14
Preparing Mathematics Education Researchers for Disciplined Inquiry: Learning from, in, and for Practice

JO BOALER
Stanford University

DEBORAH LOEWENBERG BALL
University of Michigan

RUHAMA EVEN
Weizmann Institute of Science

ABSTRACT

In this chapter we consider what it takes to learn to conduct research in mathematics education. We argue that learning any complex practice requires opportunities to unpack its components in order to see what underlies competence performance. Many of the components of successful research remain implicit and are left to new researchers to glean from finished products. In this chapter we consider: What is it that accomplished scholars do as they conduct research? What are the practices in which they engage? What is it that they have to mobilize, consider, try, and carry out? We unpack the work of research, considering such domains as reading, writing, moving from particular observations to general insights, and communicating through multiple media. We propose that focusing on these practices of research offers a promising avenue for the preparation of beginning scholars.

WHAT IS THE PROBLEM? A DIFFERENT PERSPECTIVE

The earnest Ph.D. student sitting across from the professor had come to seek help with her research proposal: "I am not sure what is wrong, but each time I try to write what it is I want to study, it falls flat, and my committee sends it back, saying that I am not framing my problem in a way that makes it possible to design a study, and I just don't understand. I am getting more and more frustrated." Having read her most recent draft, and having also heard a similar tale of frustration from her advisor, the professor was intrigued. How had this talented doctoral student become stuck at this point? Her draft sketched the problem of teachers' use of curriculum materials, and made a number of claims about how teachers should use such materials. In particular, impressed with Stein, Grover, and Henningsen's (1996) work on the use of mathematical tasks

Second International Handbook of Mathematics Education, 491–521
A.J. Bishop, M.A. Clements, C. Keitel, J. Kilpatrick and F.K.S. Leung (eds.)
© 2003 Dordrecht: Kluwer Academic Publishers. Printed in Great Britain.
Reproduced with permission of the copyright owner. Further reproduction prohibited without permission

in classrooms, she wanted to focus on improving middle school teachers' interpretation and use of academically challenging mathematical tasks. She seemed to have read widely and well, reaching back into the more general research on teaching literature (e.g., Walter Doyle's work in 1983 and 1988), and she was drawing on her own work as a teacher as well. But her committee was right: More a treatise than a design for research, her proposal identified a problem of practice and discussed ways to solve it. No question was evident. The researcher had chosen and framed an empirical study, yet she seemed to think that she could write the final version now, without collecting any data, doing any analyses, or experimenting with any interventions. What was her conception of research? How had she come so far with such an underdeveloped idea of what it means to conduct 'disciplined inquiry' (Shulman, 1997)? When the professor asked her what she wanted to focus on, she replied unhesitatingly: "When I was teaching seventh grade, I used *Mathematical Pathways* (a new curriculum), and those tasks worked so well with students. I want to study teachers working with this curriculum and show how they are able to use complex tasks with students. We didn't have the sorts of problems that the literature shows. I want to write about how this curriculum works."

Enthusiastic about or committed to one or another approach, method, or idea, novice researchers often set out to describe an approach and show that it works. This researcher already 'knew' that her approach worked. She wanted to write about the ways that it worked. She wanted to show, prove, convince. This problem comes in all shapes and sizes, from the teacher educator who wants to write about his mathematics methods course and show that his course works, to the professional developer whose use of cases produces such good results, to the mathematics curriculum coordinator convinced of the effectiveness of her district's collaborative approach to curriculum planning. Consumed with wanting to find out what works or to prove that some method or another does work, novice researchers often have difficulty framing a problem that can be investigated, designing a way to investigate it strategically, and considering what their study can claim once they do it. They read 'the literature', write 'research questions', develop questionnaires or interview protocols, and too often assume that the sum of these will yield a piece of 'research'.

This scenario illustrates one of the many problems that may be encountered by new researchers – problems that can emerge from any study, whether empirical, theoretical or philosophical. A different example is provided by new researchers who know they need to become familiar with relevant studies in their field but conceive of this task as a goal in its own sake, or even as a ritual, signalling to other members of the community that they are well read. Such researchers often include a summary of related studies without managing to draw connections among the studies or connections between the studies reviewed and the one they are conducting. Although they have acquired lots of knowledge in graduate school, these and other researchers are unprepared for the complex practice of research. In this chapter, we consider the *work* of research, using the notion of *disciplined inquiry* (Cronbach & Suppes, 1969; Shulman, 1997) to call

Reproduced with permission of the copyright owner. Further reproduction prohibited without permissio

attention to the idea that scholarship is fundamentally about the pursuit of questions and that it is distinguished from everyday curiosity and conclusion by the attentive and rigorous care with which scholars frame problems, design ways to work on them, consider results, and make claims.

The practice of research in mathematics education entails a wide range of knowledge, skills, forms of reasoning, deployment of tools, uses of imagination, and habits of mind. Researchers work to explore and understand the relationship among different factors in educational settings through a process of asking questions, framing and testing hypotheses, reading carefully, gathering evidence, analysing information, revising hypotheses, and making claims. They test their ideas by communicating them to others and improving their claims and arguments on the basis of criticism. In this chapter, we reconsider what it takes to learn to do disciplined inquiry in mathematics education. Rather than treating this question as one of what researchers should know, we turn the question on its head: What is it that accomplished scholars do as they conduct research? What are the practices in which they engage? What is it that they have to mobilize, consider, try, and carry out? We propose that focusing on these *practices of research* offers a promising avenue for the preparation of beginning scholars. If disciplined inquiry rests on such practices, then learning to do such inquiry depends on coming to see and appreciate such practices, understanding their nature and role, and being able to engage appropriately and skilfully in such practices. That is no small order.

WHAT DOES IT MEAN TO CONSIDER RESEARCH FROM THE PERSPECTIVE OF ITS *PRACTICES*?

Pickering (1995) examines the work of mathematicians and scientists and notes that there are "specific, repeatable sequences of activities on which scientists rely in their daily work" (p. 4). He refers to these repeated actions as 'practices' and describes the complex problem solving in which mathematicians and scientists engage as "the mangle of practice" (1995, p. 1). Similarly, mathematics education researchers engage in a repeated set of practices that require knowledge but, more important, that also require the successful enactment of knowledge in different situations. Because research is not merely the validation of common sense but also a set of systematic methods for improving upon everyday knowledge, researchers must develop a peculiar constellation of attitudes that include being sceptical, being open to surprise, trying to prove one's ideas wrong, and considering alternatives. Framing a researchable question is one example of a practice that lies at the heart of the enterprise. Reading widely and making good use of theory and ideas in one's own domain and others are other critical aspects of the research process. Other parts of the work include generalising from particulars – developing concepts and formulating relationships (Shulman, 1997) – revising and improving hypotheses, considering alternative interpretations, using evidence appropriately, and representing ideas in writing. These and other

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission

practices comprise the fundamental work of research – of disciplined inquiry and problem solving. Yet many of these core practices of disciplined inquiry are invisible to beginners, and even experienced scholars know and do them tacitly. Consequently, what it takes to do research remains either substantially overlooked or inadequately explicated. We propose that the close examination of core practices offers a way of reconsidering what it might take to prepare beginners to engage in scholarly inquiry.

Because the preparation of mathematics education researchers has rarely been the object of systematic investigation, our field's discussions of learning to do research tend to lack a theoretical frame. Practices are also not often the focus of deliberations about the research preparation curriculum. High-quality research entails a set of practices that depend on the use of a complex combination of knowledge, beliefs, experiences, and habits of mind, yet discussions of the preparation of beginning researchers often focus instead on debates about the necessary base of professional knowledge. This chapter seeks to add to the small but expanding collection of work (Batanero, Godino, Steiner, & Wenzelburger, 1994; Lester & Carpenter, 2001; Lester & Lambdin, in press; Lingefjärd & Dahland, 1998; McIntosh & Crosswhite, 1973; Reys & Kilpatrick, 2000; Schoenfeld, 1999) that examines ways to prepare novices to engage in the work of research in mathematics education. We begin with a brief overview of the range of problems on which researchers in this field focus, for these provide the ground for a discussion of the practices that their investigation entails, and then return to an examination of what might be offered by making the practices of the work more prominent and explicit in the preparation of mathematics education researchers.

THE EXPANSION OF THE FIELD: WHAT IS INVOLVED IN MATHEMATICS EDUCATION RESEARCH?

Researchers of mathematics education seek to gain knowledge that will support the improvement of mathematics teaching and learning, from infancy into the many mathematically demanding parts of adult life, and during the school years in between. For many researchers, that goal has meant focusing on the knowledge and beliefs of learners. Researchers have asked questions about students' mathematical trajectories – investigating the ways in which particular mathematical concepts develop over time. In the 1960s and 1970s, researchers compiled representations of students' cognitive development as it related to particular mathematical domains and to curriculum materials. Many of the studies at that time combined statistical analyses of students' responses to questions and assessments, with close analyses of pervasive student conceptions and errors. This work contributed to building an important knowledge base in mathematics education, with an accumulation of studies tracing the development of key mathematical concepts and topics, such as fractions, probability, and geometry. Many of the inquiries at that time drew from psychological theories and frameworks, and in

Reproduced with permission of the copyright owner. Further reproduction prohibited without permissio

1976 the International Group for the Psychology of Mathematics Education (PME) was established. This organization brought together researchers from across the world who were investigating students' cognitive development of mathematical ideas. PME continues, to this day, to host the only annual international conference in mathematics education, and although it has expanded its focus beyond the cognitive development of mathematical concepts, psychology continues to be the central discipline employed by researchers who attend the conference, and more generally, by the mathematics education community.

The 1980s witnessed a time of expanded inquiry in mathematics education (for a more comprehensive review of the development of the field of mathematics education research in the U.S., see Lester & Lambdin, in press). Researchers increasingly began to consider teachers and teaching in addition to curriculum and learning, reflecting a greater appreciation of the influence of teachers on students' development. Researchers observed the behaviour of teachers, counting particular teacher actions, analysing the role of their subject knowledge and other teaching variables. In the same decade, researchers increasingly turned their attention to issues of equity as they sought to understand the reasons for differential performance in mathematics, particularly among boys and girls but also among different cultural, linguistic, and ethnic groups in mathematics. Researchers developed questionnaires and assessments and conducted comparative analyses, largely quantitative, of the responses of male and female students, and students of different ethnicities. Inquiries at this time were varied, as the field of mathematics education diversified, continuing its emphasis on curriculum, teaching, and learning, but also embracing issues of assessment, policy, ethnomathematics, philosophy, teacher learning, and professional development.

During this same period, as the areas of inquiry in mathematics education expanded, so too did methods of inquiry. Whereas researchers had tended in the past to conduct their studies in universities or laboratories in an attempt to control all the variables that they observed, they began to study contexts explicitly. Researchers began to situate their inquiries within schools and other educational settings in order to capture, rather than eradicate, "the mobile, complex, ad hoc, messy and fleeting qualities of lived experience" (S.J. Ball, 1995, p. 255). Questionnaires and assessments remained popular as research tools. Researchers were also, however, increasingly turning to classroom observations and interviews to gain information about the dynamics of teaching and learning. By the start of the 1990s, mathematics education inquiries had diversified considerably. Those focusing upon learning started to move beyond students' individual cognitive development to consider socio-cultural and situated aspects of mathematics learning – what Lerman (2000) describes as the "social turn in mathematics education research" (p. 19). Additionally, researchers began to expand their frameworks and analytical perspectives beyond psychology, drawing from a wider range of disciplines including philosophy, anthropology, and sociology (Boaler, 2000; Sierpinska & Kilpatrick, 1998).

The diversification of the field of mathematics education means that beginning researchers face a broad base of perspectives, theories, and frameworks with

eproduced with permission of the copyright owner. Further reproduction prohibited without permissio

which they need to become familiar. In addition, new standards are emerging for the dissemination of educational research that are likely to have significant implications for beginning researchers. In the past, educational researchers have aimed to produce new and important knowledge but have not generally concentrated upon the ways such knowledge may be used in practice. Educational research has recently come under significant criticism for its lack of impact in educational settings. Critics assert that researchers focus on questions that are not crucial to those working in practice and produce recommendations that are not workable. In the United States, these criticisms prompted a committee of leading researchers convened by the National Academy of Education (NAE, 1999) to call for research to focus more on what they called "problem-based research and development." The group further recommended that research be judged not only on its intrinsic quality but also on the extent to which it affects practice. In an important report addressing what some perceive as a crisis in the field, the NAE urged those funding research to prioritise studies designed to impact practice – studies that included specific attention to 'travel': the degree to which research is designed to be transportable and useful to practice.

Several specific recommendations frame the National Academy report (NAE, 1999). One is that professional researchers and professional educators collaborate more with practitioners in the framing and conduct of studies, sharing accountability for the goals of the project. A second is that researchers develop materials that explicitly focus upon transporting the aims, concepts, and methods of research to new settings. Third, the report urges researchers to write directly about the results of their efforts to affect practice. These different recommendations for ways of working seem promising, but whether or not these specific ideas are adopted, researchers from across the world will in future need to pay close attention to the question of impact. That high-quality research in mathematics education will be increasingly both rooted in and connected to practice holds important implications for new researchers. This connection to practice will influence the focus of work, the questions and issues being studied, the methods chosen for studying questions, the people involved and, in important ways, the ways in which the products of research are conceived, shared, and used.

This short tour through some of the questions and issues that have occupied researchers of mathematics education over recent decades, and that present pressing issues for new researchers, reveals that our field has considerable breadth. Those preparing to do research in mathematics education in the future need at minimum to understand and respect the nature of different research traditions and paradigms, being sufficiently open to appreciate the value of different perspectives and frameworks. But this breadth of perspective does not only mean that new researchers have a lot to learn, it also means they have an exciting endeavour before them that is full of possibilities. Shulman (1997) notes that researchers may now "investigate an impressive variety of questions from a rich set of alternative social and political perspectives" (p. 18), but their ability to do so will depend, in part, upon being knowledgeable about the philosophical

Reproduced with permission of the copyright owner. Further reproduction prohibited without permissio

and epistemological foundations of different paradigms and the nature of evidence they provide. The task of research is complex, and as researchers work to "bring out the amazing complexity of what lies in, behind and beyond" (Strauss, 1987, p. 10) data, they will need to deploy considerable knowledge in specific situations, act on the basis of considered professional sensibilities, and exercise both imagination and discipline. We argue that making the development of practices of disciplined inquiry a more specific focus of research preparation could help to prepare beginning scholars with useful resources for the complex work of mathematics education research.

