

Assessing Delay Propagation in Airline Plans

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Outline

- Motivation
- Analysis Approach
 - Propagation Tree Analysis
- Optimization Model
 - Minimize Delay Propagation
- Conclusion and Future Research

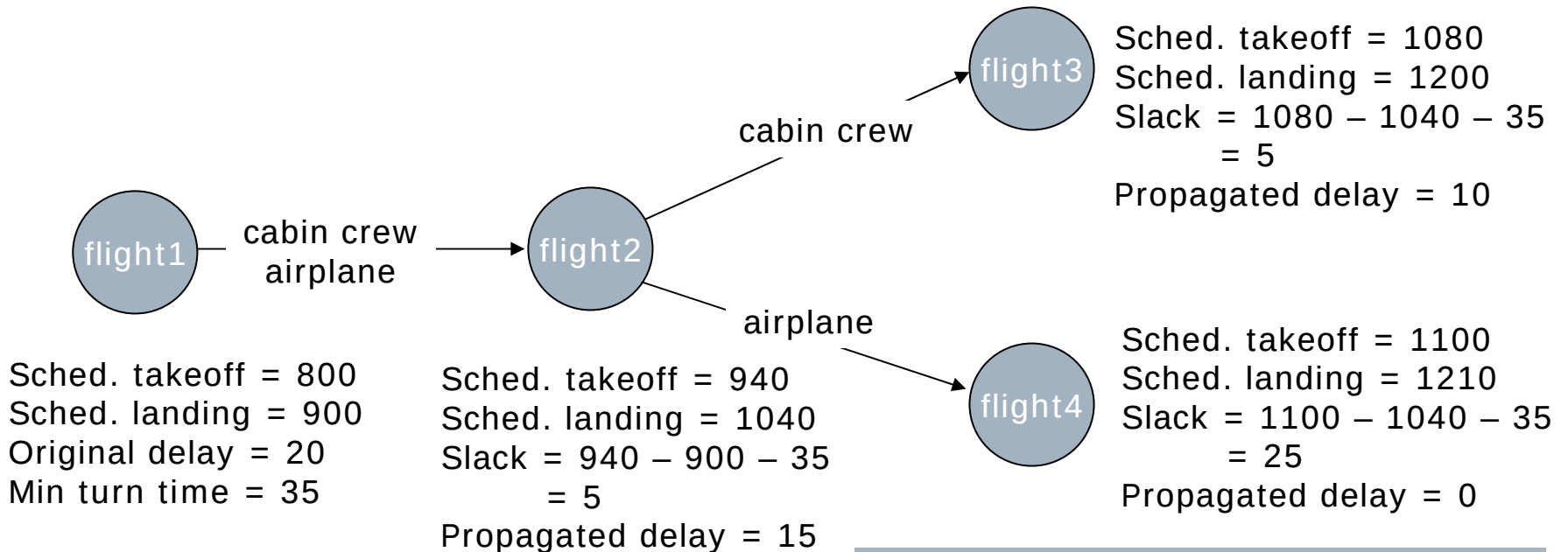
Motivation

- ❑ Airline scheduling is complex due to lots of interdependent expensive resources
- ❑ To fully utilize resources such as crews and aircraft, airlines develop schedules with minimal slack
- ❑ Plans that are efficient on paper may not be robust in practice
- ❑ Delays can propagate downstream from one flight to another, assuming there is limited buffer between the flights

Propagation Tree: Example

Nodes: flights

Arcs: connections due to transfer of resources



original delay = 20 min
Propagated delay = 25 min

Challenges of Robustness

- Stochastic nature:
 - Incorporating stochasticity makes challenging optimization problems even more difficult

- Large-scale nature:
 - Recovery models involve simultaneous consideration of multiple resources e.g. airplanes and crew

- Complicated trade-offs:
 - How much planned cost to incur in return for potential improvements in performance?

