

Improving Crew Scheduling By Incorporating Key Maintenance Routing Decisions

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Motivation

- Crew costs are the second largest operating expense faced by airlines
- Small improvements in efficiency can have significant financial impact
- Scheduling options are limited by maintenance routing decisions made earlier in the airline planning process
- Solving maintenance routing and crew scheduling simultaneously yields a large and challenging problem

Research Objective

- Our goal is to improve crew scheduling by incorporating relevant maintenance routing decisions only
- Exploit the fact that only a subset of the maintenance routing decisions impact crew scheduling to decrease problem size

Structure of Presentation

- Describe the crew scheduling and maintenance routing problems and demonstrate the relationship between them
- Present the *extended crew pairing model (ECP)* and discuss structural properties that we can exploit to enhance tractability
- Present an alternative model that can work in tandem with ECP to improve performance

Crew Scheduling

- Schedules are built by generating pairings and then assigning them to crews
- A *crew pairing* is a sequence of flights that can be flown by a single crew:
 - beginning and ending at the crew's home airport
 - sequential in space and time
 - satisfying government regulations and labor agreements such as:
 - maximum flying time in a day
 - minimum rest requirements
 - minimum connection time between sequential flights

Crew Pairing (CP)

- Crew pairing problem: Find a minimum cost set of feasible pairings such that each flight is included in exactly one pairing
- Set partitioning problem with an exponential number of binary decision variables:

$$\min \sum_p c_p y_p$$

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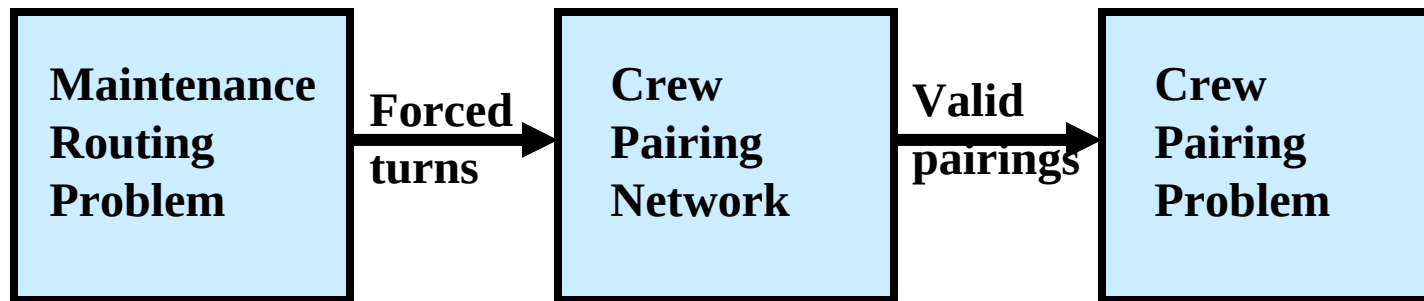
$$\sum_p \delta_{fp} y_p = 1 \quad \forall f$$

$$y_p \in \{0,1\} \quad \forall p$$

MR and Its Impact on CP

- Maintenance routing problem (MR) assigns aircraft to flights to ensure adequate maintenance opportunities
- Crews need enough time between two sequential flights to travel through the terminal -- *minimum connect time*
- If both flights are covered by the same aircraft, this connection time can be reduced -- tighter connections can be permitted
- A *forced turn* is a connection that is crew-feasible only if both flights are covered by the same aircraft

Sequential Approach



An Example

- Flights:
A B C D E F G H
- Forced turns:
A-B A-D D-G
- MR solution (x_1) uses forced turns A-D and D-G
- MR solution (x_2) uses forced turn A-B
- Potential pairings:
 - E-F-G-H (y_1) - \$1
 - B-C-E-F (y_2) - \$2
 - A-D-G-H (y_3) - \$5
 - A-B-C-D (y_4) - \$4
- Crew pairing solutions:
 - $x_1 \Rightarrow$ pairings 2, 3 -- \$7
 - $x_2 \Rightarrow$ pairings 1, 4 -- \$5

Klabjan, Johnson, and Nemhauser

- Solve a variation of the crew pairing problem first, including all forced turns in the crew pairing network
- Given the crew solution, require all forced turns included in it to be part of the maintenance solution, which is solved second
- For “good” instances, this yields the optimal solution to the integrated problem (and many instances are “good”)
- For “bad” instances, this leads to maintenance infeasibility

Cordeau, Stojković, Soumis, and Desrosiers

- Directly integrate string-based models
- Basic maintenance routing and crew pairing variables and constraints, plus linking constraints
- Benders decomposition approach using a heuristic branching strategy
- Promising computational results