WHAT IS THERE FOR NEW RESEARCHERS TO LEARN? PRACTICES OF MATHEMATICS EDUCATION RESEARCH

Deliberations about the preparation of mathematics education researchers tend to start with questions of knowledge. What and how much mathematics should future researchers know (Dossey & Lappan, 2000)? How may students be introduced to different research methods (Schoenfeld, 1999)? Which theoretical perspectives should they know? In what depth should they know them (Metz, 2001; Page, 2001; Pallas, 2001)? We contend that although such questions are important, future researchers of mathematics education may not be best served by an exclusive focus on conventional knowledge categories. Research, after all, is not *knowledge*. Research, whether empirical, theoretical or philosophical, is an *active process of investigation*, one that relies on strategic use of knowledge, in context. Because it is something people *do*, not just *know*, we turn next to examine a small but illustrative set of core practices of research: reading, formulating a research question, using data carefully to make and ground claims, moving from the particular to the general, considering mathematics, and communicating research findings. Although they are central to disciplined inquiry, these practices are frequently obscured by the products of research. We seek here to make plainer what we mean when we argue that the preparation of new mathematics education researchers may be better informed if more explicit attention is given to the work in which they will engage.

Reading

Reading is a central resource for scholarly work. Scholars read closely in their own fields, keeping up with new developments and deepening their awareness of the historical trajectory and foundations on which contemporary work is based. Mathematics education researchers also read in other fields: sociology, philosophy, psychology, gender studies, anthropology, and mathematics itself, for example. Moreover, because research demands imagination and insight, reading in a wide expanse of public literature is also part of scholarly work. Reading as various as poetry, literary commentary, news analyses, culture and the arts, biography and fiction, all animate and inspire scholars' minds and imagery, contributing to the quality of their thinking.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission

Reading places many demands on new scholars. First, the sheer volume and complexity of the reading in which they must now engage, although it may be fascinating, is also inundating. Getting through hundreds of pages a week and still making sense, making connections, and keeping track is no small task. Many scholars are transcending disciplinary, cultural, and linguistic boundaries as they engage in this central, but often untutored, practice of research. Equipped with dictionaries and highlighters, they tackle stacks of pages. But rarely is there explicit work on ways to read for different purposes, ways to make records of what one is reading, reasons to read, and ways to pursue questions using reading. For philosophers, reading is their empirical work; even empirical researchers need to come to understand the different roles played by reading.

For example, confronted with a question about the nature of discussion in a mathematics classroom, a researcher might read about mathematical reasoning, or she might survey the work on discourse analysis. Another researcher, his imagination piqued by an essay he reads in a literary magazine, might turn to work on philosophy of language or on how practices of deliberation are structured in law. Reading is both a stimulus of the creative insight and a site for pursuing it.

Reading responsibly means reading to locate one's own work in the ongoing conversation in an area. Novices often fall into one of two chasms in relation to the literature in their field. On the one hand, they may too frequently develop the view that they are the first to discover an idea or consider a relationship, pompously declaring it as 'new', as something about which nothing has been asked and nothing is known. On the other hand, though, new scholars are often tied to the research literature, leaden-footed, unable to speak with their own voice, or think beyond the boxes and categories in which the ideas have been set. What they read functions less as a tool for situating and developing their work than as the subject of their work, and they cite and cite, never bringing their ideas into the mélange. What is involved in reading maturely, giving proper respect to past work, and simultaneously pushing forward and out, contributing to the development of new findings, new ideas, new theory?

A host of auxiliary practices are important to the practice of reading. For example, what can a dictionary offer, and does it matter what sort of dictionary one uses? When, and for what sort of question, might one use a dictionary? For example, many new researchers may not know to investigate the etymology of particular words with which one becomes preoccupied: For which words and which sorts of ideas does this make sense to do? And what does one actually do? Another practice concerns the use of reference lists and footnotes. Seeing the literature as offering gateways to exploration – maps for further journeys – means studying not just what another author has written but also what she has read. Learning a variety of ways to make records of one's reading, for different purposes; storing articles and books for later access; studying a text for what there is to learn about composition, argument, and rhetoric – each of these very different skills plays an important role in the practice we call reading for scholarly work.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission

Not to be overlooked are ways to read. Graduate students who enter mathematics education from a background in mathematics are likely to read slowly, scrutinizing every word, and contemplating carefully every line. Accustomed to a mathematical precision unequalled in other fields, they may have difficulty interpreting complex texts constructed with a different aesthetic. They may not know how to skim or read for the gist of an argument or a set of results, for 'skimming' is not a practice of reading in mathematics. Similarly, many novice researchers may not know what it means to study text closely. They may read merely for what they think an author is saying and have little experience with interpretation, with examining closely what an author is doing (Schwab, 1958).

Simple and everyday as it may seem, reading is a core practice of disciplined inquiry (Brown & Dowling, 1998). Taking for granted that its skilful development will happen automatically is likely unwise. Preparing new researchers for the work of reading and for learning to use reading as central to their work would mean transcending the often-tacit knowledge of reading possessed by mature scholars to make visible the many elements of the practice.

Forming a Research Question

No practice may be as fundamental to disciplined inquiry than that of framing a productive question or problem for research (Metz, 2001; Schoenfeld, 1999). Without a well-framed question, no amount of method or analysis can lead to quality work. According to Schoenfeld (1999), "The difficulty is to find a problem that is hard enough so that its solution merits publication but not so hard that it cannot be solved. Learning to do this should be part of all doctoral training" (p. 184). Posing a good research question is a practice that may seem mysterious or even misleadingly easy to novice researchers and yet that plays a pivotal role in conducting good research. A number of constituents of this practice become more visible when we consider some different questions that have been asked in a range of studies in mathematics education. Framing a researchable question is the analogue to formulating an interesting and productive problem in mathematics, and just as important to the quality of the resulting inquiry.

Consider an example: the Third International Mathematics and Science Study (TIMSS) video study (Stigler & Hiebert, 1997). In this project, researchers asked a deceptively simple question: "How is mathematics taught in the United States, Germany, and Japan?" Considering that students in the United States score relatively poorly in international achievement tests, the researchers were curious about what U.S. teachers were doing that was similar to or different from teachers in these other countries. The value of a question depends, however, on the way in which answers to it are pursued. Stigler and Hiebert (1997) acknowledged that one approach to this question would have been to give teachers questionnaires asking them to describe their teaching. But recognizing that educators generally lack shared vocabulary when describing teaching, even within single countries, the researchers acknowledged the limits of that approach. To illustrate, they remark, "One teacher will call something 'problem solving'

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission

while her colleague next door calls the same thing a 'routine exercise'" (p. 15). They therefore chose to study and analyse a sample of lessons in each country that they captured on videotape. Through a careful process of analysis that is described elsewhere (Stigler & Hiebert, 1997, 1999), the researchers produced a range of important insights about mathematics teaching in the three countries. The success of the TIMSS video study rests upon a range of factors including an unprecedented set of records of practice and innovative analyses of those data. Significant, however, is to note that the ideas grew from a study framed by a single, well-formed core question.

Consider a second study in which K.C. Cheung (1988) asked another simple but important question: How are attitudes towards mathematics related to mathematics achievement? To answer this question, Cheung explored the Second IEA (International Association for the Evaluation of Educational Achievement) Mathematics Study data for students in Hong Kong. This study showed that three specific attitudes did correlate significantly with mathematics achievement: students' perception of their ability to do mathematics, their perception of the importance of mathematics to society, and their perception of the creativity within mathematics. Cheung uses this finding to make recommendations regarding the ways in which teachers may pay attention to the development of positive attitudes amongst students through their teaching.

