

Composite-Variable Modeling for Large-Scale Problems in Transportation and Logistics

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Transportation and Logistics Challenges

- Complex networks spanning both space and time
- Tightly constrained resources
- Elaborate feasibility rules
- Large number of inter-connected decisions

Modeling Challenges

- Non-linearities
- Many constraints, many integer variables
- Weak linear programming relaxations
- Symmetry

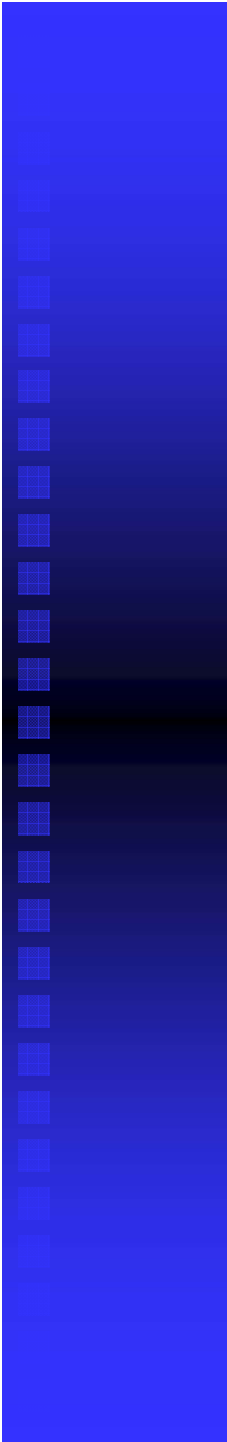
Research Objective

Developing alternative modeling approaches and solution techniques that...

- ◆ ... are tractable
- ◆ ... are extendible
- ◆ ... leverage user knowledge
- ◆ ... allow users to trade off solution time and quality
- ◆ ... are generalizable

Overview

- Introduction to Composite Variables
- Example 1: Airline Planning
- Example 2: Service Parts Logistics
- Generalizations and Conclusions



Introduction to Composite Variables

What Is a Composite Variable?

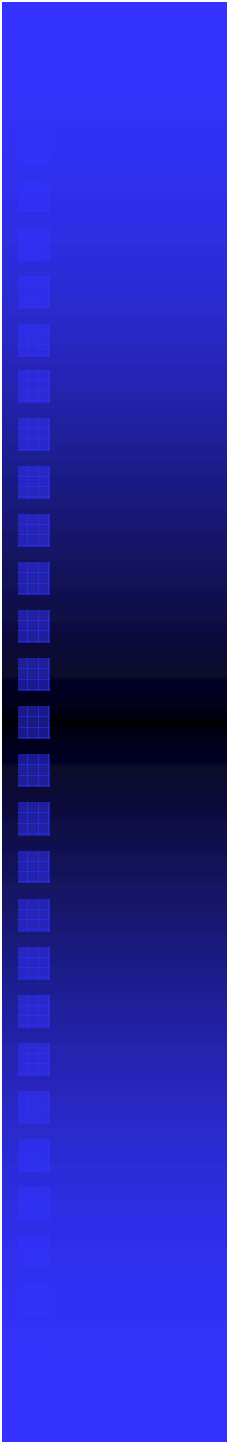
- A composite variable bundles multiple decisions into a single binary variable
- Example
 - ◆ Basic: Should I open warehouse x ? Should I assign customer y to warehouse z ?
 - ◆ Composite: Should I open warehouse x , stock products a , b , and c there, and use this warehouse to supply customers p , q , and r ?

Potential Benefits

- Removing non-linearities
- Strengthening LP relaxations
- Decreasing the number of constraints
- Exploiting problem structure
- Leveraging user expertise
- Providing user control for trading off solution time and quality

Too Many Variables?!

- Potential for an exponential increase in the number of binary variables
- We can sometimes control the number of variables needed via:
 - ◆ Feasibility
 - ◆ Dominance
 - ◆ Branch-and-price
- Design decisions are critical!