Literature Survey

☐ **Study of the delays**

- Bratu and Barnhart (2002)

☐ **Robust schedule generation**

- Ageeva and Clarke (2000)
- **Lan et al. (2003)**

☐ **Robust fleet assignment**

- Rosenberger et al (2001)
- Kang and Clarke (2003)
- Lan et al. (2003)

☐ **Robust crew scheduling**

- Chebalov and Klabjan (2002)
- Schaefer et al (2001)
- Yen and Birge (2000)

Different approaches

- Time windows
- Stochastic programming
- Simulation
- Recovery models

Our Goal

☐ **Analysis:**

- Better understand delay propagation
 - ☐ It has long been suspected that optimal plans are not optimal in performance
 - ☐ Our goal is to analyze this hypothesis quantitatively

☐ **Optimization:**

- Use the better understanding to better planning
 - ☐ Use analysis results to develop planning tools that better incorporate robustness

Analysis Metrics

□ Propagation magnitude

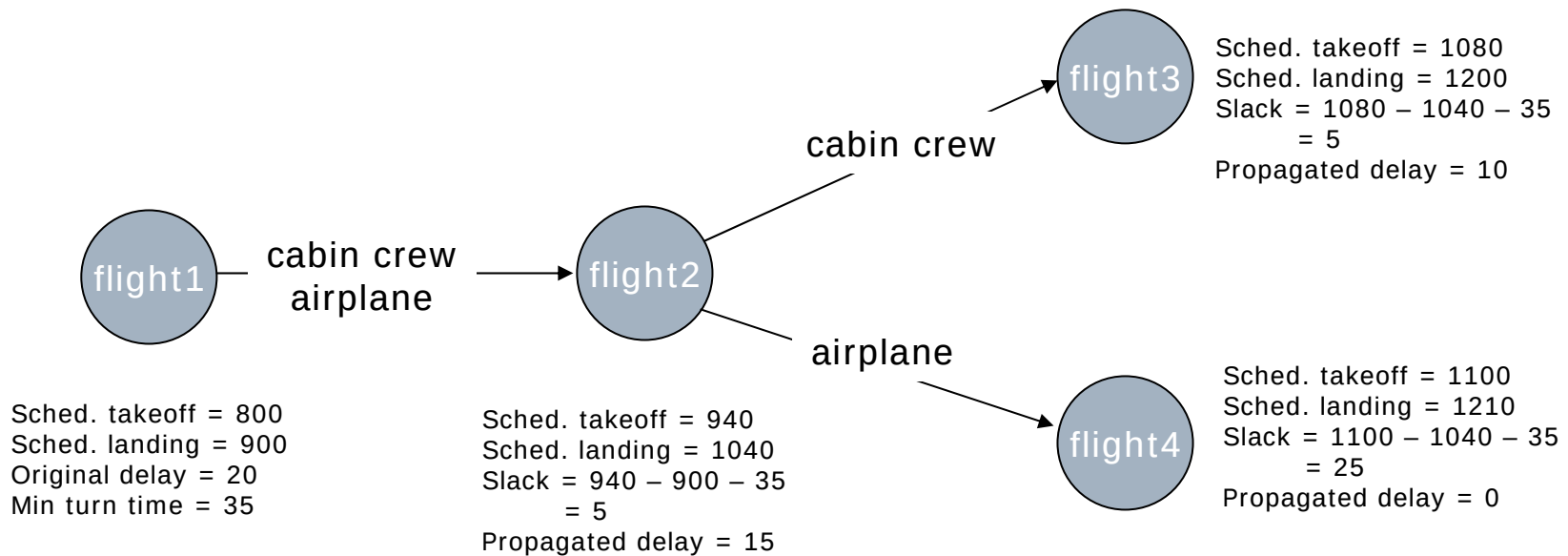
$$\text{propagation magnitude} = \frac{\text{total propagated delays}}{\text{original delay}}$$

