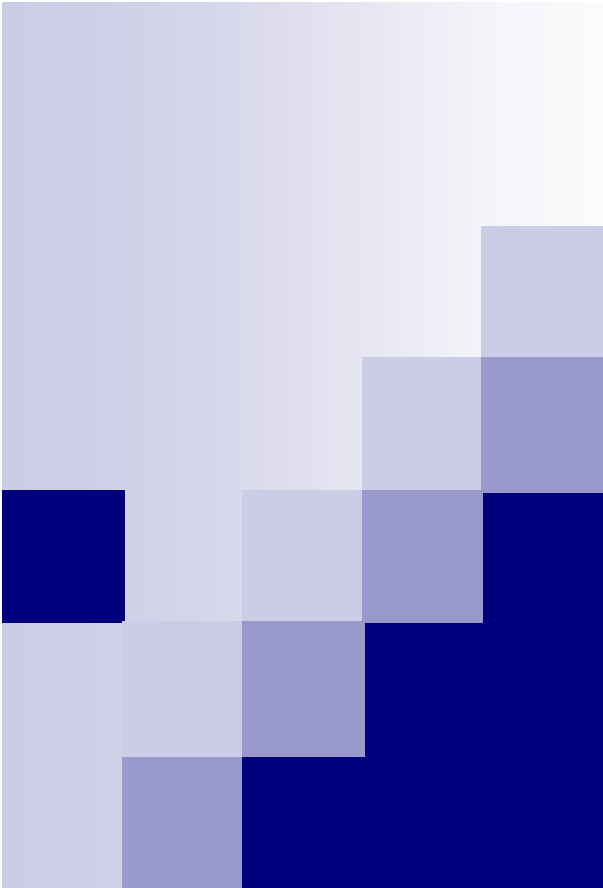




Using Composite Variable Modeling to Solve Integrated Freight Transportation Planning Problems

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Joint work with Amy Cohn



Outline

- Planning process for small package carriers
- Load matching and routing problem
 - Traditional multi-commodity flow approach
 - Alternative composite variable approach
- Integrated planning model
- Computational results
- Conclusions and future research directions
- Questions



Planning Process for Small Package Carriers

- Load planning or package routing
- Trailer assignment
- Load matching and routing
- Equipment balancing
- Driver scheduling

Planning Process for Small Package Carriers

*Load planning or
package routing*

Trailer assignment

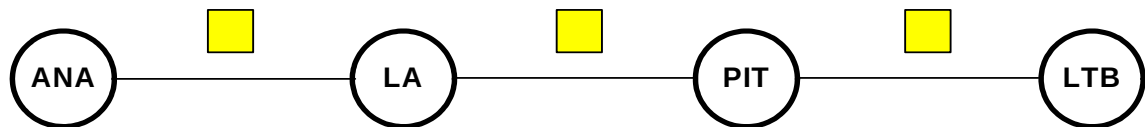
Load matching and
routing

Equipment balancing

Driver scheduling

Load planning or package routing

- Determine routing or path for each package
- Service commitments and sort capacities must not be violated



Planning Process for Small Package Carriers

Trailer assignment

- Assign routed packages to trailer type to form *loads*

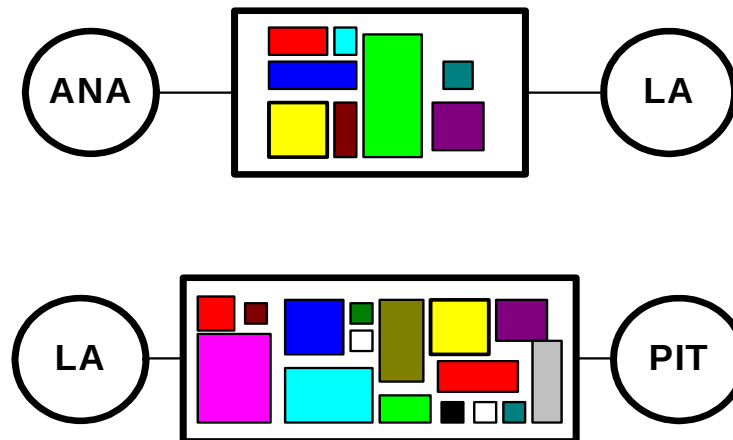
Load planning or
package routing

Trailer assignment

Load matching and
routing

Equipment balancing

Driver scheduling





Planning Process for Small Package Carriers

Load matching and routing

Load planning or
package routing

- Match loads together to leverage cost efficiencies

Trailer assignment

Load matching and routing

Equipment balancing

Driver scheduling



Planning Process for Small Package Carriers

Load planning or
package routing

Trailer assignment

Load matching and
routing

Equipment balancing

Driver scheduling

Equipment balancing

- Delivering loads from origin to destination causes some areas of the network to accumulate trailers and others to run out
- Redistribute trailers so that no such imbalances occur