Another interesting, and very different, question is one posed by Paula Ensor (2001). She surveyed the existing research on mathematics teacher education and found that it left many unanswered questions concerning the impact of teacher education upon teaching. She therefore chose to conduct a two-year longitudinal study in which she tracked seven student teachers in South Africa through a year long methods course and their first year of teaching. Ensor asked: How do students' experiences in teacher education classes affect their subsequent beliefs and actions in teaching secondary mathematics? In the first year of her study, she collected 100 hours of course observations, interviewed student teachers and teacher educators, and collected all assessed work including a teaching practice journal, a reflective journal, a class test, a curriculum project and the scripts of an examination. In the second year, Ensor interviewed the teachers who were teaching in secondary schools, videotaped some of their lessons, and interviewed their heads of department. This collection of data enabled Ensor to consider the questions she posed and to draw some important conclusions. She found that the beginning teachers "drew in two ways from the method course: they reproduced a small number of discrete tasks that had been introduced to them there, and they also deployed a professional argot – a way of talking about teaching and learning mathematics" (p. 296). Ensor also found that student teachers 'recontextualized' what they learned from the course and that the effects of teacher education were not 'washed out' (Zeichner & Tabachnik, 1981), but transformed. Ensor produced some important insights that were clearly related to the framing of her questions and to the appropriateness of the methods chosen to answer them.

What characteristics do these very different questions share that may be

Reproduced with permission of the copyright owner. Further reproduction prohibited without permissio

helpful in our exploration of the practice of asking a question? One obvious characteristic is that, at their core, they get at a fundamental issue. These researchers framed and asked questions that were central to the puzzles and problems of the field in which they were working. A second characteristic of these questions is the 'fit' between question and method. Although most of the questions could be answered in multiple ways, the methods chosen by these researchers capitalized on the questions they wished to answer. Thus when the TIMSS researchers asked the question of how mathematics is taught in three countries, they were able to answer this question by collecting actual records of teaching in the three countries. Cheung asked about the relationship between attitude and achievement and drew upon a large database including these variables that could be manipulated statistically.

Still another characteristic of these different research problems is the careful attention researchers paid to the size and scope of the questions asked. This is a crucial aspect of research questions that is linked closely to resources. A single student could not, for example, have conducted a cross-national video study. Even with the resources available, the TIMSS researchers needed to make sure that the study was manageable – making decisions to focus only on one grade level, for example, and on lessons addressing particular subject areas. At the same time, the different studies were sufficiently broad to enable the researchers to explore the data and ask different questions of them. Some new researchers pose important questions that are insufficiently focused to be the topic of a research study. Others ask questions that are too small: They can be answered, but they give insufficient space for the researcher to wade around in data, to explore, and to develop important ideas. The size and scope of a study do not have to be completely determined at the outset, as many decisions and adjustments can be made along the way, but they are an important component of any research question.

Yet another aspect shared by these different studies is the importance of the questions asked and their contribution to theory or practice. Some researchers find questions that are of an appropriate grain size and design appropriate methods to answer them, but the new knowledge they produce is not helpful to practitioners and does not contribute to the development of theory. The importance of the question to be asked is a critical dimension to consider, and consideration of its importance is aided both by experience and by careful reading.

The questions in these different studies were also grounded in theory. Ensor (2001) chose to adopt a sociological framework because of gaps she saw in the current knowledge base, and her report gives clear evidence of the ways that the theories she employed were built into the questions she asked, guiding her attention, for example, to the recontexualization (Bernstein, 1996) of pedagogical practices. Research studies are enhanced when questions lie within a coherent theoretical framework. Theory opens new ideas for researchers, helping them to see differently, to go beyond the limits of their experience, and to make sense of findings produced. At the same time, theory adds coherence, helping to bring

Reproduced with permission of the copyright owner. Further reproduction prohibited without permissio

together the different data and ideas that researchers collect and that may otherwise appear unconnected. Theoretical frameworks help researchers to achieve both the breadth needed for important insights and the focus required by disciplined inquiry. Frameworks may shift as research settings and questions become more familiar (see, e.g., Cobb, 2000). But it is important that questions are theoretically grounded and that the questions draw from a broader, well-chosen framework.

Still another important aspect of these questions is that they connected to an existing knowledge base in meaningful ways (Metz, 2001, p. 13). All of the above researchers identified important gaps in the knowledge base available and used those gaps as targets to pursue. An important part of forming a problem involves considering the place of a new study: Will it add to and build upon prior knowledge? Does it answer the questions that have been raised and not previously been answered? Or questions that have not even been considered? How may prior knowledge be built into the study, so that it provides a productive grounding and springboard for new thoughts? A researcher forming a question needs some knowledge of the research that has been conducted – its character, depth, and breadth – before deciding upon a study.

Forming a coherent question "on which legitimate progress can be made in a reasonable amount of time" (Schoenfeld, 1999, p. 170) is a practice critical for new researchers, but there are few descriptions in the literature of what that actually means, what is involved in doing it well or not well, and what people who do it well actually do. It is important then that beginning researchers receive ample opportunity to reflect upon good questions that have been asked and the ways that productive questions help to frame the problem and methods of gathering evidence. The quality of research depends equally on practices of interpreting and making sense of that evidence. It is to this domain of practice that we now turn.

Being Rigorous with Data

Although questions, theories, and frameworks are crucial, the quality of a study depends just as fundamentally on the rigor and care with which researchers treat their data. Although this domain of practice usually does receive more attention in research training, it is at times reduced to a set of technical decisions regarding methods. Shulman states, "One of the enduring problems in research methodology has been the tendency to treat selection of method as primarily a technical question not associated with the underlying theoretical or substantive rationale of the research to be conducted" (1997, p. 17). Technical decisions are important, and researchers need considerable knowledge of different methods of analysis to support those decisions. But the rigorous and careful analysis of data requires more than just methods. Researchers make many decisions regarding the analysis of data – for example, which of the data that they have gathered should be analysed in depth and which are less crucial to the development of understanding? Which representations of the information might afford analytical insights?

Reproduced with permission of the copyright owner. Further reproduction prohibited without permissio

How does one corroborate the reliability of information? What does any particular piece of information mean, and what assumptions are needed to interpret it in that way? These are some of the questions that researchers face and that are dealt with in different and generally unspoken ways.

Debates between qualitative and quantitative researchers and the concomitant dichotomy established by such debates are now a feature of the past, as most contemporary researchers appreciate that the different methodologies give access to different forms of information, all of which are useful and important. Further, they appreciate that it is more productive to consider and clarify the distinctions and similarities between qualitative and quantitative approaches than to argue over their relative worth (Ely, Vinz, Anzul, & Downing, 1997). But the questions of rigor, of what it means to make claims, to produce evidence, and to make warrants for knowledge (Pallas, 2001; Schoenfeld, 1999) persist. In this section, we consider the exploration of data and the production of research claims, including the ways researchers may reflect on the nature of evidence, on which claims are acceptable, and on what particular pieces of information might mean. These two practices – of exploring data and considering the nature of the claims produced – should coexist within studies and be attended to in equal measures by researchers employing qualitative or quantitative methods, but that is often not the case. Indeed, there is a tendency for quantitative researchers to be rigorous in their exploration of data while neglecting exploration of what the data mean. Qualitative researchers, on the other hand, are often reflective about what data mean but do not always employ rigorous methods of data analysis.