Example 1: Airline Planning

Motivation

- Crew costs are the airlines' second largest operating expense
- Even small improvements in efficiency can have large financial impact
- Earlier maintenance routing decisions limit schedulers' options
- A simultaneous approach is very big!

Crew Pairing Problem (CP)

- A crew pairing problem can be formulated as an integer programming problem:
 - ◆ ...
 - ◆ ...
 - ◆ ...
- Crew pairing problem can be formulated as an integer programming problem:
 - ◆ ...
 - ◆ ...
 - ◆ ...

$$\min \sum_p c_p y_p$$

st

$$\sum_{p: f \in p} y_p = 1 \quad \forall f$$

$$y_p \in \{0,1\} \quad \forall p$$

Maintenance Routing Problem (MR)

- min 0

st

$$\sum_{r: f \in r} x_r = 1 \quad \forall f$$
- {aircraft balance constraints}
 {aircraft count constraints}

$$x_r \in \{0,1\} \quad \forall r$$

How Are They Connected?

- Crews need enough time between two sequential flights to travel through the terminal -- minimum connect time
- This time can be decreased if the two flights have been assigned to the same aircraft
- A *short connect* is a connection that is crew-feasible only if both flights are assigned to the same aircraft

A Basic Integrated Approach (BIM)

■ Combined

■ Permit crew pair

■ Ensure for each short

◆ A pair only contains

$$\min \sum_p c_p y_p$$

st

$$\{CP\}$$

$$\{MR\}$$

$$\sum_{r:t \in r} x_r - \sum_{p:t \in p} y_p \geq 0$$

$\forall t$

$$x_r \in \{0,1\}$$

$\forall r$

$$y_p \in \{0,1\}$$

$\forall p$

erating

constraint

ect t can

Motivation for a Composite Approach

- Only *some* of the maintenance routing information is relevant for crew scheduling
- We don't want to find an integrated solution -- we want to find an optimal crew solution that is maintenance-feasible
- Can we leverage this to develop a more tractable model?

Composite Definition

- We define our composite variable to be a *complete, feasible solution to the MR problem*
- This captures all maintenance routing information
- The *extended crew pairing (ECP) model* finds an optimal crew schedule and a corresponding maintenance solution

ECP Formulation

Flights:

$$\min \sum_p c_p y_p$$

st

$$\sum_{p: f \in p} y_p = 1 \quad \forall f$$

Forced turns:

$$\sum_{s: t \in s} z_s - \sum_{p: t \in p} y_p \geq 0 \quad \forall t$$

Convexity:

$$\sum_s z_s = 1$$

$$z_s \in \{0,1\} \quad \forall s$$

$$y_p \in \{0,1\} \quad \forall p$$

Benefit: Dominance

Theorem:

- * We can replace the set of MR solutions with the set of *unique, maximal, maintenance-feasible short turn sets* and still be equivalent to BIM.

Dominance, cont.

■ Uniqueness

- ◆ Different MR solutions with the same short connect sets yield identical columns
- ◆ Example: 8,700 solutions $\Rightarrow$ **1 column**

■ Maximal sets

- ◆ Supersets dominate
- ◆ Example: millions of route string variables $\Rightarrow$ **4 UM columns**

Benefit: ECP Has No More Binary Variables Than Basic CP!

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th

$$\min \sum_p c_p y_p$$

st

$$\sum_{p:f \in p} y_p = 1 \quad \forall f$$

$$\sum_{s:t \in s} z_s - \sum_{p:t \in p} y_p \geq 0 \quad \forall t$$

$$\sum_s z_s = 1$$

$$y_p \in \{0,1\} \quad \forall p$$

$$z_s \in \{0,1\} \quad \forall s$$

can be co
nstraints.

$$\min \sum_p c_p y_p$$

st

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$$\sum_{s:t \in s} z_s - \sum_{p:t \in p} y_p \geq 0 \quad \forall t$$

$$\sum_s z_s = 1$$

$$y_p \in \{0,1\} \quad \forall p$$

~~$$z_s \in \{0,1\} \quad \forall s$$~~

$$z_s \geq 0 \quad \forall s$$

Benefit: Fewer Constraints

- Eliminate *all* maintenance routing...
 - ◆ ...cover constraints
 - ◆ ...balance constraints
 - ◆ ...count constraints
- Replace with *a single* convexity constraint

Benefit: Tighter LP Relaxation

Theorem:

- * For any problem instance, the value of the LP relaxation of ECP is greater than or equal to that of the LP relaxation of BIM.
- * In some instances, it is strictly greater.

Proof:

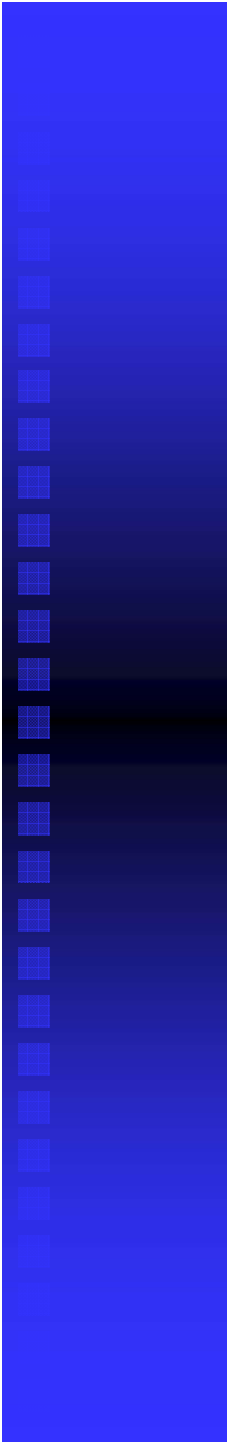
- * Based on a comparison of ECP to a Dantzig-Wolfe decomposition of BIM
- * Applies in other applications as well

Other Benefits

- Can leverage existing technology
- Extendability
- User control in trading off solution time and quality

Conclusions

- Crew scheduling is critical to airline profitability but quality can be compromised by making maintenance routing decisions independently
- A direct integration can be inflexible and difficult to solve
- ECP provides an alternative approach that exploits the fact that only *some* maintenance routing information is relevant, and uses dominance to greatly reduce the size of the problem



