

Using Mathematical Programming to Make VCG Auctions Tractable: Truckload Procurement

Amy Cohn

March 8, 2007

with Damian Beil, Amitabh Sinha,
Richard Chen, and Shervin AhmadBeygi



Michigan**Engineering**

Industrial and Operations Engineering



MICHIGAN
ROSS SCHOOL OF BUSINESS



National Science Foundation
WHERE DISCOVERIES BEGIN



Overview

- VCG Auctions
- Introduction to Truckload Procurement Problem
- Mathematical Programming Approach to Bidding and Winner Determination
- Applying to Truckload Procurement
 - ◆ Operational Considerations
 - ◆ Stochastic Case
- Conclusions and Future Research
- Questions

A Brief History of Auctions

- Date as far back as 500 BC in Babylon
- Records from Roman auctions in 193 AD
- Sotheby's and Christie's art auctions from 1700's
- eBAY – baseball cards to diamonds to Pez dispensers to rare stamps to basketball tickets

Auction Characteristics

- Many types of auctions
- Our focus is on
 - ◆ Reverse (i.e. procurement)
 - ◆ Closed bid
 - ◆ Single round
 - ◆ Multi-item (combinatorial)

Combinatorial Auctions

- Many items being auctioned simultaneously
- Costs are not necessarily additive
 - ◆ Economies of scale
 - ◆ Energy auctions
 - ◆ Economies of scope
 - ◆ Telecommunications
- Importance of bidding on *bundles* of items

Vickrey Auctions: Mechanism

- Vickrey (second-price) procurement auction
 - ◆ Single-round, closed bid
 - ◆ Each bidder places a bid
 - ◆ Lowest bidder wins
 - ◆ Payment is *second lowest* bid

Vickrey Auctions: Bidding Strategy

- Optimal Vickrey bidding strategy:
- ...Bid your true cost
 - ◆ If you bid higher...
 - ◆ ...you decrease your chances of winning
 - ◆ ...you don't improve your payment if you do win
 - ◆ If you bid lower...
 - ◆ ...you might win, but lose money!

Vickrey Auctions: Benefits

- Benefits of a Vickrey auction:
 - ◆ Economically efficient allocations
 - ◆ Well-defined optimal bidding strategy

VCG Auctions: Characteristics

- VCG combinatorial mechanism
 - ◆ Bidders bid on *bundles* of items, capturing costs accurately
 - ◆ Winner determination problem
 - ◆ Payment determination problem
- Challenges
 - ◆ Combinatorial bidding
 - ◆ Combinatorial winner, payment determination problems

VCG Auctions: Literature

- Current approaches to VCG combinatorial auctions in the literature:
 - ◆ Limit the combinations on which bids can be placed (Pekeč and Rothkopf 1998, 2003, 2006)
 - ◆ Iterative (multi-round) auctions to replicate VCG results (Kwon, Anandalingam, Ungar 2005; Ausubel, Cramton, Milgrom 2006; Sandholm and Boutilier 2006)
 - ◆ Approximations and special cases (Sandholm 2002; Tennenholtz 2002)
 - ◆ Algorithms to solve winner determination across all bids (Sandholm, Suri, Gilpin, and Levine 2005; Sandholm 2002)
- None completely capture the benefits of a fully enumerated combinatorial auction – problems are too big!

Truckload Procurement: A Motivating Example

- Shippers auction off *loads* – origin/destination pairs – to trucking companies to transport for them
- A shipper might have many different loads to offer at the same time
- The costs trucking companies incur depends on the distance, the company's cost-per-mile...
- ... *but also on the other loads the trucking company is moving*

Truckload Procurement: Economies of Scope

- In bidding for a load, your cost depends on what other loads you win as well



Cost of carrying a load from A to B is:

$\$/\text{mile} * d(A,B)$

$+ \$/\text{mile} * d(B,A)$

(driver needs to be returned home!)