Kenneth Ruthven's (1990) exploration of the impact of graphic calculators offers an example of a study in which the researcher explores data and their meaning carefully. Ruthven employed a classic design in this study. He found four schools that each offered two advanced-level mathematics classes, one of which used a graphic calculator, and the other did not. He then gave students a questionnaire – collecting background information – and a 40-minute test of mathematics. Ruthven found that the students using graphic calculators scored significantly higher grades on the test. One approach to this result could be to communicate test scores and link them with characteristics from the questionnaire, but Ruthven engaged in two important practices that went further. First, he spent time addressing the question of what information was collected, giving careful consideration to the questions asked in the test and the reasons for them. Second, he reflected on the limitations of the data, saying, for example, that the students' responses to the tests "cannot be interpreted as accurate records of the reasoning processes of individual students" (p. 439). This exploration of the *meaning of the data* is an important process that is often neglected in reports.

A second example of careful data analysis is evident in a study conducted by James Hiebert and Diane Wearne (1993). In this study, the researchers considered students' learning in six second-grade classrooms that used different instructional approaches. To consider the impact of different teaching approaches, they collected achievement data and analysed them according to the classroom tasks and discourse patterns observed in the different classes. In their analysis of tasks,

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission

the researchers considered the problems used in the classes, collecting data on the number of problems posed, the kinds of problems – their contextual and mathematical features – the time spent on each problem, and the physical materials that were available to support problem solving. Classroom discourse was analysed in two ways: The researchers considered who talked and how much they talked, and they considered the kinds of questions teachers asked. For the analysis, they recorded and then coded all of the teachers' questions during class discussion and teacher presentation. After questions were identified, two coders classified them into nine question types that differed in the amount of self-explanation and kinds of cognitive activity they elicited. The researchers then investigated whether the different classroom tasks and questions were associated with differences in learning. The resulting analysis is rich, detailed, and informative. Because of the careful collection and analysis of different forms of data, the researchers were able to provide quantitative comparisons of relationships between tasks, questions, and learning, as well as qualitative descriptions, with examples of particular discourse from the classrooms used to illuminate the general categories found. Capturing the relationship between teaching and learning is extremely difficult work that can be superficial and misleading if it is not performed carefully. Many studies compare classes taught using different curriculum with no evidence of the teaching and learning interactions in classrooms, but Hiebert and Wearne provide insights into these relationships through a careful collection and analysis of data.

A noteworthy aspect of Hiebert and Wearne's (1993) data analysis is the rigor with which it is conducted and explained. One of the most important developments in mathematics education research of recent decades is a move towards in-depth case study research. Strauss (1987) has argued that researchers who employ quantitative methods have an array of standard analytic methods from which to choose, but researchers employing qualitative or mixed methods do not. He also asserts that qualitative researchers have paid considerable attention to the data collection stage but very little to the process of data analysis, with analyses being conducted implicitly by experienced researchers but rarely given explicit attention or taught to new researchers. Learning careful practices of qualitative analysis requires explicit attention to what are often subtle and refined methods of working with narrative data. Some beginning researchers come to regard qualitative methods as an easier option, conducting interviews or observations from which they simply quote in order to support their arguments. This lack of rigor comes about partly when researchers are not taught disciplined procedures for the analysis of qualitative data. One of the greatest contributions of the grounded theory approach has been the development of analytic methods that may constitute the 'microscopic examination' (Strauss, 1987) of qualitative data, including coding – a central analytic method for the analysis of qualitative data that is set out in a number of research manuals (Miles & Huberman, 1994). Data may be examined in many different ways, some of which are superficial. Some researchers read through observation notes or interviews without coding them, merely developing ideas from the notes that

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission

they support using extracts from data. But such methods are insufficiently rigorous to reveal the patterns and relationships that lie within data. Hiebert and Wearne's careful study of classroom problems and teacher questioning, including categorical classification and multiple coding, provides an example of the insights that may be achieved when data are explored fully.

An important stage of data analysis, whether researchers employ qualitative or quantitative methods, is that of verification, whereby researchers subject their emerging ideas to careful scrutiny. Different methods exist for verifying, such as searching for counterexamples, conducting respondent validation, and triangulating data. After discovering and naming a phenomenon, researchers are naturally eager to verify its existence, but such fervour may run counter to intellectual integrity and the pursuit of knowledge. It is essential, therefore, to consider the knowledge that researchers may gain of the different methods of verification as well as the beliefs that may encourage the pursuit of such knowledge. At every stage of research, but particularly at the stage of verification, researchers should be encouraged to ask searching questions of their data: Where are these ideas coming from? What evidence do I have for them? What counter-evidence? What would it take to make me think in the *opposite* way from that which I do now? (Lester & Wiliam, 2002). What would be an alternative interpretation? The asking and answering of such questions is an important – but generally hidden – aspect of the research process.

We have considered in this section some ways in which researchers may employ rigor in their work and the different methods and practices that may encourage that. In Schoenfeld's (1999) consideration of the preparation of researchers, he highlights as central the need for "researchers to develop a deep understanding of what it means to make and justify claims," continually asking such questions as "what is the scope of that claim? What kinds of evidence can be taken as a legitimate warrant for that claim?" (p. 180). We contend that this kind of careful inquiry may be supported through more explicit consideration of the work of data analysis. Developing a more careful and nuanced understanding of the ways data may be explored to extract their complexity and of the assumptions behind such work will encourage rigor as well as enable new researchers to move their reports beyond description to *analysis*, an important stage of research to which we now turn.

Moving from the Particular to the General

Perhaps one of the most elusive practices of research is the making of concepts and ideas. Empirical research involves close examination of particulars. How does the analysis of these particulars relate to the questions that researchers seek to investigate? Making sense of the specific details in data, looking for patterns and meaning, and formulating general insights are both important and difficult. Novice researchers often collect a range of data and conduct standard methods of analysis but produce insights that do not extend beyond their particular examples. They offer cases, or tables, and summarize their data.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission

Description predominates, with little analysis beyond the instances at hand. Leading researchers appear sometimes effortlessly to produce key insights that are generative for other researchers. Such insights rarely come from a standard application of research methods, and they often leave new researchers baffled as to their origin. Where are such insights found? How are they generated? At what stages should they emerge?

Fred Erickson (1977) considers the work of qualitative researchers and concludes that probably 'the most important thing' they do is locate 'key incidents' in their data. Such incidents are elucidated and described so that readers know why the incident is important, how it relates to the broader setting and to other theories, and what about it is general. Although Erickson writes of qualitative research, the process he describes seems important to researchers of all methodologies:

> I think what qualitative research does best and most essentially is to describe key incidents in functionally relevant terms and place them in relations to the wider social context. ... In the research report the generic features of these incidents are highlighted with as much concrete detail as is necessary to make a statement of the relation of the instance to the pattern of the whole. The qualitative researcher's ability to pull out from field notes a key incident, link it to other incidents, phenomena and theoretical constructs, and write it up so that others can see the generic in the particular, the universals in the concrete, the relation between the part and whole may be the most important thing he does. It involves massive leaps of inference over many different kinds of data from different sources – field notes, documents, elicited texts, demographic information, unstructured interviews, and very possibly survey data. This is a decision process analogous to that of the historian or biographer deciding which incidents among many in a person's life to describe. (p. 61)

Erickson describes the way that 'key incidents' may be used to illustrate salient relations in a setting, to elucidate the 'generic in the particular' and even to tie together a whole account. But beginning researchers, often faced with an array of data, frequently find it difficult to appreciate the more critical issues and ideas within data or the ways that these ideas relate to other cases and situations. The more data they collect, the harder the task of making general connections may become. To make this idea more concrete, we turn now to a classic research study in mathematics education conducted by Guy Brousseau.