Example 2: Service Parts Logistics

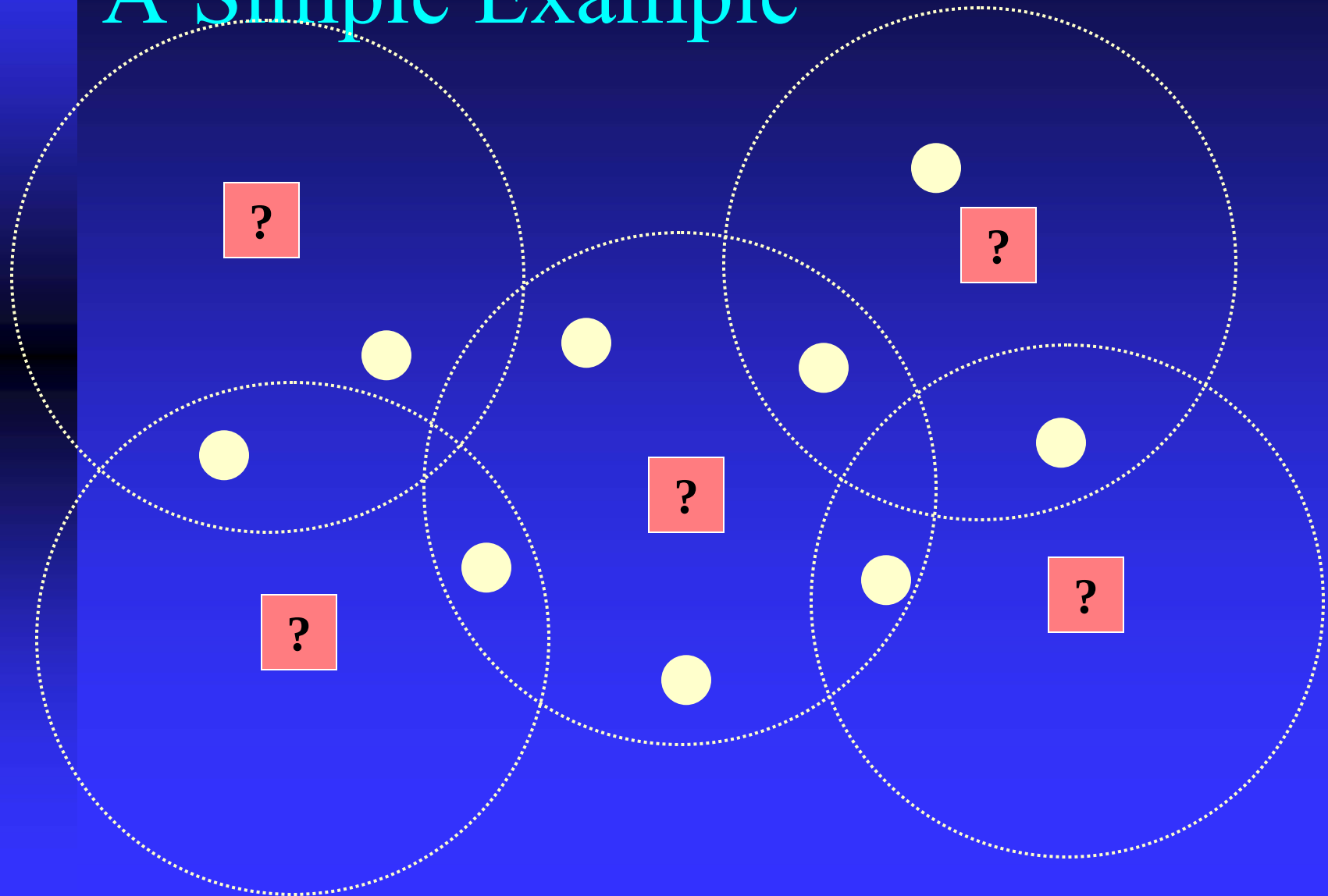
What Is Service Parts Logistics?

- Service contracts guarantee that repair parts/technician will arrive in a given amount of time to fix broken equipment
- Time window may be as short as two hours
- *Service parts logistics* manages the stocking and transporting of repair parts to satisfy these commitments