Our Objectives

- Guarantee maintenance feasibility
- Allow the user the flexibility to trade off between solution time and quality
- Leverage the fact that only a fraction of the maintenance routing decisions are relevant to the crew pairing problem

Approach

- In the sequential approach, the crew scheduler is given an MR solution and solves the corresponding CP
- We'd like to allow the crew scheduler to choose from a collection of MR solutions the one which contains the most useful set of forced turns
- Problem: We don't want to solve one CP for each MR solution
- Solution: Extended crew pairing model (ECP)

The Extended Crew Pairing Model (ECP)

- In addition to choosing crew pairings, select one maintenance routing solution from a given set of feasible solutions
- Add constraints that prohibit pairings containing a forced turn from being selected unless the chosen maintenance solution also contains that forced turn

Matrix Representation

	x_1	x_2	y_1	y_2	y_3	y_4		rhs	
Flights:	0	0	0	0	1	1	=	1	A
	0	0	0	1	0	1	=	1	B
	0	0	0	1	0	1	=	1	C
	0	0	0	0	1	1	=	1	D
	0	0	1	1	0	0	=	1	E
	0	0	1	1	0	0	=	1	F
	0	0	1	0	1	0	=	1	G
	0	0	1	0	1	0	=	1	H
Forced turns:	0	1	0	0	0	-1	$\geq$	0	A-B
	1	0	0	0	-1	0	$\geq$	0	A-D
	1	0	0	0	-1	0	$\geq$	0	D-G
Convexity:	1	1	0	0	0	0	=	1	Conv.

General Formulation

$$\min \sum_p c_p y_p$$

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Flights: $\sum_p \delta_{fp} y_p = 1 \quad \forall f$

Forced turns: $\sum_s \alpha_{ts} x_s - \sum_p \beta_{tp} y_p \geq 0 \quad \forall t$

Convexity: $\sum_s x_s = 1$

$$x_s, y_p \in \{0,1\} \quad \forall s, p$$

Does ECP Meet Our Goals?

- Maintenance feasibility: ✓
- User flexibility: ✓
 - One column for the original MR solution -- sequential approach
 - One column for each MR solution -- optimal approach
 - Any number of columns in between -- a priori, via column generation, or both
- Leveraging key MR decisions: ✓
 - Only forced turn information appears in ECP

ECP Enhancements

- Dramatically reduce the *number* of MR columns:
 - *Uniqueness*: Eliminate redundant columns
 - Example: 41 flights, single set of forced turns $\Rightarrow \gg 8,700$ solutions $\Rightarrow$ 1 column
 - *Maximal independence*: Eliminate dominated columns
- Only need one column per *unique, maximally independent, maintenance feasible* forced turn set
- Theoretical bounds and computational observations
 - Example: 61 flights $\Rightarrow \gg 25,000$ solutions $\Rightarrow$ 4 required columns

ECP Enhancements, cont.

- Relax the *integrality* of MR columns:
 - Same number of binary variables as original CP
- LP relaxation of ECP is tighter than LP relaxation of a basic integrated approach
 - Dantzig-Wolfe decomposition
 - Convex hull of MR LP vs. IP

Generating MR Solutions

- Algorithm to generate UMI columns -- can generate k UMI columns in at most the time needed to solve k MR problems
- Can generate columns using delayed column generation to take advantage of dual information -
 - pricing problem still yields UMI columns
- Can still use existing crew pairing generators

Computational Experiment

- **Problem A:**

Lower bound: 31,396.10

ECP with 16 MR columns: 31,396.10

Optimality gap: 0%

- **Problem B:**

Lower bound: 25,076.60

ECP with 20 MR columns: 25,498.60

Optimality gap: 1.7%

Observations

- Zero optimality gap in instance A doesn't necessarily imply sequential approach would yield an optimal solution – many equivalent crew pairing solutions, some maintenance feasible and some not
- Number of forced turns in an optimal solution is small relative to total number – UMI sets often capture many of them
 - A: 58 max; ~38 per column; 9 used in solution
 - B: 68 max; ~37 per column; 10 used in solution

Benefits of ECP

- Ensures maintenance feasibility
- Can be solved heuristically or to optimality, allowing user to trade off solution time and quality
- Leverages the fact that only forced turn decisions from the maintenance routing problem impact crew pairing

Benefits of ECP, cont.