Planning Process for Small Package Carriers

Driver scheduling

Load planning or
package routing

Trailer assignment

Load matching and
routing

Equipment balancing

- Take output of load matching and equipment balancing problems and assign drivers to each tractor movement

Driver scheduling



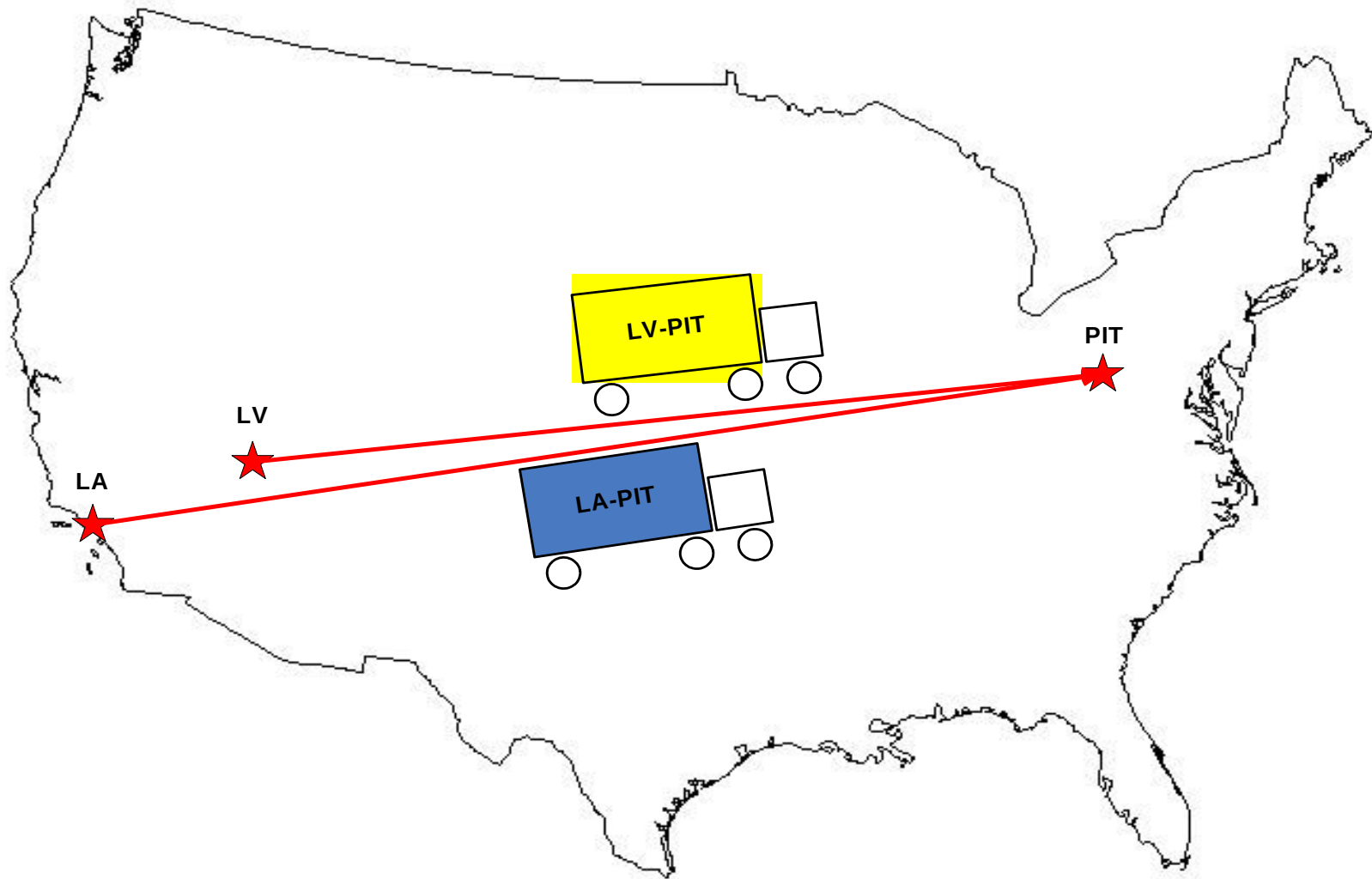
Load Matching and Routing Problem

- Non-linear cost structure: *single trailer combination* vs. *double trailer combination*
 - May incur circuitous mileage to move load as part of double combination
- Moves must be time feasible
- Example – 2 loads must be moved