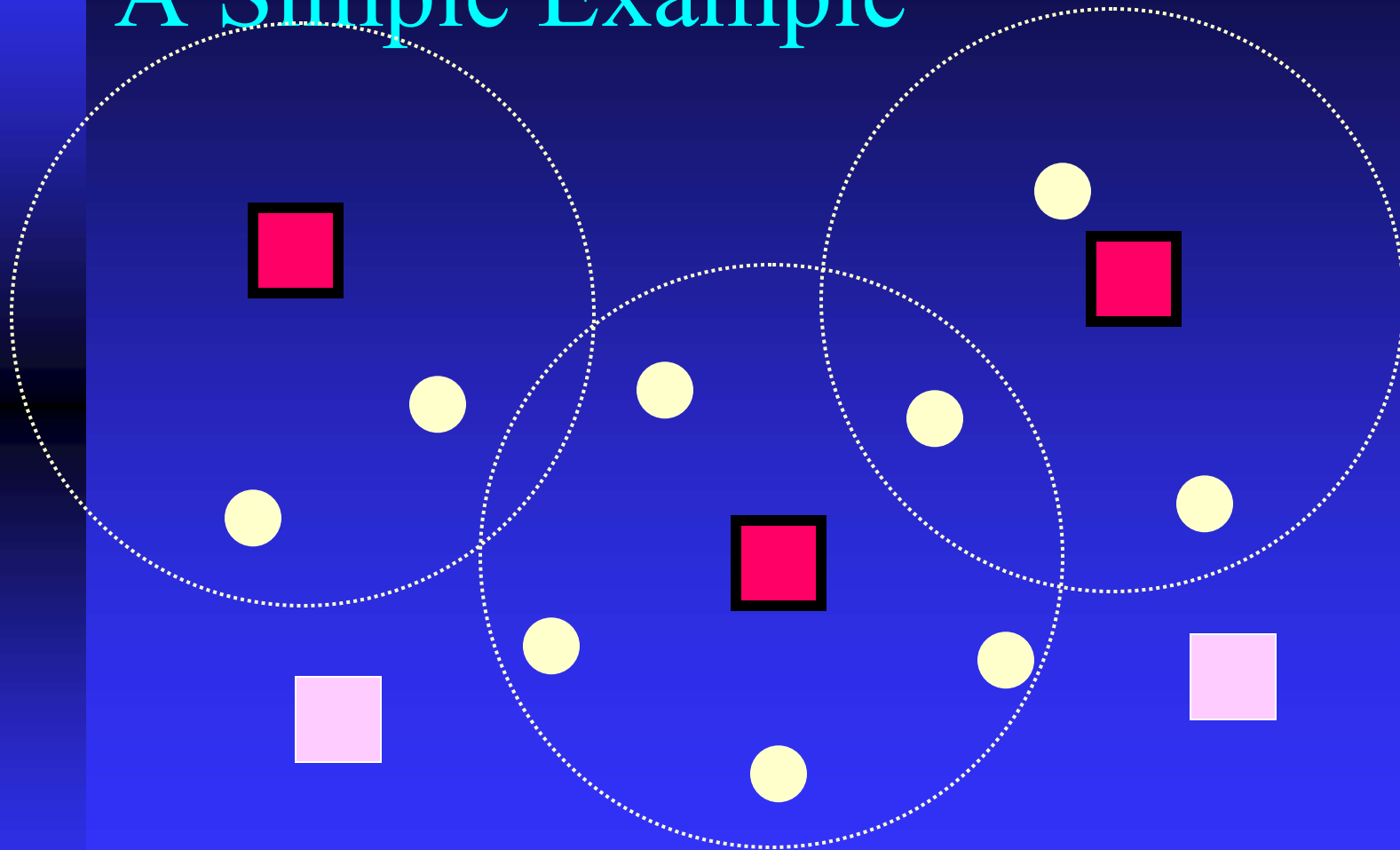
The High-Cost, Low-Demand Stocking Problem

Given a high-cost, very low-demand SKU, find the smallest set of warehouses in which to stock it, such that every potential demand for that part is within range of at least one warehouse in the chosen set.

A Simple Example



A Simple Example



Set Covering Model

$$\min \sum_{w \in W} c_w x_w$$

st

$$\sum_{w \in r(d)} x_w \geq 1 \quad \forall d \in D$$

$$x_w \in \{0,1\} \quad \forall w \in W$$

The Capacitated High-Cost, Low-Demand Stocking Problem

Given a set of HCLD SKUs, a demand set for each SKU, and a set of warehouses with limited capacity, find the minimum cost assignment of SKUs to warehouses such that each demand is covered and capacities are not exceeded.

Basic Formulation

$$\min \sum_{s \in S} \sum_{w \in W} c_{sw} x_{sw}$$

st

$$\sum_{w \in r(d)} x_{sw} \geq 1 \quad \forall s \in S, d \in d(s)$$

$$\sum_{s \in S} w_s x_{sw} \leq CAP_w \quad \forall w \in W$$

$$x_{sw} \in \{0,1\}$$

Why Is This Hard?

