

Composite Variable Models for Complex Problems in Transportation and Logistics

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1) My research is in the field of *large-scale optimization*. An *optimization problem* is a problem in which we have a collection of decisions to make, a set of rules governing how these decisions interact, and a means of quantitatively comparing possible solutions. Our goal in such a problem is to find the best set of decisions that collectively satisfy all of the rules.

We encounter optimization problems in all aspects of our lives, from the most mundane to the most critical decisions that we face. In some cases, we can easily analyze the situation to make good decisions (which route to drive to work, what to cook for dinner). However, many optimization problems are too complex and their impact too important to solve simply by observation or using rules-of-thumb. Consider, for example, the airline industry. An airline cannot make a decision about whether or not to offer a particular flight in isolation. Instead, they must simultaneously consider what type of aircraft and which crew to assign. Furthermore, the value of this flight depends on what other flights are offered, because many of the passengers on that flight may be connecting to other flights. This is just a small subset of the airline decisions that are impacted by the addition of a single new flight.

When faced with a complex system in which decisions cannot be made by observation, an alternative approach is to represent the optimization problem as a mathematical model, often called a *mathematical program*. In a mathematical program, a variable represents each of the decisions, a mathematical constraint enforces each rule, and an objective function captures the goal of finding the best solution. Mathematical programming has been used to solve an enormous class of real-world problems, from structuring a transportation system to constructing a financial portfolio to determining the most effective way to orient beams when administering radiation to a cancer patient.

I am particularly interested in applying optimization techniques to problems in transportation and logistics. Such problems typically center on a complex *network*, in which each decision impacts every other decision, making global studies of the system both necessary and difficult. Not only are transportation and logistics problems typically complex; they also tend to suffer from *discrete decisions* (for example, the number of workers to hire for a given shift must be an integer number) and *non-linearities* (for example, the cost of moving freight is not necessarily a linear function of the *amount* of freight to be moved, but instead of the integer number of trucks needed to move that freight). Mathematical programs with discrete variables and non-linear constraints or objective function are typically very difficult to solve. In many cases, there is no known solution approach that can yield an optimal solution to such models.

The focus of this proposal is on developing alternative models, based on *composite variables*, that allow us to solve some of these difficult problems. A composite variable is a variable that encompasses multiple decisions. For example, instead of a defining a variable to represent the question “Should I include flight f in my airline schedule?” a composite variable might encompass the set of decisions, “Should I include flight f in my schedule, assign it to fleet type t , and staff it with crew c ?”

Taking a composite variable approach typically results in a very large increase in the number of variables. This is because we replace the variables associated with individual decisions with variables representing *combinations* of decisions. A composite variable model might have an exponential number of variables – literally, billions or more. Thus, a poor composite variable formulation simply replaces one mathematical program that is intractable because of characteristics such as non-linearities with another that is intractable because of its size.

A strong composite variable formulation, on the other hand, can yield substantial benefits. The appropriate formulation can eliminate complicating constraints, remove non-linearities, and decrease the impact of discrete variables on the performance of the solution algorithms. Furthermore, large-scale optimization techniques exist which can in some cases allow very large models to be solved in very short periods of time. Thus, the challenge is to find a model that, on the one hand, addresses the problems plaguing the basic formulation but, on the other hand, lends itself to solution through the use of these large-scale techniques.

I am proposing to continue my efforts in how such modeling approaches can be used specifically in the context of transportation and logistics problems, focusing on issues of network interactions, large numbers of discrete decisions, and non-linear cost functions.

2) The next several months of this project will be a “proof-of-concept” – we will be focusing on a single, specific real-world application, working towards developing generalizable insights. A graduate assistant would participate in all stages of the project: identifying the initial application on which to focus, developing candidate models, analyzing and comparing the models, gathering data, implementing the promising models for computational testing, and developing theoretical insights from the results of the applied research. If sufficient progress is made by early Fall, the student would also have the opportunity to present the preliminary results at a major conference in October.

3) This project is based on my dissertation research, in which I investigated how composite variable modeling could be used to solve an important problem in integrated airline planning. My current focus is on extending and generalizing these ideas to develop a theoretical foundation for a much broader class of problems in transportation and logistics. During the summer, I hope to invest the majority of my time on this project, developing a substantial proof-of-concept to demonstrate the broad applicability of these ideas. With the assistance of a graduate student, I believe that we can make enough progress to begin attaining external funding, potentially from both industry and government agencies.

4) This is an ideal opportunity for a junior graduate student to begin using the concepts taught in first-year courses in a concrete way, as well as the opportunity to learn more sophisticated techniques. Furthermore, it gives the student the chance to influence the project direction early on, establishing his or her own niche for a future dissertation topic. Finally, I hope that this work will lead to long-term funding opportunities for the student in the ensuing years.

5) I am considering two students for this funding, Melinda Davey and KoMing Liu, both of the IOE department. They are currently working with me for course credit, Melinda in logistics and KoMing in airline planning. This semester coursework is part of the requirement for all first-year PhD students in the department. Both students have demonstrated great aptitude and enthusiasm for research, and I hope to be able to fund them over the summer to maintain their momentum. Rackham funding would be invaluable in helping me to make this happen.