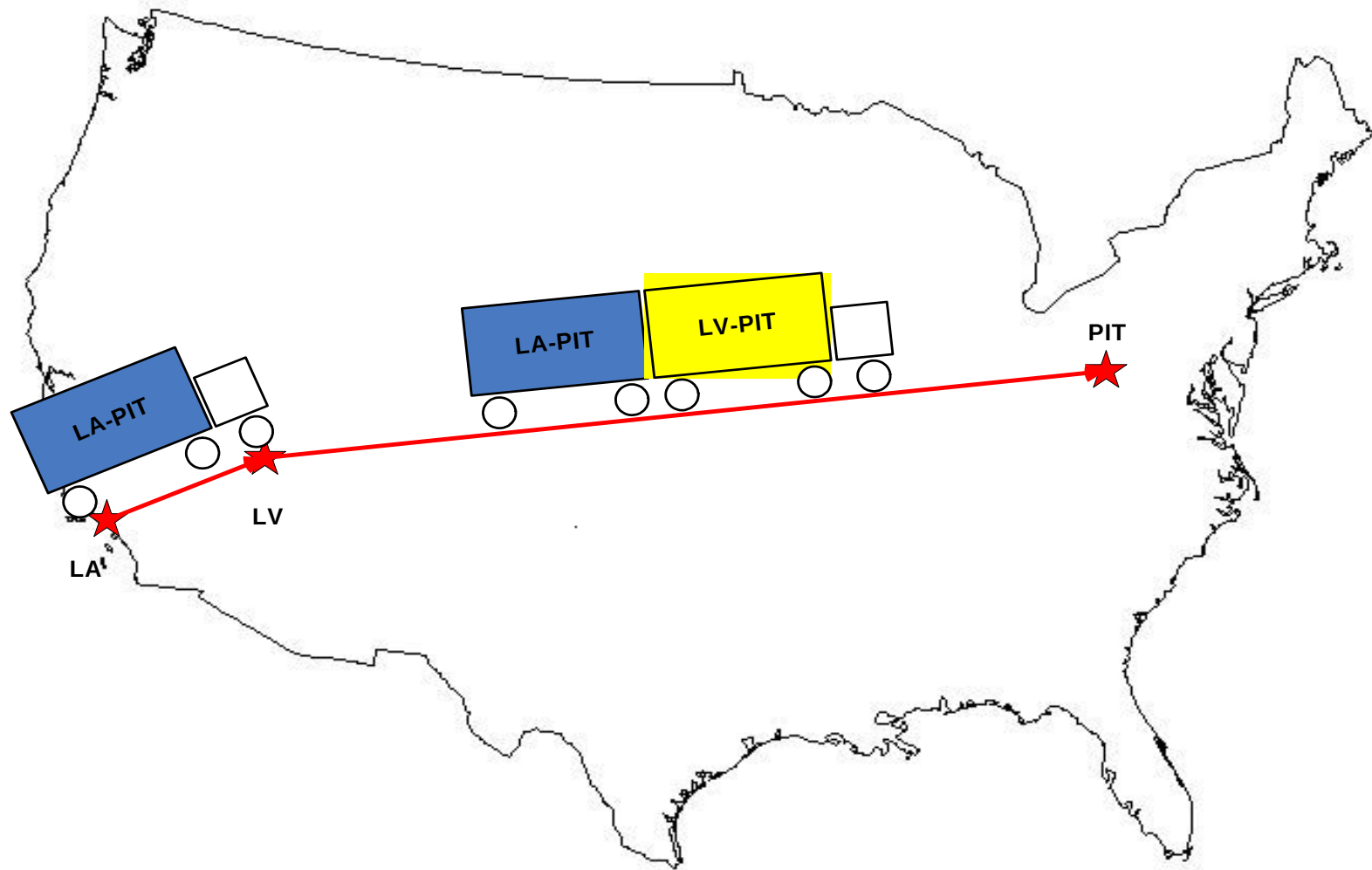
LA-PIT

LV-PIT

Load Matching and Routing Problem



Load Matching and Routing Problem





Multi-commodity Flow Based Model

- ***Commodity*** is an origin, destination, time window combination
- ***Time-space network***: each node represents a facility at a time

