming** problem with recourse

Stochastic Programming Formulation

$$\min \sum_{k \in \mathcal{K}} \sum_{a \in \mathcal{A}^{\mathcal{B}}} d_a^k x_a^k + \mathcal{Q}^P(x)$$

$$\sum_{k \in \mathcal{K}} x_a^k = 1 \quad \forall a \in \mathcal{A}^{\mathcal{B}}$$

$$x_a^k \in \{0, 1\} \quad \forall a \in \mathcal{A}^{\mathcal{B}}, \quad \forall k \in \mathcal{K}$$

- The **first-stage** decision, x assigns loads to carriers
- $\mathcal{Q}^P(x)$, is the objective function of the **primal recourse problem** corresponding to the first stage decision x

Decomposition of the Recourse Problem

- The recourse problem can be decomposed by **scenario**:

$$\mathcal{Q}^P(x) = \sum_{s \in \mathcal{S}} p^s \mathcal{Q}_s^P(x)$$

- The recourse problem can be decomposed by **carriers**
- Then, we can breakdown the set of scenarios ($\mathcal{S}$) and reduce them to **carrier-specific** scenario sets ($\mathcal{S}^k$):

$$\mathcal{Q}^P(x) = \sum_{k \in \mathcal{K}} \sum_{s \in \mathcal{S}^k} p^s \mathcal{Q}_{ks}^P(x)$$

- As we decompose the scenarios by carriers, the total number of combinations we need to consider drops dramatically from $|\mathcal{S}| = |\mathcal{S}^1 \times \mathcal{S}^2 \times \dots \times \mathcal{S}^k|$ to $\sum_{k \in \mathcal{K}} |\mathcal{S}^k|$

The Structure of the Recourse Problem

The decomposed recourse problem corresponding to carrier k under scenario s can be formulated as:

$$Q_{ks}^P(x) = \min \sum_{a \in \mathcal{A}^E} e_a^k y_a^k + \sum_{a \in \mathcal{A}^C} c_a^{ks} z_a^k$$

$$\begin{aligned} & \sum_{a \in \mathcal{A}^E : O(a)=i} y_a^k - \sum_{a' \in \mathcal{A}^E : D(a')=i} y_{a'}^k + \sum_{a \in \mathcal{A}^C : O(a)=i} z_a^k - \sum_{a' \in \mathcal{A}^C : D(a')=i} z_{a'}^k \\ &= \sum_{a' \in \mathcal{A}^B : D(a')=i} x_{a'}^k - \sum_{a \in \mathcal{A}^B : O(a)=i} x_a^k \quad \forall i \in \mathcal{N} \\ & z_a^k \leq u_a^k \quad \forall a \in \mathcal{A}^C \end{aligned}$$

$$\begin{array}{ll} y_a^k \in \mathbb{Z}^+ & \forall a \in \mathcal{A}^E \\ z_a^k \in \mathbb{Z}^+ & \forall a \in \mathcal{A}^C \end{array} \Rightarrow \begin{array}{ll} y_a^k \geq 0 & \forall a \in \mathcal{A}^E \\ z_a^k \geq 0 & \forall a \in \mathcal{A}^C \end{array}$$

Discussion

- The proposed formulation for the recourse problem is a **min-cost flow** problem, therefore:
 - The recourse problems are **very easy** to solve
 - As the first stage decisions are set, the LP-relaxation of the recourse problem has **integer solutions**
 - Therefore, there is **no duality gap**
- **Next step:** How can we solve the stochastic programming formulation?
- We can use an **algorithmic approach** based on Bender's decomposition, known as the L-shaped algorithm

Solution Algorithm

- **Step (0)** Initialize the main problem with $\theta_{ks} \geq 0 \quad \forall k \in \mathcal{K}, \forall s \in \mathcal{S}^k$ and solve it to get x .
- **Step (1)** Using vector x , solve the dual recourse problem $Q_{ks}^D(x) \quad \forall k \in \mathcal{K}, \forall s \in \mathcal{S}^k$.

- **Step (2)** Use the extreme point (α^v, π^v) found in step 2 to generate a new cut:

$$\theta_{ks} \geq \mathbf{B}(x)\pi^v + \mathbf{u}\alpha^v \quad \forall k \in \mathcal{K}, \forall s \in \mathcal{S}^k$$

and add it to the main problem.

- **Step (3)** Re-solve the main problem. If the new solution x is the same as the one found in the previous iteration, stop. Otherwise, go to step 1.

TRUCKLOAD AUCTIONS

- Perform empirical analysis of solution quality and characteristics
- Use this approach to study VCG and other auction designs, and assess their quality
- Develop a tour-based approach to address length and cardinality of tour considerations (submitting to CIEG for parallel implementation of solution approach)
- Implement STPA algorithm (submitting to CIEG for parallel implementation of solution approach)

GENERAL

- Apply the implicit bidding approach to other application domains such as: wireless spectra, energy markets, and operations procurement