NERS555: Introduction to Radiological Physics and Radiation Dosimetry
Assignment 2: **The Compton interaction** Fall 2018

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Due: October 4: 2:30 pm. Bring it to class, or slip it under my office door anytime before that.

The Klein-Nishina differential cross section is given by:

$$\frac{\mathrm{d}\sigma_{\mathrm{KN}}}{\mathrm{d}\Omega_\theta} = \frac{r_e^2}{2}\left(\frac{\alpha_1}{\alpha}\right)^2\left(\frac{\alpha_1}{\alpha}+\frac{\alpha}{\alpha_1}-\sin^2\theta\right) \quad \text{where} \quad \frac{1}{\alpha_1}-\frac{1}{\alpha}=1-\cos\theta \quad \text{or} \quad \frac{\alpha_1}{\alpha}=\frac{1}{1+\alpha(1-\cos\theta)}$$

where r_e is the "classical electron radius" (Wiki it!), θ is the angle between the scattered photon and incoming photon directions, α is the incoming photon energy divided by the rest mass energy of the electron, and α_1 is the similarly scaled scattered photon energy.
The unnormalized angular probability distribution is:

$$p(\mu) = \left(\frac{\alpha_1}{\alpha}\right)^2\left(\frac{\alpha_1}{\alpha}+\frac{\alpha}{\alpha_1}-\sin^2\theta\right) \quad \text{where} \quad \mu\equiv\cos\theta \quad \text{and} \quad -1\le\mu\le 1$$

1. **[20%]**
   (a) **[10%]** Using conservation of relativistic energy and momentum, derive the equations:

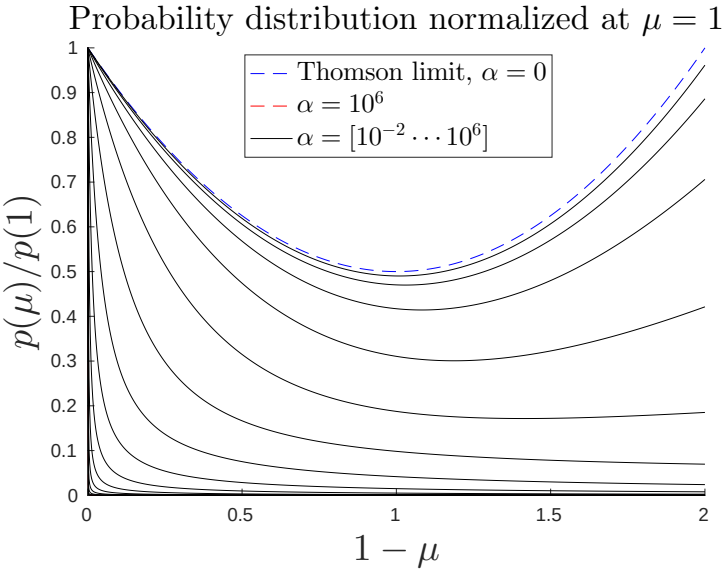
   $$\frac{1}{\alpha_1}-\frac{1}{\alpha}=1-\cos\theta \quad \text{and}$$

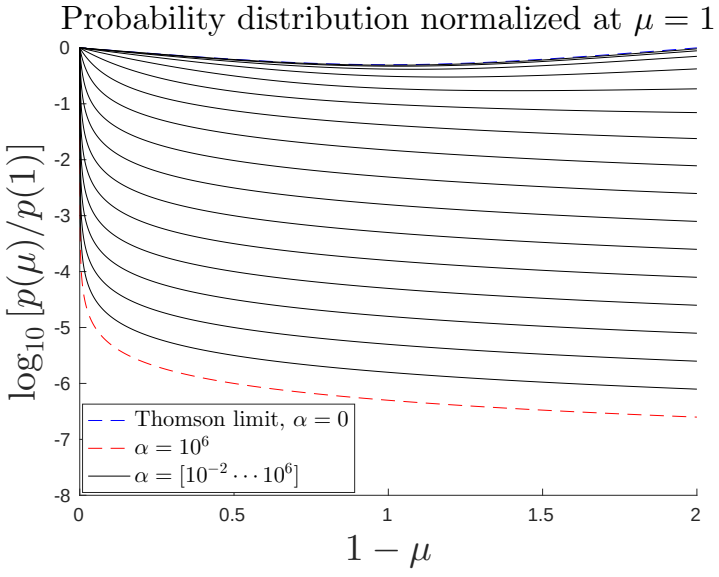
   $$\cot\varphi = (1+\alpha)\tan\left(\frac{\theta}{2}\right)$$

   and discuss the $\theta=0$ and $\theta=\pi$ limits.

   Consult the lecture notes for description of the variables.

   (b) **[10%]** Plot the cross section as a probability distribution in $\mu=\cos\theta$. It should resemble the following:





2. **[80%]** Find expressions, plot *vs.* α (low energy and high energy limits must be seen clearly) and discuss the high and low energy limits:

   (a) **[20%]** $\langle \alpha_1/\alpha \rangle$

   (b) **[20%]** $\langle (\alpha_1/\alpha)^2 \rangle - \langle \alpha_1/\alpha \rangle^2$

   (c) **[20%]** $\langle \mu \rangle$

   (d) **[20%]** $\langle \mu^2 \rangle - \langle \mu \rangle^2$

   Hint: If $p(x)$ is an unnormalized probability distribution defined between x_0 and x_1, then

$$\langle x \rangle = \left(\int_{x_0}^{x_1} \mathrm{d}x \; x p(x) \right) \Big/ \left(\int_{x_0}^{x_1} \mathrm{d}x \; p(x) \right)$$

$$\langle x^2 \rangle = \left(\int_{x_0}^{x_1} \mathrm{d}x \; x^2 p(x) \right) \Big/ \left(\int_{x_0}^{x_1} \mathrm{d}x \; p(x) \right)$$

$$\langle f(x) \rangle = \left(\int_{x_0}^{x_1} \mathrm{d}x \; f(x)\, p(x) \right) \Big/ \left(\int_{x_0}^{x_1} \mathrm{d}x \; p(x) \right)$$

   Your results should resemble the following:

