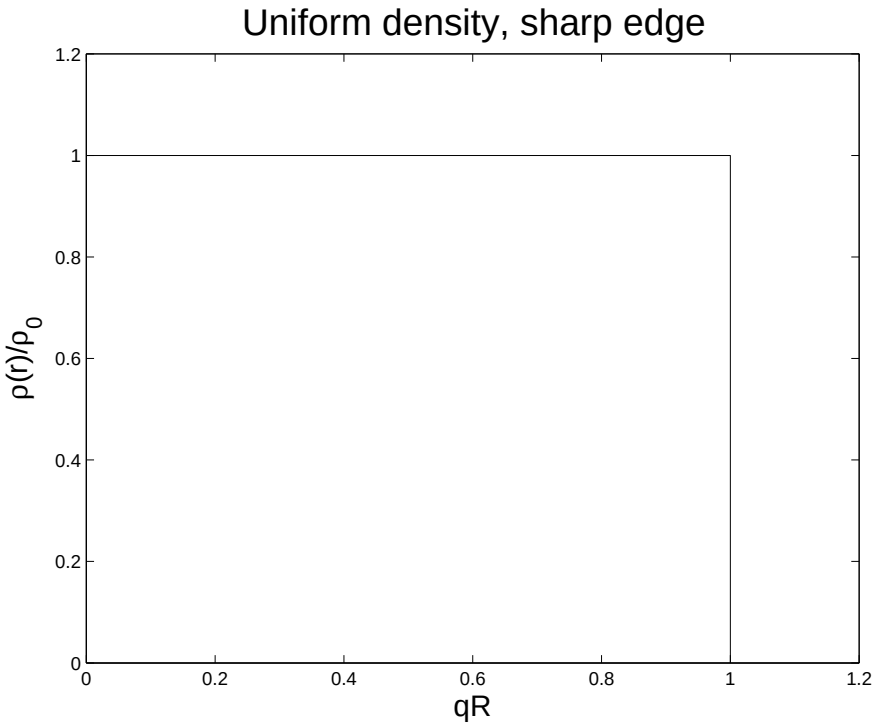
NERS 312 Winter 2019
Assignment 1: Nuclear Properties

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Due: January 28, 2019 at 11:00 am (at the start of class)

There are 3 questions with sub-parts, spread over 4 pages. Please read the instructions carefully. The assignment must be handed in by the due date given above, at the start of class. There are some extra credit opportunities. These extra credit parts are graded to a much higher standard, and there is no help available for the extra credit parts except for clarification.

1. **[10%]** If all the matter on earth collapsed to a sphere with the same density of the interior of a nucleus, what would the radius of the earth be? State all assumptions you make, and cite all sources of data you used.
2. **[50%] The effect of the nuclear edge on** $|F(\vec{q})|^2$
   (a) **[This is review, but worth 5%]** Find $F(\vec{q})$ for a uniformly charged sphere. For a uniform sphere, $\rho_{\rm p}(r) = \rho_0\Theta(R-r)$, where R is the radius of the nucleus.[1]



Plot[2] $\log(|F(\vec{q})|^2)$ *vs.* qR.

[1] Mathematical note:

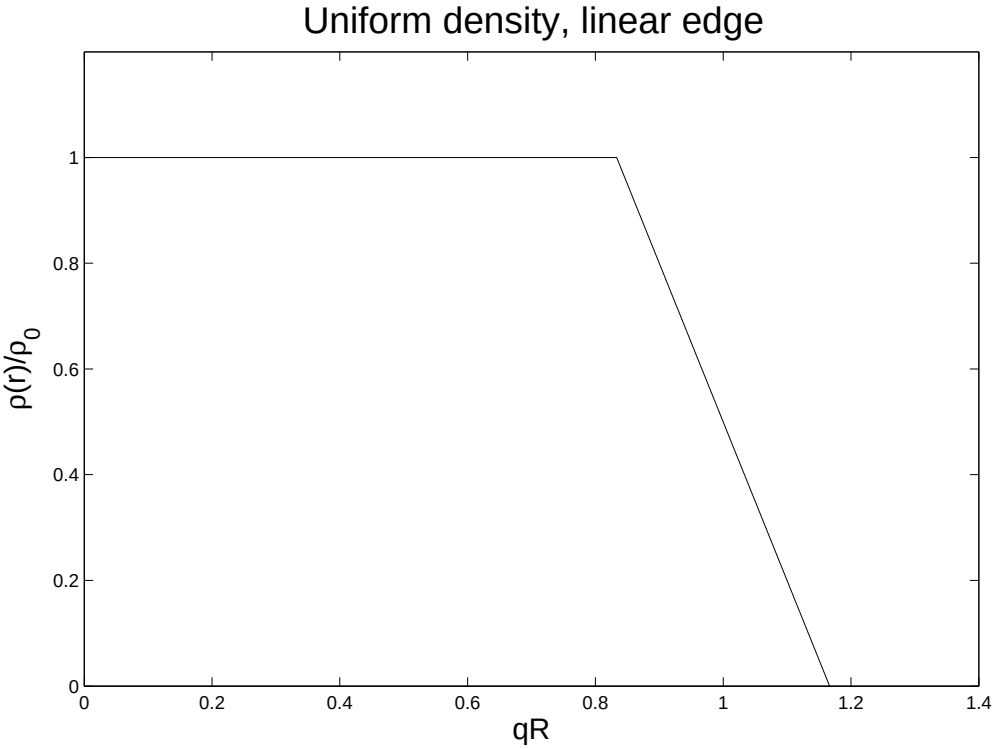
$$\Theta(z) = 1\ ;\ z>0$$
$$= 0\ ;\ z<0$$