□ Propagation severity

- Total number of disrupted flights

□ Note: both metrics are functions of the original delay value

Example: Revisited



- Magnitude (flight1,20) = $25/20 = 1.25$
- Severity (flight1,20) = 2

original delay = 20 min
Propagated delay = 25 min

Analysis Procedure

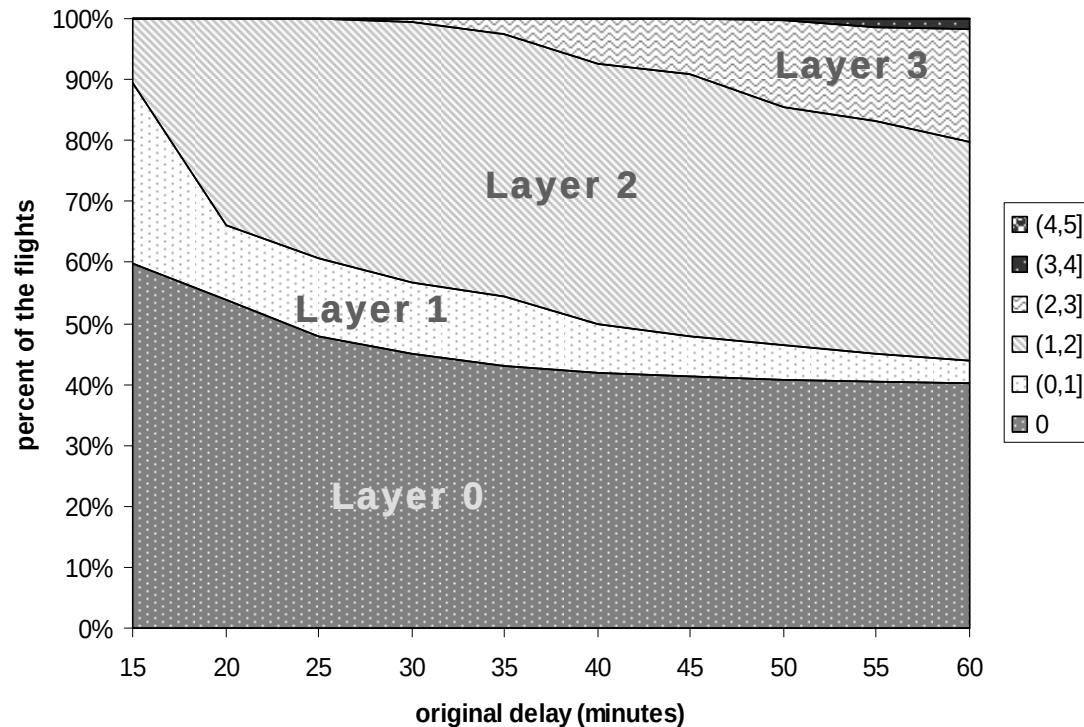
- For each flight and each value of the initial delay (15, 20, ... 60) minutes
 - Construct the propagation tree
 - keep track of the analysis metrics