- No more binary variables than the basic crew pairing model alone
- Tighter LP relaxation than a basic integrated approach
- Flexible
 - Can take advantage of advances in maintenance routing solvers and crew pairing generators
 - Can incorporate new maintenance constraints

Alternate Approach -- CCP

- Begin by solving unconstrained crew pairing problem (i.e. permit all forced turns)
- If solution is feasible, stop -- optimal
- If not, add a cut to rule out infeasible solution and repeat -- *constrained crew pairing problem* (CCP)

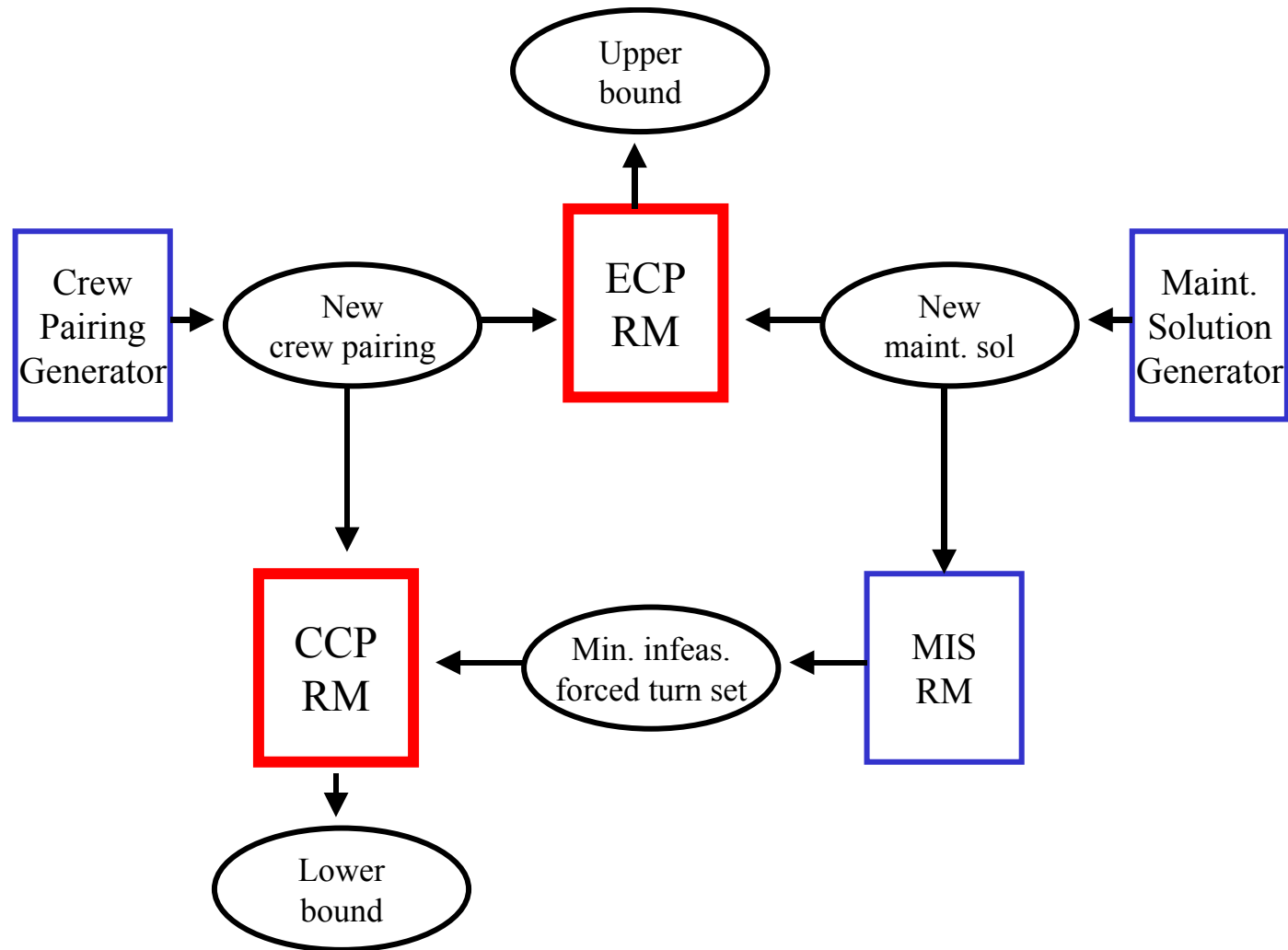
Identifying Cuts -- MIS

- Basic cut:

$$\sum_{p \in P} \pi_{Fp} y_p \leq |F| - 1$$

- Want cuts to be *minimal infeasible forced turn sets*
- Sets are found by solving an optimization sub-problem (MIS)
- MIS is solved via cut generation

Model Synergies



Conclusions

- Crew scheduling is critical to airline profitability but quality can be compromised by making maintenance routing decisions independently
- A direct integration can be inflexible and difficult to solve
- ECP provides an alternative approach that exploits the fact that only some maintenance routing information is relevant
- CCP can be used in tandem to provide tighter lower bounds while seeking optimal solutions