[2] Technical note: $|F(\vec{q})|)^2$ can be zero, so its logarithm would be $-\infty$, causing the plots to look strange. We need the logarithm to see the full structure of $|F(\vec{q})|^2$. So, in order to make the plots look reasonable, you will probably have to adjust the scale on the y-axis in an appropriate fashion.

(b) Do the same as in (a) but include a linear edge in the model of the nucleus. That is,

$$\begin{aligned}\rho_{\rm p}(r) &= \rho_0\Theta([R-t/2]-r) \\ &= \rho_0[1/2-(r-R)/t]\ ;\ R-t/2 \le r \le R+t/2\ ,\end{aligned}$$

where t is the nuclear "skin depth", and, mathematically, can take any value between 0 and $2R$. Note that $\rho_{\rm p}(R-t/2)=\rho_0$ and $\rho_{\rm p}(R+t/2)=0$. Note also, that when $t=0$, the nuclear shape is the same as in part (a).



Plot $\log(|F(\vec{q})|^2)$ *vs.* qR as in part (a) showing several values of t over the entire domain of t, $0 \le t \le 2R$. Compare with the result of part (a).

(c) Compare and discuss your results. Does the elimination of the sharp edge eliminate the sharp minima in $\log(|F(\vec{q})|^2)$? What else could be contributing to the reduction of the sharpness of these minima?

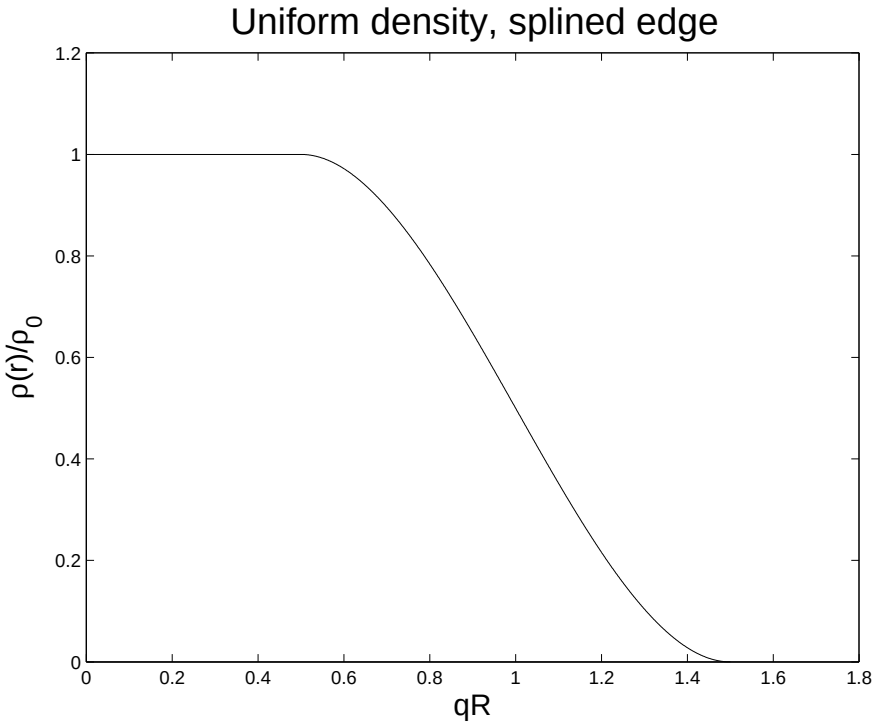
(d) **10% extra credit** Do the same as in (b) and (c) but include a cubic edge in the model of the nucleus. That is,

$$\begin{aligned}\rho_{\rm p}(r) &= \rho_0\Theta([R-t/2]-r) \\ &= \rho_0(A+Br+Cr^2+Dr^3)\ ;\ R-t/2 \le r \le R+t/2\ .\end{aligned}$$