Truckload Procurement: Economies of Scope



Cost of carrying a load from B to A is:
 $\$/\text{mile} * d(A,B) + \$/\text{mile} * d(B,A)$

Truckload Procurement: Economies of Scope



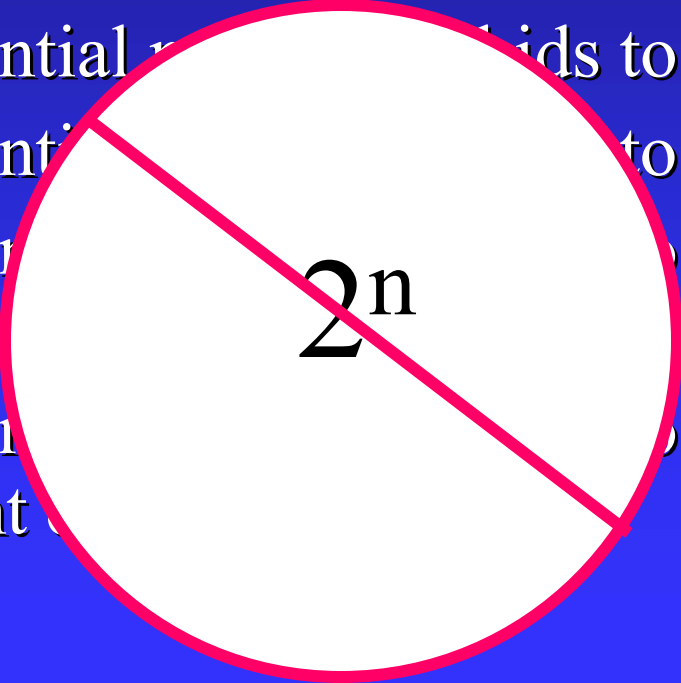
Cost of carrying a load from A to B *and* B to A is:
 $\$/\text{mile} * d(A,B) + \$/\text{mile} * d(B,A)$

Truckload Procurement: Bidding Challenges

- If you only bid on individual loads, you lose this economy of scope
 - ◆ Bid your true cost for just that load, and you may over-bid and lose to a competitor
 - ◆ Bid your estimated cost, assuming other loads to create a *continuous move*, and you may win the load but lose money (because you don't win the other loads)

Truckload Procurement: Tractability Challenges

- To capture the true cost, you should bid on continuous moves – bundles of loads pulled in sequence to minimize empty mileage
 - ◆ Exponential number of bids to compute
 - ◆ Exponential number of messages to communicate
 - ◆ Exponential number of bundles to consider in winner
 - ◆ Exponential number of bundles to consider in payment



2^n

Challenges of Combinatorial Auctions



Our research seeks ways
to overcome
this “combinatorial crisis”

Canonical Approach to VCG

- Given a set of bidders and a set of items, enumerate all possible subsets
- Each bidder provides their true cost for each subset
- Optimization problem: Find the cheapest collection of bids such that every item is in exactly one bid (winner determination)
- Optimization problem: For every bidder who is awarded at least one item, strip out this bidder's bids and re-solve the optimization problem (payment determination)

Traditional Winner Determination Problem

- Exhaustive bidding approach:
 - ◆ N – set of bidders
 - ◆ M – set of services

$$\text{Min} \quad \sum_{n \in N} \sum_{s \subseteq M} c_s^n x_s^n$$

$$\text{St} \quad \sum_{n \in N} \sum_{s \subseteq M: i \in s} x_s^n = 1 \quad \forall i \in M$$

$$x_s^n \in \{0,1\} \quad \forall s \subseteq M, n \in N$$

Our Approach: Exploiting Underlying Cost Structure

- Bidders cannot explicitly compute all 2^n bids, nor can the auctioneer process them, because of the exponential size
- But bids are not always “exogenously endowed” – they are often generated using an underlying function – can we embed this function in a mathematical program?
 - ◆ Mathematical programming is about finding the best solution without considering each solution explicitly...

Alternative Winner Determination Problem

- Bid-generating function approach:

$$\text{Min} \quad \sum_{n \in N} f^n(x^n)$$

$$\text{St} \quad \sum_{n \in N} x_m^n = 1 \quad \forall m \in M$$

$$x_m^n \in \{0,1\} \quad \forall m \in M, n \in N$$

- Same number of constraints
- $|N| * |M|$ variables instead of $|N| * 2^{|M|}!!!$
- But...what is f^n ?

Problem Description

- Given a set M of loads to be auctioned and a set of N carriers (i.e. bidders), assign the loads to bidders such that the overall system cost is minimized
- How can we compute the cost function $f^n(x^n)$?
- That is, for a given carrier n , what is the cost of carrying the set of loads defined by the vector x^n , which indicates whether load m is included in the set ($x_m^n = 1$) or not ($x_m^n = 0$)

Bid-Generating Function – Notation

■ Notation:

- ◆ c_m^n is the *direct* cost to carrier n of carrying load m
- ◆ $e_{i,j}^n$ is the *estimated* cost to carrier n of moving from the destination of load i to the origin of load j
- ◆ $y_{i,j}^n$ is a binary variable that takes value 1 if carrier n moves from load i to load j

Bid-Generating Function – Formulation

$$f^n(x^n) = c^n x^n +$$

$$\text{Min} \quad \sum_{i \in M} \sum_{j \in M} e_{i,j}^n y_{i,j}^n$$

$$\text{St} \quad \sum_{j \in M} y_{i,j}^n = x_i^n \quad \forall i \in M$$

$$\sum_{j \in M} y_{j,i}^n = x_i^n \quad \forall i \in M$$

$$y_{j,i}^n \in \{0,1\} \quad \forall i \in M, \forall j \in M$$

■ Minimum cost flow problem

- ◆ Relax integrality
- ◆ Network simplex

Winner Determination Problem

$$\text{Min} \quad \sum_{n \in N} \sum_{i \in M} c_i^n x_i^n + \sum_{n \in N} \sum_{i \in M} \sum_{j \in M} e_{i,j}^n y_{i,j}^n$$

$$\text{St} \quad \sum_{n \in N} x_i^n = 1 \quad \forall i \in M$$

$$\sum_{j \in M} y_{i,j}^n = x_i^n \quad \forall i \in M, \forall n \in N$$

$$\sum_{j \in M} y_{j,i}^n = x_i^n \quad \forall i \in M, \forall n \in N$$

$$y_{j,i}^n \in \{0,1\} \quad \forall i \in M, \forall j \in M, \forall n \in N$$

$$x_j^n \in \{0,1\} \quad \forall i \in M, \forall n \in N$$

- Relax integrality of y
- Multi-commodity flow problem

Computational Results

Load Based Model

K	N	BL	Time (sec.)		B&B nodes		Binary	Continuous	Constraints
			Mean	SD	Mean	SD			
50	50	50	84	88	6	8	2,500	125,000	5,050
50	50	100	666	456	7	6	5,000	500,000	10,100
50	50	200	6,134	8,274	4	3	10,000	2,000,000	20,200

Location Based Model

K	N	BL	Time(sec)		B&B Nodes		Binary	Continuous	Constraints
			Average	SD	Average	SD			
50	100	10,000	477	482	4	7	500,000	500,000	15,000
50	100	20,000	1,243	313	2	2	1,000,000	500,000	25,000
50	100	40,000	907	29	0	0	2,000,000	500,000	45,000
50	50	80,000	1,714	885	0	0	4,000,000	125,000	82,500

Operational Considerations

- Limitations (min, max) on number of carriers

$$w_n \in \{0,1\} \quad \forall n \in N$$

$$w_n \geq x_i^n \quad \forall n \in N, \forall i \in M$$

$$w_n \leq \sum_{i \in M} x_i^n \quad \forall n \in N, \forall i \in M$$

$$\text{min_carriers} \leq \sum_{n \in N} w_n \leq \text{max_carriers}$$

Operational Considerations

- Limitations (min, max) on number of loads per carrier

$$\text{min_loads} * w_n \leq \sum_{i \in M} x_i^n \leq \text{max_loads} * w_n$$
$$\forall n \in N$$

Operational Considerations

- Upper bound on length of tour
 - ◆ More challenging than the others, because it is continuous move-specific, not carrier specific
 - ◆ y variables define explicit tours
 - ◆ Broad literature on limiting length of tour

Problem Uncertainty

- There is uncertainty in this problem – carriers don't always know whether they'll have a backhaul opportunity on a lane
- We can think of this as a special case of the multi-commodity flow problem where the costs of some arcs (the “connection arcs”) are uncertain
- Can we minimize the expected outcome?