- Many more binary decision variables
- Very large number of constraints
- Greater tendency towards fractional solutions to the LP relaxation

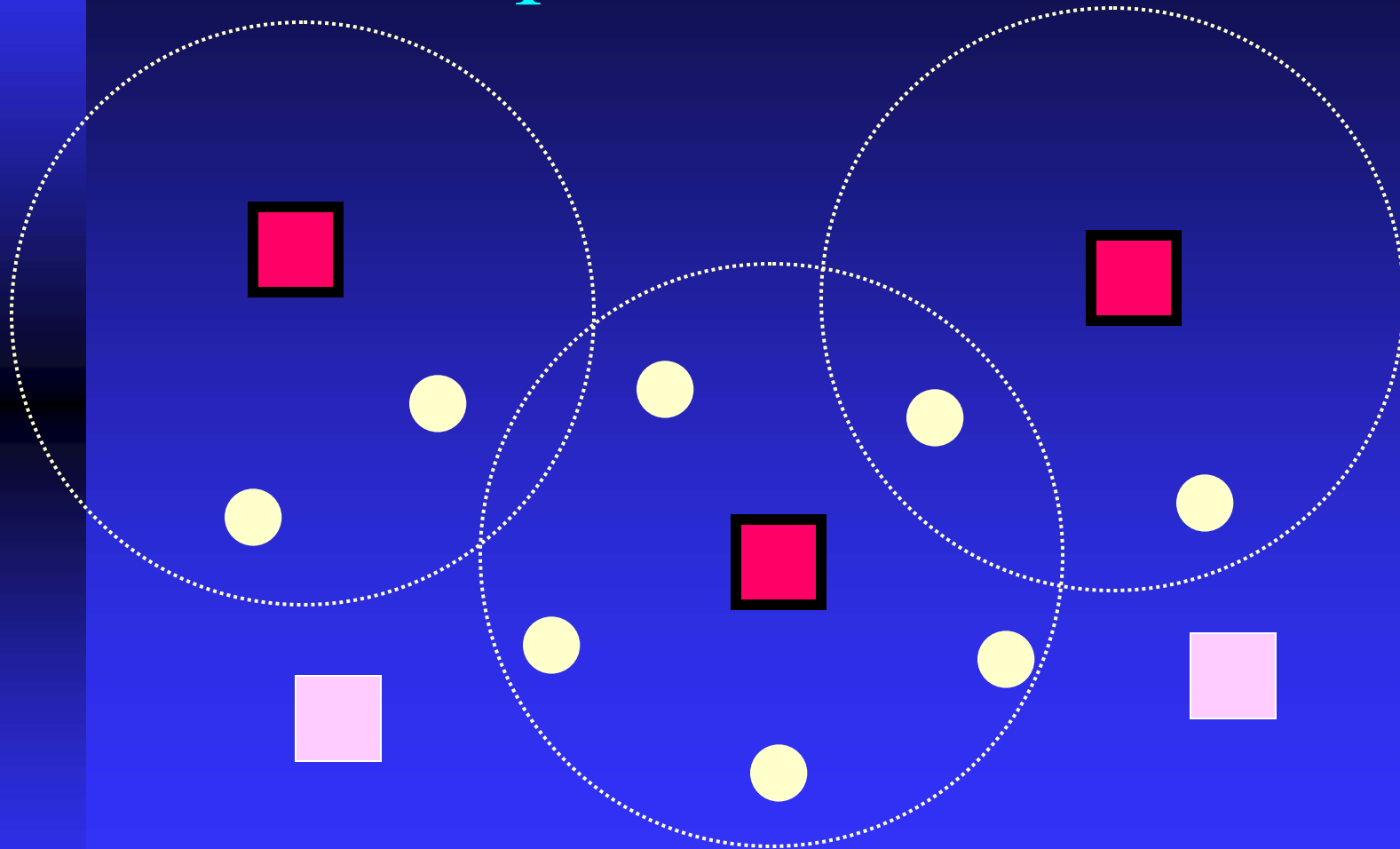
Composite Motivation

- Leverage what we know about individual parts
- Decrease number of constraints
- Tighten LP relaxation
- Provide a flexible framework -- potential for further extensions

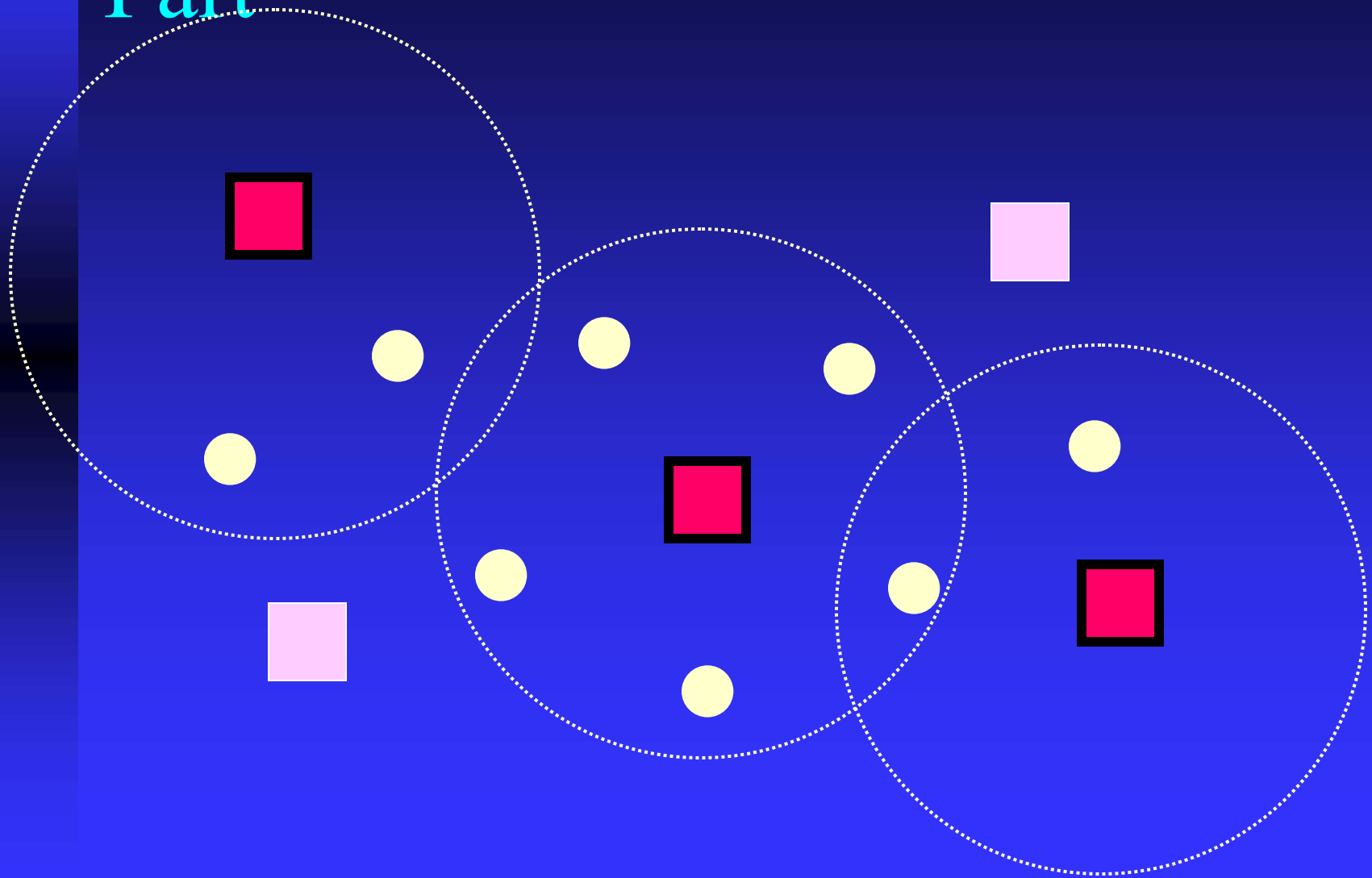
Composite Definition

- We define x_{sg} to be a composite variable:
Should I stock part s in exactly those
warehouses in group g ?
- We only define a variable for a pair s/g if g
provides total coverage for demand for s
- G_s denotes the set of all warehouse groups
that cover SKU s