Variables

- x_{ijk} – number of commodity k flowing on arc (i,j)
- s_{ij} – number of single combinations flowing on arc (i,j)
- d_{ij} – number of double combinations flowing on arc (i,j)

Multi-commodity Flow Based Model

$$\min \sum_{(i,j) \in A} c_{ij}^s s_{ij} + \sum_{(i,j) \in A} c_{ij}^d d_{ij}$$

$$\text{s.t.} \quad \sum_{i:(i,j) \in A} x_{jik} - \sum_{j:(i,j) \in A} x_{jik} = \begin{cases} -d_{ij} & \text{if } i \in V, k \in K \\ s_{ij} & \text{if } i \in V, k \in K \end{cases}$$

$$s_{ij} + 2d_{ij} = 2 \sum_{k \in K} x_{jik}$$

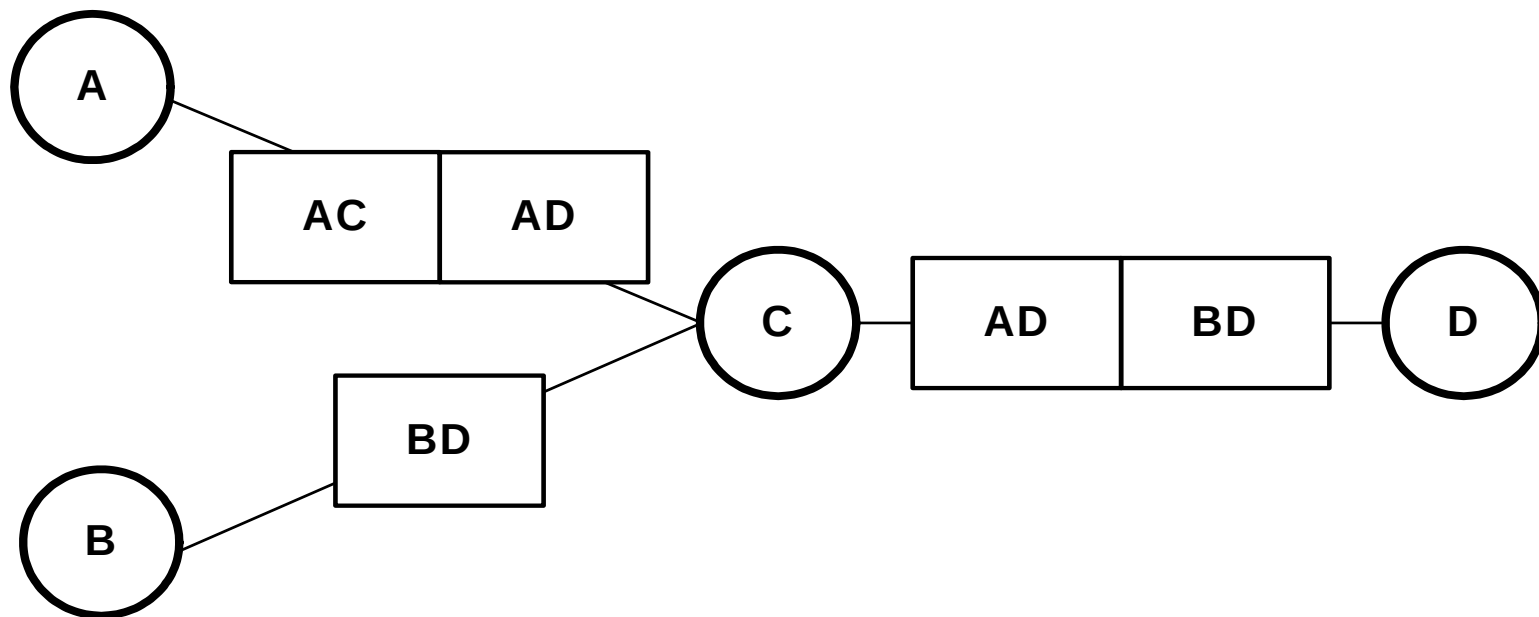
**ALWAYS
CHEAPER TO
SEND TRAILERS
AS ½ DOUBLE
RATHER THAN A
SINGLE!**



Composite Variable Modeling Approach

- ***Composite variables*** embed complexity implicitly within the variable definition
- Instead of considering the movement of trailers along each arc, consider groups of trailers which move together
- A ***cluster*** is a set of loads, the routes they take, and the tractor configurations that pull them
 - Every load in the cluster moves completely from origin to destination
 - All loads in the cluster interact in some way
 - Only define clusters which are feasible

Composite Variable Modeling Approach



Composite Variable Modeling Approach

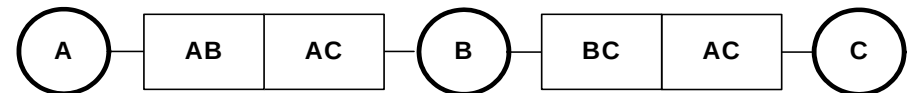
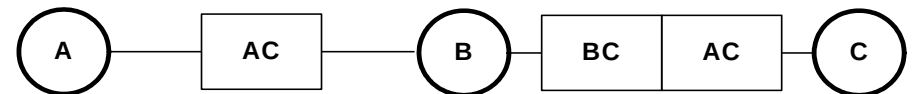
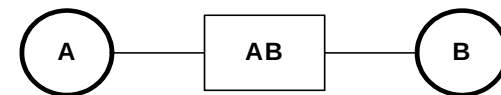
- Create clusters using pre-defined templates

- Limits the number of variables

- No math program required to generate potential clusters

- Can leverage user expertise to create templates

- Difficult to capture characteristics can be incorporated





Composite Variable Modeling Approach

Parameters:

- c_c – cost cluster c
- δ_c^k – number of commodity k moved in cluster c
- b_k – number of commodity k to be moved through the network

Variables

- x_c – number of cluster c used in the solution

$$\begin{array}{ll}\min & \sum_c c_c x_c \\ \text{s.t.} & \sum_c \delta_c^k x_c = b_k \quad \forall k \text{ in } K \\ & x_c \text{ in } Z^+\end{array}$$



Composite Variable Modeling Approach

- Promising computational results
 - Real-world instance with 2500 loads and 2500 links
 - Converges to an integer solution within seconds
 - Within minutes, very small optimality gap
 - Demonstrated improvement relative to solution used in practice