- Part I – Legacy carrier
 - More than 500 domestic flights daily

- Part II – Low fare carrier
 - Less than 500 domestic flights daily

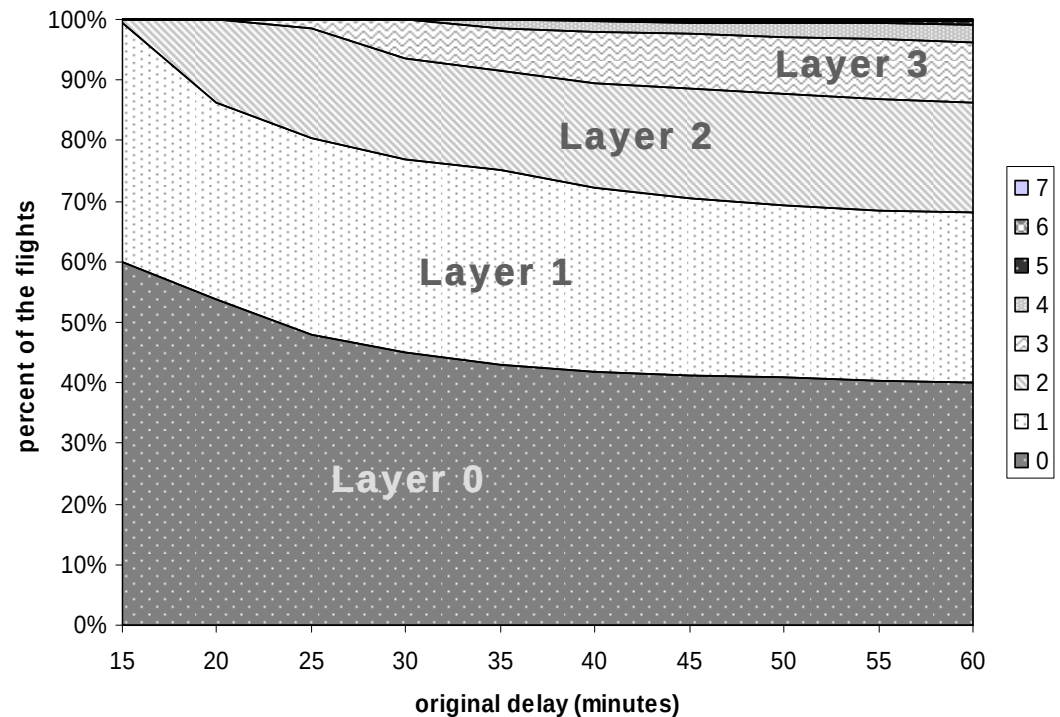
Preliminary Results – Carrier 1

- Propagation magnitude
 - In the case of a 60 minute original delay, about 40% of the flights show no propagation



Preliminary Results – Carrier 1

- Propagation severity
 - A 60 minute delay can potentially disrupt up to 7 more flights in the flight network



