NERS555: Introduction to Radiological Physics and Radiation Dosimetry
Assignment 4 (**double-weight**) Fall 2018

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1 Fundamental concepts

1. What is meant by radiation equilibrium? Why is this concept important in dosimetry?
2. Give two real-life examples where radiation is nearly in a state of equilibrium.
3. Without equations, describe the major result of Fano's theorem.
4. Discuss the important consequences of the combination of Fano's theorem and radiation equilibrium.

2 Basic interaction physics

1. Ignoring photonuclear processes, what are the 3 most important photon interactions to the general area of medical physics?

   In what energy range is each dominant?

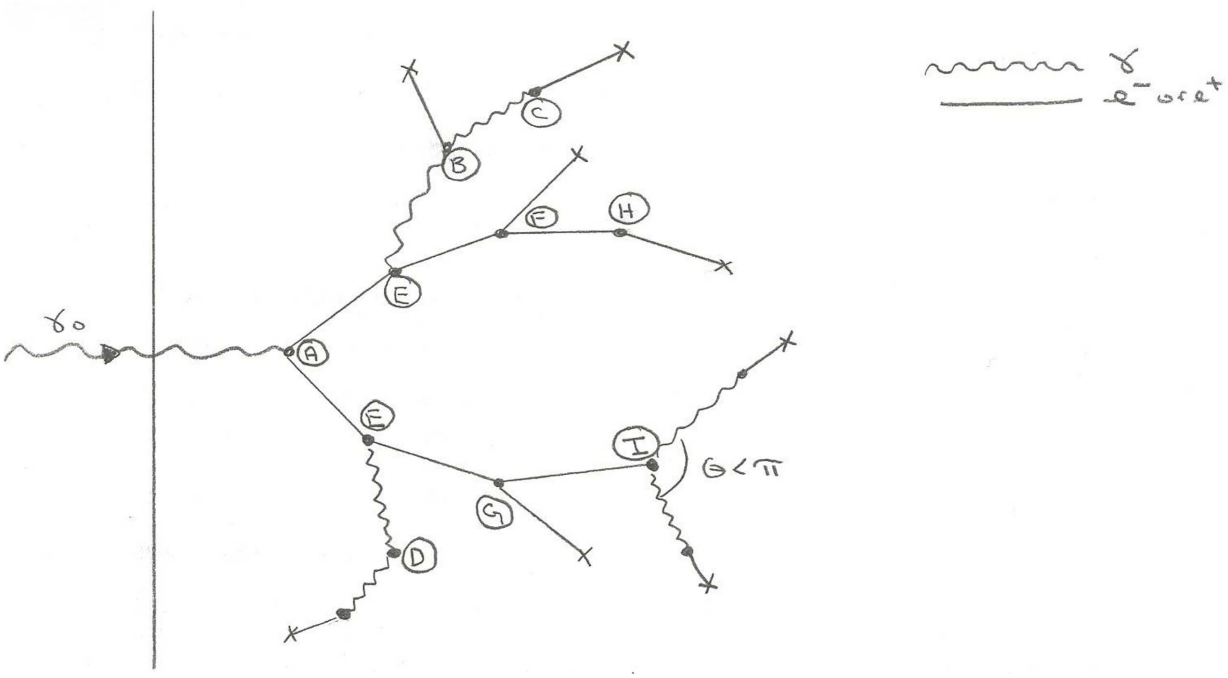
   What is the atomic number dependence of each?
2. Ignoring electronuclear processes, what are the 2 most important electron interactions to the general area of medical physics?

   In what energy range is each dominant?

   What is the atomic number dependence of each?
3. Ignoring electronuclear processes, what are the 3 most important positron interactions to the general area of medical physics?

   In what energy range is each dominant?

   What is the atomic number dependence of each?
4. A 50 MeV photon enters a thick Pb slab. Draw a representative picture of the entire life of this photon and all of its daughter particles. Make sure that you show all the interactions described above.
5. Repeat the previous question but with a 1 MeV photon. What processes can not occur?
6. Consider the following diagram. The interaction points are labeled by uppercase letters. For each interaction type, name the interaction process (*e.g.* Compton) and give the atomic number dependents of each cross section (*e.g.* Z/atom). What is the approximate energy dependence of each [(*e.g.* E_γ^{-1}].



	Interaction name or type	Z dependence (/atom)	Energy dependence
A			
B			
C			
D			
E			
F			
G			
H			
I			

7. Describe, in words, what the Compton interaction is.

   Provide a drawing illustrating it.