“Rules of the Game”

- Stage One Decisions: The auctioneer assigns bid loads to carriers
- The carriers realize their actual connection costs
- Stage Two Decisions: The carriers construct their routes, based on the bids they were awarded and the actual costs of their connection arcs
- Our goal is to minimize expected value of overall system cost

Winner Determination Problem

$$\begin{aligned} \text{Min} \quad & \sum_{n \in N} \sum_{i \in M} c_i^n x_i^n + \\ & \sum_{s \in S} \sum_{n \in N} \sum_{i \in M} \sum_{j \in M} p^s e_{i,j}^{ns} y_{i,j}^{ns} \end{aligned}$$

$$\begin{aligned} \text{St} \quad & \sum_{n \in N} x_i^n = 1 && \forall i \in M \\ & \sum_{j \in M} y_{i,j}^n = x_i^n && \forall i \in M, \forall n \in N, \forall s \in S \\ & \sum_{j \in M} y_{j,i}^n = x_i^n && \forall i \in M, \forall n \in N, \forall s \in S \end{aligned}$$

$$\begin{aligned} y_{j,i}^n &\in \{0,1\} && \forall i \in M, \forall j \in M, \forall n \in N, \forall s \in S \\ x_j^n &\in \{0,1\} && \forall i \in M, \forall n \in N \end{aligned}$$

Winner Determination Problem

$$\text{Min } \sum_{n \in N} \sum_{i \in M} c_i^n x_i^n + Q(x)$$

$$\text{St } \sum_{n \in N} x_i^n = 1 \quad \forall i \in M$$

$$x_j^n \in \{0,1\} \quad \forall i \in M, \forall n \in N$$

Where $Q(x)$ is the expected value of the second stage decisions, i.e. the min-cost flow problems

Winner Determination Problem

$$\text{Min } \sum_{n \in N} \sum_{i \in M} c_i^n x_i^n + \theta$$

$$\text{St } \sum_{n \in N} x_i^n = 1 \quad \forall i \in M$$

$$\theta = Q(x)$$

$$x_j^n \in \{0,1\} \quad \forall i \in M, \forall n \in N$$

Winner Determination Problem

$$\text{Min} \quad \sum_{n \in N} \sum_{i \in M} c_i^n x_i^n + \theta$$

$$\text{St} \quad \sum_{n \in N} x_i^n = 1 \quad \forall i \in M$$
$$\theta = D(x)$$

$$x_j^n \in \{0,1\} \quad \forall i \in M, \forall n \in N$$

where D is the dual of Q (this is true because, once we fix x , we can relax the integrality of y !)

Winner Determination Problem

$$\text{Min} \quad \sum_{n \in N} \sum_{i \in M} c_i^n x_i^n + \theta$$

$$\begin{aligned} \text{St} \quad & \sum_{n \in N} x_i^n = 1 && \forall i \in M \\ & \theta \geq D^i(x) && \forall i \in \{\text{e.p.'s of } D\} \end{aligned}$$

$$x_j^n \in \{0,1\} \quad \forall i \in M, \forall n \in N$$

where $D^i(x)$ is the value of the i^{th} extreme point of the dual of Q , evaluated at objective function x (these extreme points can be identified via cut generation)

Structural Properties

- In general, 2-stage IP's with recourse can be very hard to solve
 - ◆ The integrality of x enforces integrality of y , making the second stage an LP rather than an IP
 - ◆ We can decompose the second stage problem by carrier
 - ◆ We can also dis-aggregate scenarios
 - ◆ We can dualize based on individual scenarios/carriers
 - ◆ If a carrier's bids don't change from one iteration to the next, don't need to solve the sub-problem again
- Computational results to come...

Conclusions

- VCG auctions are powerful in theory but have been intractable in application because of their combinatorial explosion
- Using mathematical programming to embed bidders' true cost functions may allow for the implicit bidding on and consideration of *all* combinatorial bundles, without the computational cost of explicit bidding (under certain conditions on the true cost function)

Conclusions

- In truckload procurement, the underlying bid-generating function is a simple min-cost flow problem
- Embedding this structure in the WDP leads to a multi-commodity flow problem
 - ◆ Only the bid-load assignment variables are integer
- Leverage this MIP structure allows us to:
 - ◆ Solve large problem instances
 - ◆ Add operational side constraints
 - ◆ Consider the stochastic version of the problem

Areas for Future Research

- Auctions
 - ◆ Other mechanisms
 - ◆ Fire-walls, trust issues
- Math programming
 - ◆ Large-scale, column generation approach
 - ◆ Stochastic problem
- Freight transportation
 - ◆ Estimation of arc costs

Questions and Comments