Preliminary Results – Carrier 1

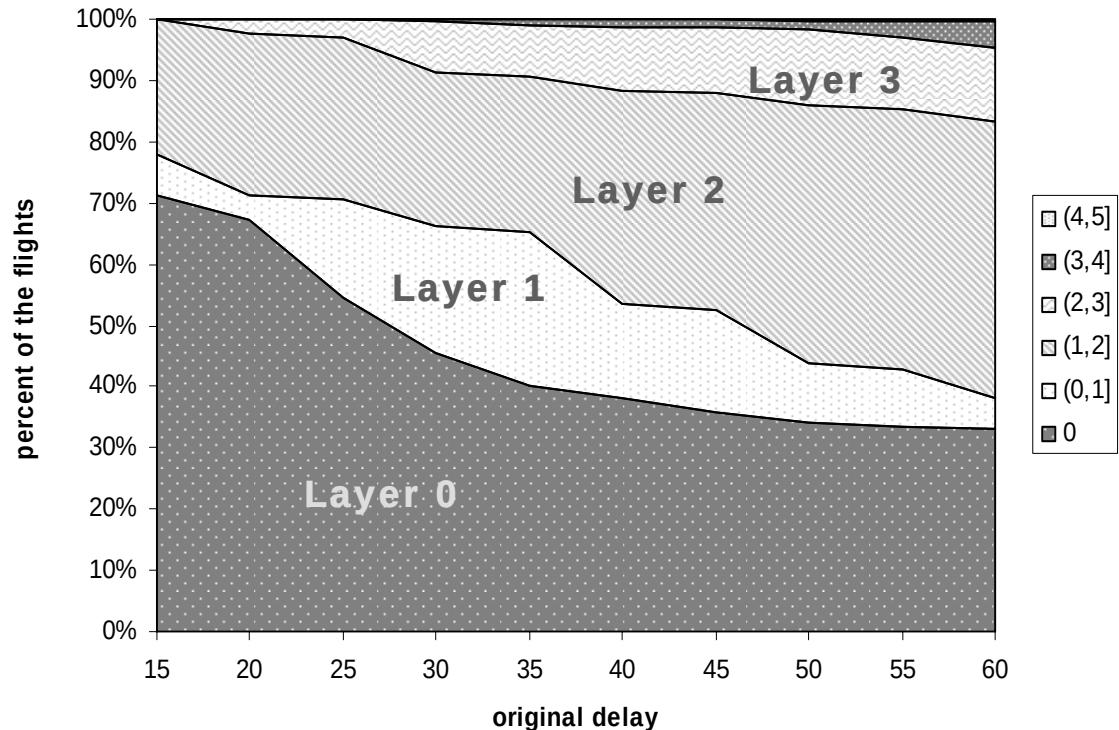
Initial delay = 60 mins

- Sorted by origin or destination airport

	Origin Airport	Total Delay		Delay Per Flt.	Dev.
Top 5	AAA	505		100.6	-9.6%
	BBB	240		100.6	-9.6%
	CCC	74		87.8	-21.1%
	DDD	72		87.3	-21.5%
	EEE	70		101.1	-9.1%
Bottom 5	VVV	2		110.0	-1.1%
	WWW	2		109.0	-2.0%
	XXX	2		79.0	-29.0%
	YYY	1		110.0	-1.1%
	ZZZ	1		94.0	-15.5%
	Average			111.26	

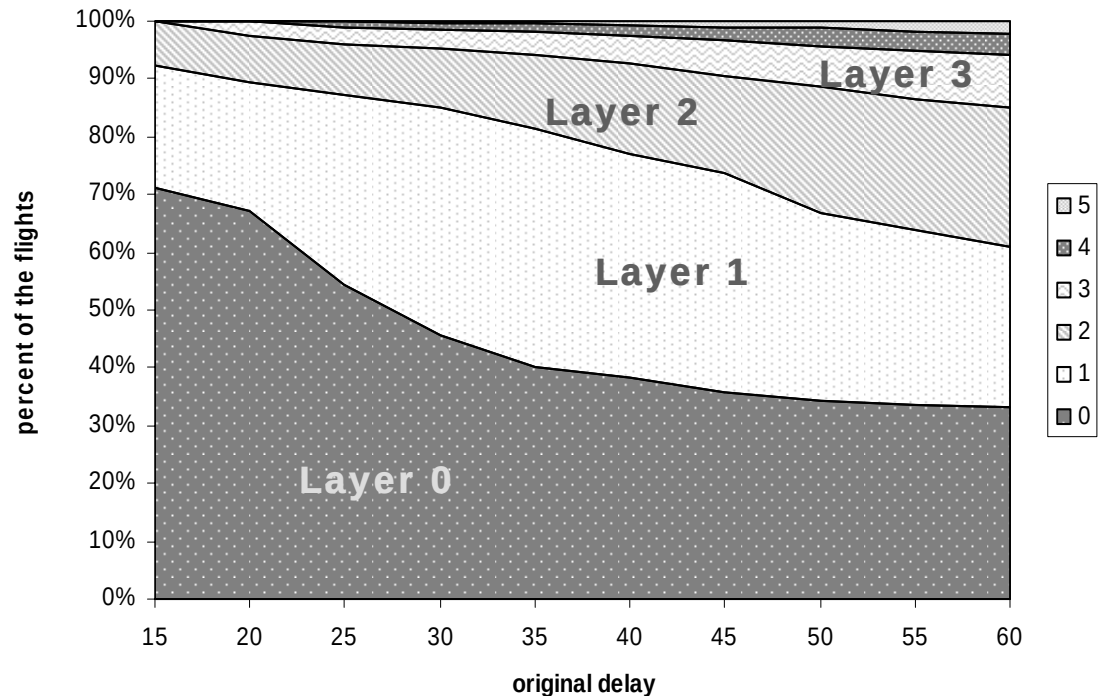
Preliminary Results – Carrier 2

- Propagation magnitude
 - Here we can observe that even in the case of a 60 minute original delay, about 60% of the flights show no propagation



Preliminary Results – Carrier 2

- Propagation severity
 - A 60 minute delay can potentially disrupt up to 3 more flights in the flight network