8. For the Compton interaction, show that the energy of the incoming photon, E_γ, and the energy of the scattered photon, E'_γ, are related to the scattering angle of the scattered photon, θ, by the relation:

$$\frac{1}{E'_\gamma} - \frac{1}{E_\gamma} = \frac{1}{m_e c^2}(1 - \cos\theta)$$

   where, $m_e c^2$ is the rest mass energy of the electron.

3 Fluence, energy fluence, and Dose

1. What is the ICRU definition of fluence?
2. What is the definition of planar fluence?

3. What does Chilton's theorem on fluence state?

4. A fluence, differential in energy, E, has the form:

   $$\varphi(E) = \frac{A}{E^3} \quad E_{\min} \leq E \leq E_{\max}$$

   and zero everywhere else. A is a constant.

   (a) What is the total fluence?

   (b) What is the energy fluence, differential in energy?

   (c) What is the total energy fluence?

4 Fluence and planar fluence detectors

Imagine a radiation field in vacuum with particles going in one direction only. The particle fluence is Φ_0, as measured by a spherical fluence detector with radius a. Now, imagine that you have a disk-shaped fluence detector. It has radius a and depth d.

1. Using directly Chilton's definition of fluence,

   $$\Phi = \frac{\sum_{i=1}^{N} \Delta l_i}{\Delta V}$$

   show that the detector measures a fluence Φ_0, when its symmetry axis (axis of rotational symmetry) is aligned with the direction of particles in the field. ($\theta = 0$). You may assume that there are enough particles in the field to allow you to evaluate the sum above as an integral.

2. Using directly Chilton's definition of fluence, show that the detector measures a fluence Φ_0, when its symmetry axis (axis of rotational symmetry) is perpendicular to the direction of particles in the field. ($\theta = \pi/2$). Hint: $\int_{-\pi/2}^{\pi/2} \mathrm{d}\phi \, \cos^2 \phi = \pi/2$.

3. Without calculation, what would you expect if the fluence detector has an angle θ_0 with respect to the field direction?

Now, imagine that a detector is an ideal planar fluence detector.

What does this detector measure for

1. $\theta = 0$?

2. $\theta = \pi/2$?

3. θ at an arbitrary angle?

5 Kerma and Dose

1. How is Kerma, K defined? Describe in words and then proviude a figure.

2. How is Collision Kerma, K_c defined? Describe in words and then proviude a figure.

3. How is Dose, D defined? Describe in words and then proviude a figure.

4. A photon with energy E_0 enters a region V, interacts via a Compton interaction, changing its energy to E_1. The Compton recoil electron that was created leaves V with kinetic energy E_e and comes to rest outside of V. During its slowing down process, the electron interacts twice via the bremsstrahlung process, producing a photon E_2 that originated inside V, and another photon with energy E_3 outside of V. What is Dose, Kerma and Collision Kerma in this case?

6 Finite detectors in non-equilibrium fields

The fluence in vacuum due to a point source that emits N particles is:

$$\Phi(r) = \frac{N}{4\pi r^2}$$

1. Imagine that an ideal disk-shaped fluence detector with finite radius a and negligible thickness, is centered a distance r from the source and is perpendicular to it. It measures a fluence $\Phi_{\mathrm{M}}(r)$. Show that,

   $$\Phi_{\mathrm{M}}(r) = \Phi(r)\frac{r^2}{a^2}\ \log\left(1+\frac{a^2}{r^2}\right)\ .$$

   Hint: It's much easier to use the Chilton definition of fluence.

2. Is the fluence continuous and finite everywhere along z? Prove your result.

3. Show that

   $$\lim_{z\to\infty}\phi(z) \longrightarrow \frac{N_0}{4\pi z^2}[1+k(a/z)^2+...]$$

   Determine the constant k.
   Hint:

   $$\lim_{\epsilon\to 0}\log(1+\epsilon) \approx \epsilon - \epsilon^2/2$$

   when $\epsilon \ll 1$.

   What does this mean?

4. Adapt your result for the first part to account for attenuation in the air between the radiating disk and the measurement point. Assume that the attenuation is a constant characterized by the attenuation constant μ. Note, just state the resulting integral and do not attempt to solve it. Assuming $\mu a \ll 1$ and $\mu z \ll 1$, attempt a first order correction to the first parts of this solution that accounts for attenuation.