One Composite Variable



Another Composite for the Same Part



Composite Formulation

$$\min \sum_{s \in S} \sum_{g \in G_s} c_{sg} x_{sg}$$

st

$$\sum_{g \in G_s} x_{sg} = 1 \quad \forall s \in S$$

$$\sum_{s \in S} \sum_{g \in G_s : w \in g} w_s x_{sg} \leq CAP_w \quad \forall w \in W$$

$$x_{sg} \in \{0,1\}$$

Theoretical Observations

- Many fewer constraints
- LP relaxations are tighter
- Dominance

Solving the Composite Model

Solve each LP with
column
one price
per SKU
part stock

Branch on SKU/
pairs

Depth-first until first
integer solution found,
then best-bound to
search the tree

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•

•

•

Computational Results

Problem Instance	Solve? Basic	Solve? Comp	Opt. Value	# Nodes Basic	# Columns Comp.	# Nodes Basic	# Nodes Comp.	# Columns Comp.
1	✓	✓		20	1	10	1	20
2		✓		1,000	90	1,000,000	1	90
3		✓		837	260	837,500	3	260
4	✓	✓	Infe	260	1	1	1	260
5	✓	✓		20	1	14	1	20
6	✓	✓		84	1	1200	1	84
7	✓	✓		412	163	412,939	1	163
8	✓	✓	Infe	258	1	1	1	258
9	✓	✓	Infe	267	1	1	1	267
10	✓	✓		20	1	10	1	20
11	✓	✓		20	1	10	1	20
12	✓	✓		20	1	1	16	67
13	✓			67	16	2800	NA	NA
14	✓	✓		NA	NA	174,400	91	131
15	✓	✓		174	74	100	33	74

33 FSBs, 10 SKUs, ~5000 demands

Computational Results (cont.)

Problem Instance	Solve? Basic	Solve Comp.	Op Val	# # Cols. Comp.	Root LP Comp.	# Nodes Basic	# Nodes Comp.	# Cols. Comp.
16	√	√		17,200	NA	0	0	17,200
17		√		297	3,744	NA	1	297
18		√		594	4,032	NA	129	594
19		√		351	3,441	NA	7	351
20		√		933	3,473	NA	152	933
21		√		3,432	NA	NA	0	3,432
22		√		2,988	15,029	NA	498	2,988
23		√		1,000	19,224	NA	1	1,000

99 FSBs, 100 SKUs, 1000 demands

99 FSBs, 500 SKUs, 1000 demands

Extendibility

- Structure leverages our knowledge of how to stock individual parts
- Potential to incorporate:
 - ◆ Transportation costs
 - ◆ High-demand parts
 - ◆ Part interactions
 - ◆ Simulation

Conclusions

- HCLD part stocking decisions are easy for one part, but not for many parts simultaneously
- The composite model improves tractability by
 - ◆ Decreasing the number of constraints
 - ◆ Tightening the LP relaxation
 - ◆ Leveraging our knowledge of the single-part problem
 - ◆ Using dominance and column generation
- The composite model also has greater promise for extendibility

Generalizations, Conclusions, and Further Research

Composites Don't Always Help!

- Example: A warehouse-based composite model for the SPL problem
- What's wrong?
 - ◆ Still a very large LP
 - ◆ No dominance

When Do Composites Work?

- Non-composite model is intractable
- Composite model addresses causes of intractability
- Dominance
- Good pricing problem

What Are Good Problem Environments?

- Contexts where we want to expand problem scope or have flexibility in rule definition
- Integrating planning problems
- User-controlled optimization
- Real-time environments
- Stochastic environments

Future Research

■ Applications

- ◆ Airline planning
- ◆ Service parts logistics
- ◆ Manufacturing
- ◆ Other aspects of the supply chain

■ Problem scope

- ◆ Robustness