Preliminary Results – Carrier 2

Initial delay = 60 mins

- ❑ Sorted by origin or destination airport

	Origin Airport	Total Delay		Delay Per Flt.	Dev.
Top 5	aaa	151		70.63	4.9%
	bbb	48		72.63	5.3%
	ccc	28		65.00	-3.5%
	ddd	28		72.17	7.2%
	eee	24		66.59	-1.1%
Bottom 5	vvv	2		60.00	-10.9%
	www	1		60.00	-10.9%
	xxx	1		60.00	-10.9%
	yyy	1		60.00	-10.9%
	zzz	1		60.00	-10.9%
	Average			67.34	

Improvements

- Probability of initial delay
 - Some flights are more likely to be delayed, hence we can weigh the propagation effects by the probability of initial delay
- Correlations
 - Delays are sometimes correlated
- Passenger itinerary, cabin crew connections
 - These can be easily added to the analysis
- Recovery considerations
 - Based on the recovery strategies, the propagation effects may be different

Optimization Model

- ❑ The idea is to change the flight times marginally in order to better utilize the slack
 - This change:
 - ❑ Does not impose any cost
 - ❑ Does not impact crew assignment/fleeting/routing
 - ❑ Does not capture all the opportunities to improve robustness
- A starting point that does not require explicit assignment of costs or values to delay

Optimization Model

- The goal:
 - Minimize the expected delay propagation
 - We only deal with one level of propagation to efficiently target the greatest effects

- Constraints:
 - Change the departure times of the flights within a given time window
 - Keep the original connections still feasible

- The idea of time windows:
 - Levin (1971), Rexing et al (2000), **Lan et al** (2003)

Linear Programming Formulation

□ Sets:

- Set of flights (nodes) $\mathcal{F}$
- Set of connections (arcs) $\mathcal{A}$
 - Union of four other sets: cockpit crew ($\mathcal{P}$), cabin crew ($\mathcal{C}$), equipment ($\mathcal{E}$) and itinerary ($\mathcal{I}$) connections
 - $\mathcal{A} = \mathcal{P} \cup \mathcal{C} \cup \mathcal{E} \cup \mathcal{I}$
- Set of possible delay values (minutes) $\mathcal{M}$

□ Parameters:

- Probability of the original delay $0 \leq p_f^m \leq 1 \quad \forall f \in \mathcal{F}, \forall m \in \mathcal{M}$
- The original slack on each arc $s_{f_1, f_2} \geq 0 \quad \forall (f_1, f_2) \in \mathcal{A}$

Linear Programming Formulation

□ Decision variables:

□ The change in the departure times

$$k_f^- \leq x_f \leq k_f^+ \quad \forall f \in \mathcal{F}$$

■ $0 \leq x_f^+ \leq k_f^+ \quad \forall f \in \mathcal{F}$

■ $0 \leq x_f^- \leq k_f^- \quad \forall f \in \mathcal{F}$

□ The new slack on an arc

$$y_{f_1, f_2} \geq 0$$

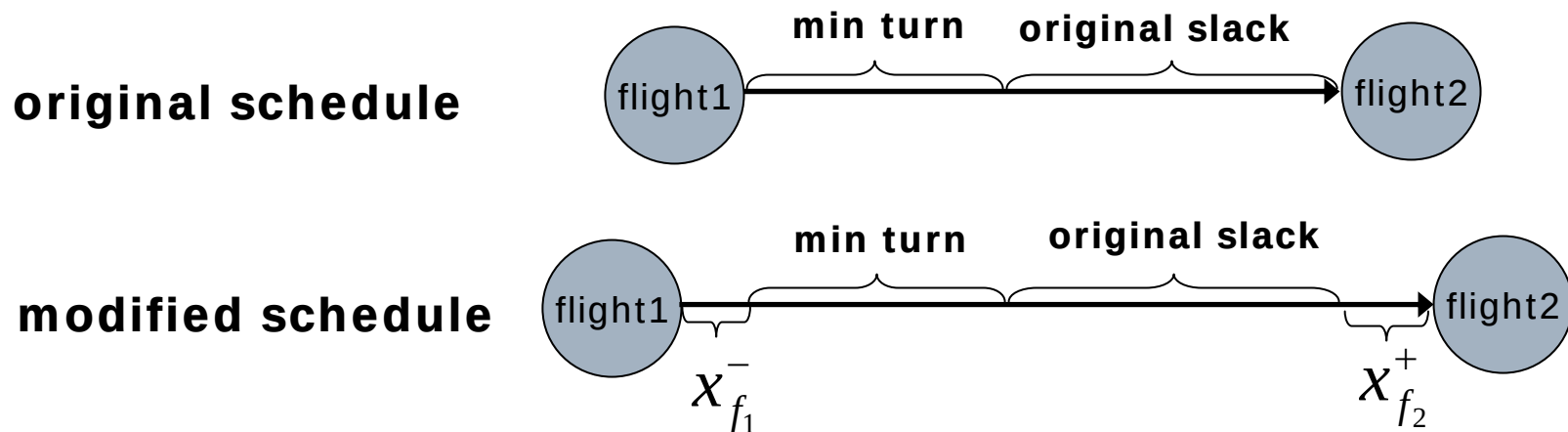
□ Delay propagated from f_1 to f_2
when f_1 is delayed by m minutes

$$d_{f_1, f_2}^m \geq 0$$

Linear Programming Formulation

□ Visualizing the constraint

$$y_{f_1, f_2} = s_{f_1, f_2} + x_{f_1}^- + x_{f_2}^+ - x_{f_1}^+ - x_{f_2}^- \quad \forall (f_1, f_2) \in \mathcal{A}$$



$$\text{new slack} = \text{original slack} + x_{f_1}^+ + x_{f_2}^-$$

Linear Programming Formulation

- Minimize the expected value of delay propagation while keeping the connections feasible

$$\text{Min} \quad \sum_{m \in \mathcal{M}} \sum_{(f_1, f_2) \in \mathcal{F}} p_{f_1}^m d_{f_1, f_2}^m$$

$$y_{f_1, f_2} = s_{f_1, f_2} + x_{f_1}^- + x_{f_2}^+ - x_{f_1}^+ - x_{f_2}^- \quad \forall (f_1, f_2) \in \mathcal{A}$$

$$d_{f_1, f_2}^m \geq m - y_{f_1, f_2} \quad \forall (f_1, f_2) \in \mathcal{A}, \quad \forall m \in \mathcal{M}$$

$$0 \leq x_f^+ \leq k_f^+, \quad 0 \leq x_f^- \leq k_f^- \quad \forall f \in \mathcal{F}$$

$$y_{f_1, f_2} \geq 0 \quad \forall (f_1, f_2) \in \mathcal{F}$$

$$d_{f_1, f_2}^m \geq 0 \quad \forall (f_1, f_2) \in \mathcal{F} \quad \forall m \in \mathcal{M}$$

Implementation

- Implemented the model using CPLEX10.0/C++
 - Sun Fire 280R
 - 1.2 GHz processor
 - Real memory: 8 GB
 - Swap memory: 1 GB

- Assumptions
 - Delays follow a discrete uniform distribution between 0,60 minutes
 - Equal time windows $k_f^+ = k_f^- = 15 \quad \forall f \in \mathcal{F}$

Computational Results

❑ Carrier 1

- Expected propagation drops from 22,929 to 14,088 (38% decrease)

❑ Carrier 2

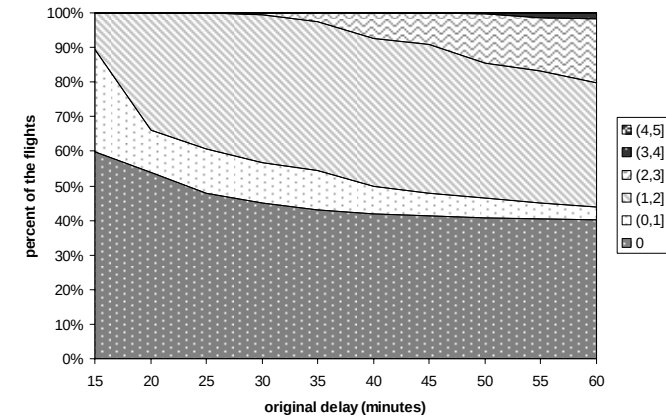
- Expected propagation drops from 5,028 to 3,255 (35% decrease)

❑ Running time for both data sets

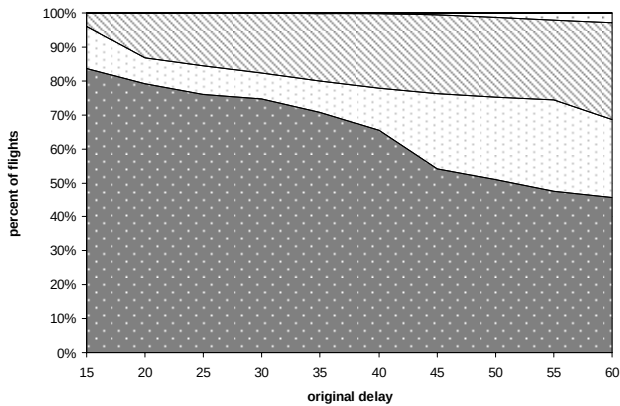
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Results – Carrier 1

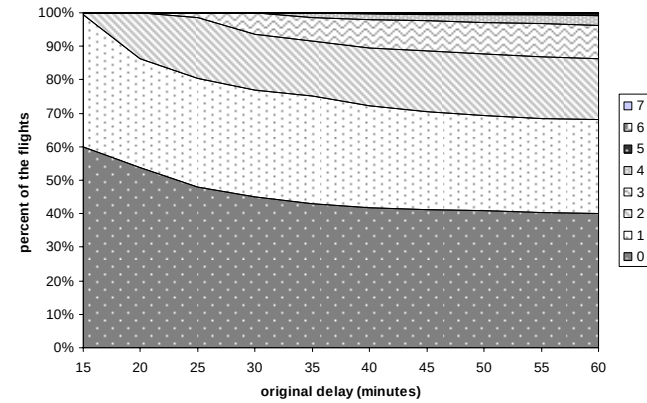
magnitude



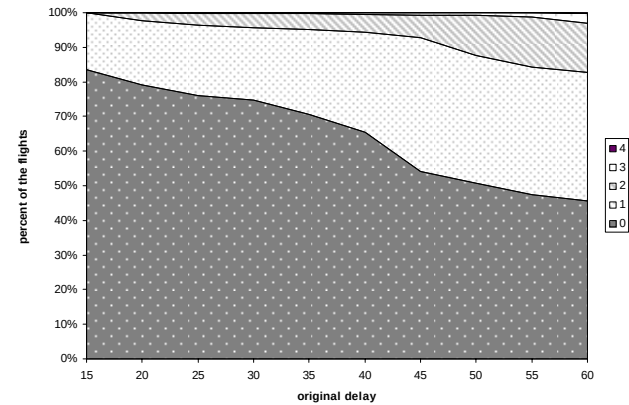
before



after



severity



Conclusion

☐ Analysis:

- In many cases, delay **doesn't** propagate
- But when it does, it can **significantly** increase system delay
- Most propagation seems to be **broad** rather than deep

☐ Optimization:

- Introducing slack in the **correct** places
 - ☐ Can **diminish** the propagation
 - ☐ Does **not** impose significant cost

Future Research

- ☐ Realistic probability distribution for the original delays
 - Correlations
- ☐ Other objective functions
 - Minimize maximum propagation instead of the expected value
 - ☐ Smaller model
 - ☐ Does not rely on the probability distribution of the original delays
 - Minimize total number of disrupted passengers
 - ☐ More sophisticated model (IP)

Questions