Brousseau (1984) performed 'clinical observations' of teachers and students to describe a phenomenon that has been generative for many other researchers (Artigue, 1999; Balacheff, 1990; Cobb, Wood, Yackel, & McNeal, 1992; Laborde, 1989). In a short chapter, tucked away in an obscure publication, Brousseau first offered his analysis of the 'didactical contract', (or *contrat didactique*, translated from his native French). As part of his analysis, Brousseau describes a scenario typical in mathematics classrooms: A teacher gives a student a

Reproduced with permission of the copyright owner. Further reproduction prohibited without permissio

mathematics problem that the student cannot immediately solve. The student asks for help, and the teacher takes the student through a series of questions. The teacher, in making the questions attainable for the student, empties the task of much of its cognitive demand, thereby reducing the opportunities for learning. The 'contract' that Brousseau describes conveys particular roles for teachers and students that reflect deeply held cultural assumptions about the work of teachers and students in classrooms. As part of this didactical contract, the teacher is expected to ask questions that students can answer and that will enable the student to develop new knowledge. The student is expected to answer the questions and learn. But Brousseau points out that this contract may be paradoxical, as the student who accepts it often fails to learn. Brousseau explains:

> The pupil ... is also confronted with a paradoxical injunction; if he accepts that the teacher, according to the contract, teaches him the results, he will not attain them himself and thus will not learn mathematics, i.e., he will not make mathematics his own. *If the pupil accepts the contract, he will refuse it – no learning takes place.* To learn, for him, implies [rejecting] the contract, and to accept being himself engaged in the problem. In fact learning will not be based on the correct functioning of the contract, but rather on breaching it.
>
> (Brousseau, 1984, p. 113, italics in original)

In subsequent years, the idea of the didactical contract, in which teachers and students tacitly collaborate to empty learning situations of their cognitive demand, has been used by numerous researchers to describe and understand interactions in mathematics and other subject classrooms. In the development of the idea of a didactical contract, Brousseau reflected upon one or more incidents he observed and regarded as critical and portrayed them in general terms. Krieger (1991) describes the social science tradition as being one of speaking "specifically in order to speak generally" (p. 4) and "understanding unique and particular phenomena in terms of more general processes" (p. 149). This move from the particular to the general, which is captured by Brousseau's work, is achieved through the identification and naming of a common phenomenon. In his clinical observations, Brousseau presumably witnessed many different interactions between students and teachers, but rather than describe each of them, he generalizes their features into a new and potent concept that can be used to view and understand the interactions of teachers and students in other classrooms. Developing and naming such a concept constitute a process that separates analysis from description. Often researchers describe a case without analysing what it is a case of, what makes it special or interesting, or how it relates to other situations and instances. But it is the extra thought that goes into such analysis that enables the generation of analytical and theoretical insights. Accounts that are purely descriptive cannot by themselves provoke the theoretical understanding of social phenomenon offered by accounts with conceptual density in which researchers look not only for instances in data but also

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission

for dimensions and categories that may be explained. Brousseau's didactical contract did not fall out of a standard research method, nor did it emerge from large amounts of data. Instead, it was the product of the practice of "understanding unique and particular phenomena in terms of more general processes" (Krieger, 1991, p. 149).

The practice of identifying and naming general explanatory ideas in research requires a range of capabilities. Naming a phenomenon involves considering what it is, what it is a general case of, and how it relates to other ideas in the literature. The process involves an understanding of the specifics of the context together with the ability to abstract from that context. Developing general ideas from specifics depends also on familiarity with other scholarship – within and outside the field of mathematics education – and of the theories that have been produced in the past that may inform the developing analyses being constructed. Skill with a variety of analytic methods and techniques, such as particular statistical procedures, coding, statistical analysis, textual analysis, and triangulation, can help as researchers can work in multiple ways with different kinds of information. Ideas and insights are also helped, although not produced, by methods of analysis. Statistical procedures, coding, and textual analysis are examples of methods that help researchers notice patterns in data that are invisible to mere inspection. Identifying a central idea requires a substantive perspective – appreciating that not all data are equally important. Indeed, experienced researchers expect and look for unevenness and discrepancies in data. Developing general ideas and concepts also requires particular dispositions and habits of mind. These include the desire to think deeply, to go beyond the surface features of an event to the connections that lie beneath, to wonder, to seek ideas and patterns that might offer a new lens or way of understanding the complexities of the particulars, their dynamics, and their variability. Another disposition is the act of creativity: the generation of ideas and names that open new ways of thinking. The knowledge and dispositions that lie within the practice we have described – of moving from the particular to the general – become more visible as one examines examples of this particular practice. Such close examination is one way that beginning researchers may learn about the practices of research.

Considering the Mathematics

Being a disciplined researcher in mathematics education means knowing when and how to take account of *mathematics* in investigating particular questions. In other words, how does the fact that the learning, the teaching, or the policy being examined is about *mathematics* figure in the ways in which data are collected or analysed? How does it affect the research question itself or the interpretation of the data? Consider the fact that studies of classroom interactions and dynamics can – and often do – proceed independently of the specific content being discussed. How turns are taken, how ideas are validated as correct, whose ideas are used and in what ways, and the role of the teacher – these are all

Reproduced with permission of the copyright owner. Further reproduction prohibited without permissio

elements of classroom life that researchers seek to understand. But when does it matter that the subject under discussion is mathematics and not literature? And, even more specifically, when does it matter that it is algebra and not probability? Researchers in mathematics education must be sensitive to when the 'subject matters' (Stodolsky, 1988) and when more general issues are more centrally at play. When the mathematics matters, disciplined research requires researchers to know and use mathematics in ways specific to the tasks of systematic inquiry. Knowing mathematics for doing research involves knowledge of specific ideas and practices: the importance of the unit in fractions or what constitutes a mathematical argument, for instance. It also involves sensibilities about what is important mathematically: the role of a particular term, for example, or the need to examine whether and how two representations are equivalent. This sort of mathematical appreciation and understanding is important to being able to make judgments about where to focus on a specific content issue and where to stand back and consider other important elements less focused on the content.

What are some of the specific practices in which mathematics education researchers engage when focusing on mathematics or seeking to uncover the place of the subject matter in their investigations? One such practice is the strategic seeking and use of comparisons and contrasts across disciplines. Stodolsky, in her 1988 book *The Subject Matters*, investigated patterns of instruction in upper elementary classrooms and uncovered sharp differences in those patterns as a function of the subject matter. She continued this work in later collaborations with Pamela Grossman in which the researchers compared teachers' professional interactions and practice in different high school subject matter departments. That life in mathematics departments was different from the professional environment and practice in other subjects was only revealed because of the researchers' explicit and strategic attention to comparisons and contrasts.

In the work on pedagogical content knowledge that began in the mid-1980s at Stanford University, Shulman and his colleagues systematically promoted cross-subject comparisons as a tool for clarifying the issues involved in this new research domain. Some researchers studied the knowledge involved in teaching history and other social studies (e.g., Wilson & Wineburg, 1988), while others investigated the teaching of English (e.g., Grossman, 1990) or science (Carlsen, in press; Hashweh, 1985/1986, 1987). Work on the mathematics knowledge needed for teaching (e.g., Ball, 1988, 1990, 1991; Leinhardt & Smith, 1985; Marks, 1990; Steinberg, Haymore, & Marks, 1985) profited from direct interaction with these related studies in neighbouring subject matters. That one of the dimensions of subject matter knowledge for teaching might be knowledge of the epistemology of the field – how ideas are validated, for instance – was stimulated and advanced by work done on this dimension in history, English, biology, and physics. Across the content areas, many of the researchers drew on ideas developed by Schwab (1958, 1961/1978), and that common theoretical foundation supported productive comparative analyses. When those researchers studying English and history argued that knowledge about different theoretical orientations to the

eproduced with permission of the copyright owner. Further reproduction prohibited without permissi

field (Schwab's, 1961/1978, *substantive structures of a discipline*) was a component of a teacher's pedagogical content knowledge, those studying mathematical knowledge came up short, for no clear correspondence existed in mathematics. This finding, too, was helpful in advancing the particular work on mathematical knowledge for teaching, pressing scholars to notice new things specific to mathematics – for example, that the epistemological demands for teaching varied across mathematical domains – from number theory to discrete mathematics, for example. This brief sketch of one domain of mathematics education research – work on teachers' mathematics subject matter knowledge – illustrates the productive use of cross-subject comparisons as a means to develop with precision certain ideas within mathematics education.