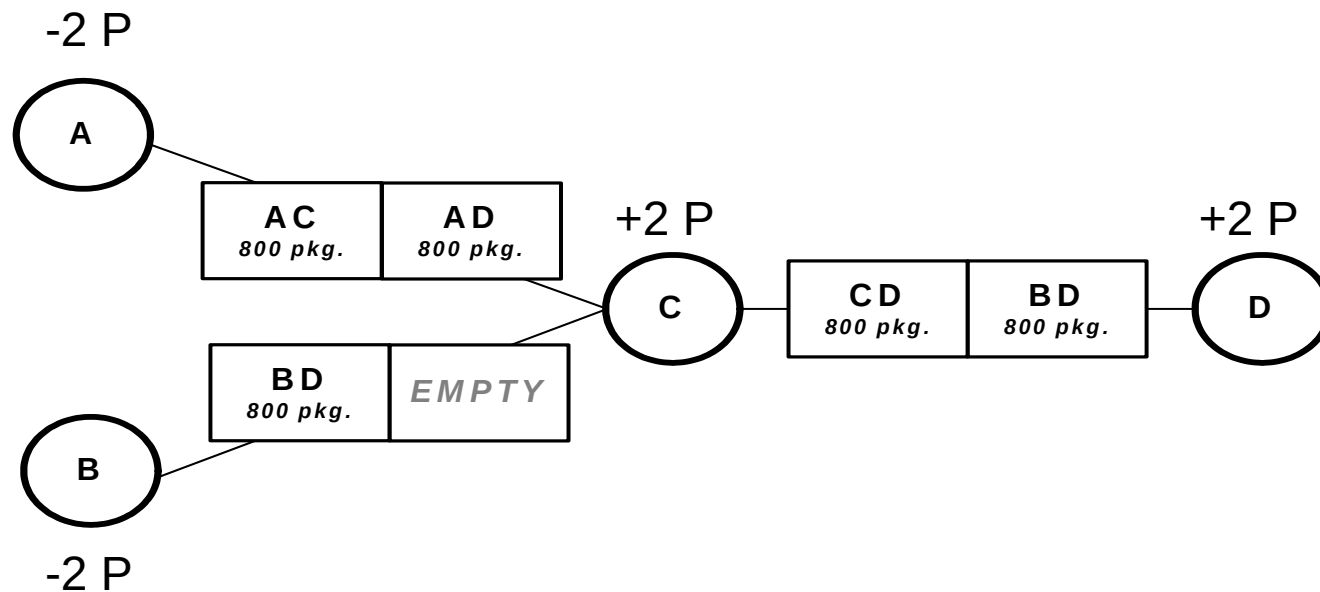


Planning Process for Small Package Carriers

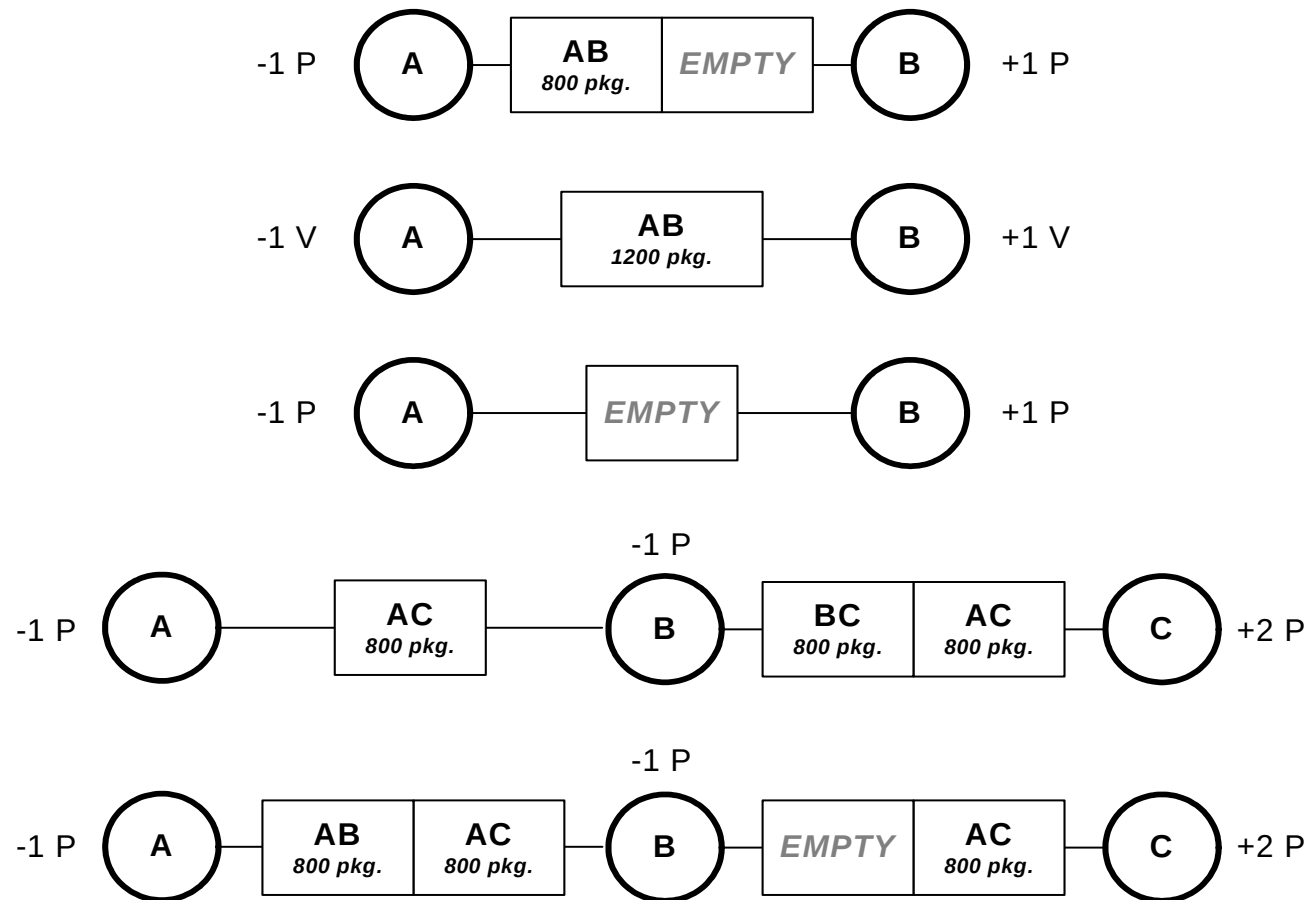
- Load planning or package routing
- Trailer assignment
- Load matching and routing
- Equipment balancing
- Driver scheduling