Arrange the four constants A, B, C, D such that:

$$\begin{aligned}\rho_{\rm p}(R-t/2) &= \rho_0 \\ \rho'_{\rm p}(R-t/2) &= 0 \\ \rho_{\rm p}(R+t/2) &= 0 \\ \rho'_{\rm p}(R+t/2) &= 0\end{aligned}$$

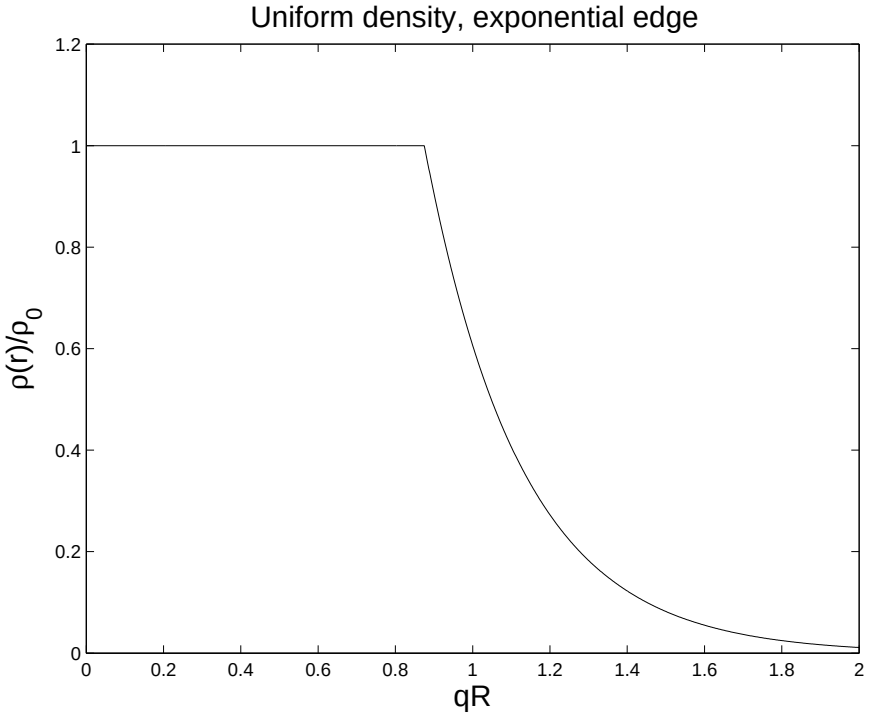
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Plot $\log(|F(\vec{q})|^2)$ *vs.* qR as in part (b) showing several values of t over the entire domain of t, $0 \le t \le 2R$. Compare with the result of parts (a) and (b).

(e) **10% extra credit** Do the same as in (b) and (c) but include an exponential edge in the model of the nucleus. That is,

$$\begin{aligned}\rho_{\rm p}(r) &= \rho_0\Theta([R-t/2]-r) \\ &= \rho_0\exp(-(r-R+t/2)/t); \quad r \geq R-t/2\ .\end{aligned}$$



Plot $\log(|F(\vec{q})|^2)$ *vs.* qR as in part (b) showing several values of t over the entire domain of t, $0 \leq t \leq 2R$. Compare with the result of parts (a) and (b).

3. **Perturbation of atomic energy levels [40%]**

   (a) Read carefully and understand the text on pages 49–55 in Krane on this topic. Krane makes the assertion that ΔE_{2p} can be ignored for atomic electron transitions. Verify this assertion by repeating the calculation for ΔE_{2p} and obtain a relationship for $\Delta E_{2p}/\Delta E_{1s}$ for atomic electrons and muons. Evaluate numerically for ^{12}C and ^{208}Pb.

   (b) **extra credit** The results in (a), for muons, is suspect, because the muon's wavefunctions have significant overlap with the physical location of the nucleus, and the shape of the edge of the nucleus may play a significant role. Repeat the analysis of part (a), but introduce a skin depth using one of the models in Question 2) or some other model of your choosing. What do you conclude?