A second way in which attention to the specifics of how the subject matters can be realized in mathematics education research is by using mathematical lenses to analyse data on learning, teaching, policy, and implementation. Researchers therefore must know mathematics in ways that enable such uses in their research. Consider, for example, work on equity in mathematics education. A large body of work exists on the differentiated opportunities to learn and outcomes often produced in school. Researchers who study the dynamics of these outcomes sometimes draw exclusively on sociological, economic, anthropological, psychological, and political perspectives in developing their theoretical frames and methods of analysis. But how students and teachers interact in school and the ways in which opportunities to learn are constructed is fundamentally a matter of *what* they are interacting about and the different histories and experiences that they bring to their joint work. That the subject under discussion is mathematics has an impact on students' presentation of self, their teachers' 'reading' of them, and the unfolding of students' consequent opportunities to learn. Probing this phenomenon requires the researcher to know mathematics in ways that enable analyses of curriculum and classroom dynamics that would otherwise fall to the background (e.g., Boaler, 2002a, 2002b; Lubienski, 2000). For example, in investigations of the demands of mathematical discourse in classroom, the precision of the research benefits from an understanding of the role of error analysis in mathematics, of how claims are proved and disproved, and of how tentative ideas are put forth, used, and improved. What does engagement in mathematical argument require of students and their teachers, and how might that differ for different students? Without this understanding, scholars are likely to misinterpret or misdirect their analysis of the course of a particular student interaction in class. Similarly, in applying mathematics to and using it in specific contexts, students may encounter differentially the interface between their knowledge and experience from outside school and what is called for in school. A deep understanding of the role of abstraction and generalization, as well as of localization and application, can help ground studies of students' engagement in school better in the specific entailments of mathematical work. This sort of analysis and perspective implies that researchers need a flexible and unpacked understanding of mathematics, one that can be readily fashioned for

Reproduced with permission of the copyright owner. Further reproduction prohibited without permissio

use in looking at students' work, listening to their talk, and observing their teachers' moves.

Consider, for example, the development of research tools for probing teachers' or students' learning, or for analysing policy documents or curriculum materials. To design and refine instruments such as student interview protocols, teacher surveys, or document analysis tools, researchers would need a refined understanding of the content, one that permits them to isolate key junctures and elements strategic for observation, measurement, probing, and attention. Information collected in the pursuit of a research question is only as good as the insights that undergird the data-collection design. To develop good insights, researchers need to understand and be able to use the subject in ways not so different from the demands of mathematical knowledge for teaching (Ball & Bass, 2000). This observation suggests an agenda for the development of researchers' skills and capacities that has typically been addressed through the mathematics requirements for the doctoral degree, but these are rarely determined on the basis of an analysis of the mathematical knowledge entailments of research.[1]

Communication of Research

"Research begins in wonder and curiosity but ends in teaching. The process of research is incomplete until the researcher can communicate his or her understandings clearly, persuasively, and effectively" (Shulman, 1997, p. 6). The teaching to which Shulman refers – communicating about research – is the process through which research may become useful, informing the practice and theory of mathematics education. Communication transforms the researchers' amalgam of descriptions, data, and analyses into knowledge, for acts such as writing or drawing diagrams do not just represent ideas that have previously been formed. Communication is a generative act that itself *produces* ideas. Indeed, it could be argued that there is no more critical aspect of the research process. Data, whether they are sets of numbers or reflections from interviews, are the raw constituents of research, but it is the act of representation that transforms data into ideas, and it is in this act of communication that ideas are formed. Communicating ideas, whether through written text or video records, is an act of knowledge generation – of higher order thinking. It is central to the work of disciplined inquiry, and yet many novice researchers have a limited sense of and interest in the act of communication. Although their training may offer some direct guidance about communication, the breadth of what is involved in the act is rarely the subject of explicit analysis and opportunity to learn.

Myriad purposes exist for communicating about research – to different audiences and with different goals. For example, researchers might offer evidence about the viability of an intervention to help those watching it take root improve it and also have the patience to let it develop more fully. Researchers might offer evidence on the effects of maturely developed approaches to provide insight about alternative approaches or regimes. Or researchers might offer conceptual

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission

tools for reframing a problem or understanding a question better. Each of these offerings represents a different agenda and requires different skills, methods, modes of presentation, and framing. Being clear about purposes and methods of communication can help make communication more effective, and research therefore also more broadly useful.

Traditionally, mathematics education research has been communicated through written media: in particular, research articles, books, chapters, essays, technical reports, commentaries, and editorials. These are important venues for communication, but as researchers strive more actively to cross the traditional divide between research and practice, they are turning to newer and more varied forms of communication, including the use of cases, classroom video records, and the Internet. Deciding on the nature of the medium through which research knowledge will be communicated is an important aspect of the communication process to which we pay attention in this section. But the practice of communication is varied, with many entailments. Once the medium has been decided on, researchers need to decide which information should be conveyed. They need to consider language that is particularly appropriate for the audience and construct ideas in such a way that they are logically connected, believable, and interesting. Writing is a critical constituent of the communication process, and we turn now to consider this crucial practice and the knowledge, skills, and sensibilities it entails before moving to examine and unpack two other aspects of communication, as well as two particularly interesting examples that exemplify their use.

Writing

Jean-Paul Sartre, at 70, said, "I still think, but because writing has become impossible for me the real activity of thought has in some way been repressed" (cited in Ely, Vinz, Downing, & Anzul, 1997, p. 14). Writing is not merely an act of representing ideas but also of generating them. It is a process of thinking and an act of creation. It can be exhilarating in one minute, frustrating the next. Tom Robbins once described it as "a cross between flying to the moon and taking a shower in a motel", capturing both the magical and the mundane. But for all its mystery and magic, the act of good writing may be taught and learned. It requires a range of knowledge, sensibilities, and skills that may be examined and explicitly encouraged among new researchers. As researchers write, they must consider what their intended audience knows, cares about, believes, and assumes. The writer's job is to capture the essence of her or his work and to convey it with meaning and with accuracy. Writers need to consider how much information they need to portray, providing sufficient details for readers to decide upon its validity. A writer seeks to convey with care the relationship between fact, analysis, truth, and opinion. Shaping the ideas into forms comprehensible to the reader requires the researcher to have a firm grip on the ideas and be able to represent them flexibly. The writer must consider the language that will be accessible to readers as well as the forms of representation that will convey the greatest meaning to particular audiences. Writing entails a fluent use of language at the word, sentence, and text level, as well as the capacity to make

Reproduced with permission of the copyright owner. Further reproduction prohibited without permissio

a set of ideas accessible using linguistic and rhetorical tools. Shulman (1999) points out that the communication of research cannot mean the retelling of all that was discovered, for "research is about the compressions, transformations and reconstitutions of experience into forms that can be readily stored, displayed, and exchanged" (p. 161). These and other aspects of the writing process may be unpacked and examined so that different opportunities for analysis and learning will be created.