Integrated Planning Problem

- Slightly redefine variables to be is a set of *volumes*, *empty trailers*, the routes they take, and the tractor configurations that pull them



Integrated Planning Problem





Composite Variable Modeling Approach

Parameters:

- c_c – cost cluster c
- v_c^k – volume of commodity k moved in cluster c
- b_k – total volume of commodity k to be moved through the network
- m_c^{tf} – impact of cluster c on balance of trailer type t at facility f

Variables

- x_c – number of cluster c to be moved through the network

$$\begin{aligned} \min \quad & \sum_c c_c x_c \\ \text{s.t.} \quad & \sum_c v_c^k x_c \geq b_k && \forall k \text{ in } K \\ & \sum_c m_c^{tf} x_c = 0 && \forall t \text{ in } T, f \text{ in } F \\ & x_c \text{ in } \mathbb{Z}^+ \end{aligned}$$



Computational Results

- Composite variable approach vs. MCF approach
- *Instance 1* – 1,257 loads; 1,296 links; 263 facilities

	CV	MCF
# variables	16,606	4.8 million
# constraints	2,297	1.9 million
Time	32 sec.	<i>(intractable)</i>



Computational Results

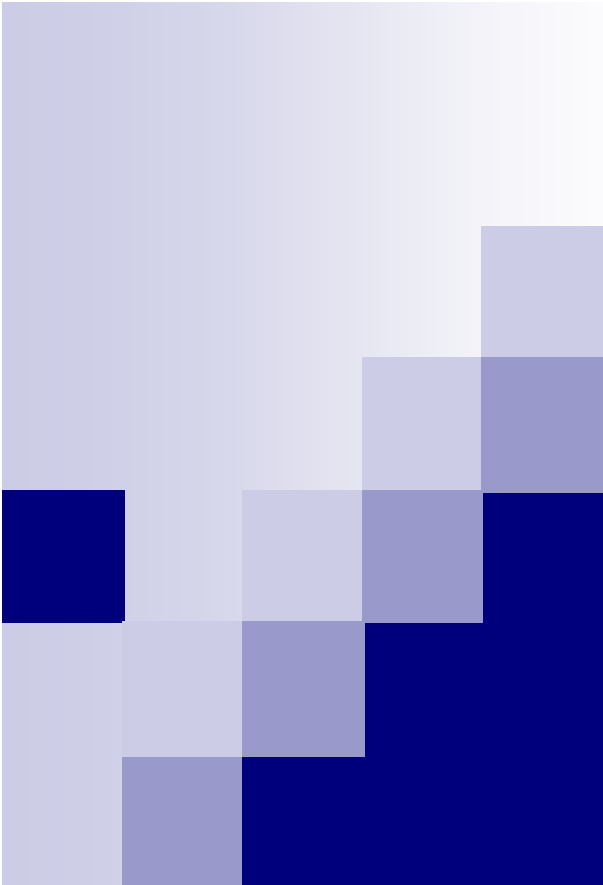
- Scalability of composite variable modeling approach

	<i>Instance 1</i>	<i>Instance 2</i>	<i>Instance 3</i>
Instance size	1257 loads 1296 links 263 facilities	2426 loads 2492 links 352 facilities	6394 loads 6596 links 568 facilities
# variables	16,606	38,127	172,444
# constraints	2,297	3,970	11,254
Time	32 sec.	144 sec.	1 hr. 14 min.



Conclusions and Future Research Directions

- Initial computational results for integrated planning problem promising
- Benefits offered by composite variable models
 - Linearize cost structure
 - Strengthen LP relaxation
 - Implicitly capture real-world detail and difficult constraints
 - Can address problems of large scope
- Future research directions
 - Further expansion of problem scope
 - Understand how applicable this is to other LTL problems to possibly generalize approach



Questions?

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