One set of skills is entailed by conventional research writing, when researchers strive to meet the standards of their own disciplinary community and to publish in the journals closest to their own work: mathematics education research journals, for example. Other skills and sensibilities are involved when the purpose is to write for a broad public audience, who share neither the theoretical perspectives nor the language commonly used to express the ideas. The need for communication that crosses into different communities is increasing, for there are new and more demanding calls for researchers to communicate their findings more broadly and to make serious attempts to affect practice. In the next section, we look more closely at broad communicative forms, drawing upon a particular example of two researchers who set out to cross the traditional research-practice divide and were extremely successful in doing so because of the broad forms of communication they employed.

Communicating research in different media

Researchers are often called upon to do more than present research findings to other researchers; they frequently must aim to cross the traditional and enduring divide between educational research and practice, or the one between research and policy. This aim raises new issues about research communication as a domain of skilled practice. Scientific research writing has its own standards and demands, but as researchers write for broader audiences – through newspapers, books, and professional journals – they will need to synthesize and animate their ideas in different ways. They may need to bring more life into their ideas, as they cannot rely upon a shared professional interest in their importance; they may need to convince in different ways, to draw upon different examples, and to arrange the knowledge in more concise forms without losing the integrity or meaning of the research. But this work, whilst unacknowledged in many universities and difficult to achieve, is important, as it frequently serves as the main vehicle by which research ideas are transported into the worlds of educational practitioners.

Two researchers in the United Kingdom – Paul Black and Dylan Wiliam – noted that countries throughout the world were paying significant attention to issues of standards and accountability whilst largely ignoring the most important aspect of learning: the work that takes place in classrooms. They focused upon formative assessment, a process by which teachers use information that they gain from classroom activities as evidence of student learning, which they then use to adapt their teaching. Black and Wiliam asked whether there was evidence that an improvement in formative assessment would raise standards and whether

eproduced with permission of the copyright owner. Further reproduction prohibited without permissi

there was evidence about ways to improve formative assessments. Rather than conducting research to answer these questions, they conducted an extensive survey of research literature, including nine years' worth of more than 160 journals. This survey yielded approximately 580 articles to study. The answer to both of the questions was yes, and the evidence that they gained from this review was overwhelming. The evidence suggested that an implementation of formative assessment in classrooms would increase learning to such a degree that a country scoring in the middle of the pack in international comparisons would rise to a place in the top five. Black and Wiliam also found that formative assessment helped low achievers more than high achievers, thus reducing the range of achievement whilst increasing achievement overall. The evidence for these claims was detailed and complex, and the initial communication of the results was in a research article in *Assessment in Education* that, along with six short responses, comprised an entire edition of the journal (Black & Wiliam, 1998a). But the authors believed that the results they had found were important for practitioners to know about and to understand, so they engaged in a number of other forms of communication that extended beyond the traditional writing of journal articles.

One of these forms of communication was an article in the professional journal *Phi Delta Kappan* (Black & Wiliam, 1998b). This journal is published in the United States and read by a large professional audience. The authors needed to communicate their findings convincingly and clearly in a much reduced space and to an audience largely unfamiliar with the jargon of educational research or technical assessment. The authors noted, "Because we are presenting a summary here, our text will appear strong on assertions and weak on the details of their justification. We maintain that these assertions are backed by evidence and that this backing is set out in full detail in the lengthy review on which this article is founded" (p. 140). Black and Wiliam produced a powerful and convincing article, absent of most of the details, illustrating the "compressions, transformations and reconstitutions" that Shulman (1999, p. 161) notes as central to the process of communication.

Black and Wiliam also took a further, highly significant step by arranging for their university to publish a small booklet communicating the main ideas of their study, specifically written for teachers and sold for two British pounds (approximately US$3). The authors also gave a series of presentations on the ideas. In each of these presentations, they combined a summary of research evidence with practical ideas to use in classrooms. News of the importance of the ideas in the booklet and presentation spread throughout the UK, and soon they had sold 20,000 copies, an unprecedented sale of an educational publication (approximately one copy for every school in the country). Black and Wiliam were asked to make hundreds of presentations and in two years addressed over 400 groups of teachers, advisors, and policy makers, making contact with over 20,000 people. Research presentations are a particular form of teaching, and they require substantial design and rehearsal. Whereas there are established

Reproduced with permission of the copyright owner. Further reproduction prohibited without permissio

norms for written communication, little guidance exists for presentations, sessions, or talks, and learning to do these well is yet another part of the practice of research.

Black and Wiliam also worked closely with government agencies, helping their findings cross the divide between research and policy. They communicated their ideas through the phrase 'assessment for learning', which became a key feature of the Framework for the Inspection of Schools in the UK, an important forum for policy advice given to schools. The varied ways in which Black and Wiliam's results influenced policy and practice would not have been achieved if they had limited their communication to a journal article, and their careful use of forms of media that directly addressed teachers and other educational practitioners, with appropriate writing, use of examples, and summaries of evidence, provides an important illustration of the role of well-crafted communication practices in the transportation of research ideas.

New possibilities for communication

The media of the research enterprise have also been changing. New possibilities exist for representing ideas and engaging people in them, making for better communication and for questions about the ways new technologies may be employed to cross the divide between research and practice. Pea (1999) asserts that new forms of representation, made possible by new communication channels such as the Internet, have the potential to change "fundamentally how education research is conceived, conducted, authored, and critically responded to by its audiences" (p. 336). Such changes are evident in the intense and frequent dialogue about research that is taking place on the World Wide Web and through electronic mail.

One researcher who has grappled with the potential of alternative media for communicating about research is Magdalene Lampert. Lampert, who studies the dynamics of classroom teaching of mathematics and who has contributed exquisitely written articles about the challenges of teaching (Lampert, 1990, 1992), became fascinated with the problem of representing practice. Teaching and learning, which are complex and interactive over time, defy typical forms of representation. Many representations of teaching compress and omit crucial elements of the work for teachers and students: for example, the multiplicity of foci, the ways in which time functions to shape the work, and the kinds of local knowledge and ways of knowing on which teachers' and students' moment-to-moment moves rely. The problem of representation became a direct object of inquiry for Lampert. Drawing on records of her own practice across an entire school year, she explored what might be involved in representing aspects of practice more adequately, paying attention to the numerous considerations teachers take from moment to moment. She strove to represent the complexity of teaching: for example, that teachers work simultaneously with individuals and with groups of students, and that learning occurs over time. She considered varieties of representation and communication that would capture the multiple and layered dimensions of the work. Framing her research problem as that of

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission

representing the fundamental unit of practice – the school year – Lampert used multiple media and forms of graphical representation along with text different from classic research writing. Lampert's resulting book (2001) on the problems of teaching illuminates in unprecedented ways what teaching and learning entail, and it also creates a new image of what it might mean to represent – and thus, communicate about – practice. With chapters on different slices of the work – teaching while leading a discussion, teaching students to be people who study in school, teaching with problems, for instance – the book combines diagrams, transcripts, scanned excerpts from the students' notebooks and the teacher's notes, reproductions of the chalkboard across a single class discussion, and a panoply of other images that contribute to representing the threads that weave together dynamically through time and through complex human interactions and relationships to produce the ordinary work we call classroom teaching. What her book achieves is to make vivid the ways in which teaching is far from ordinary work, and it contributes to scholars' and practitioners' grasp of the elements that contribute to the complexity of teaching and that make it possible to do.

But new forms of communication also raise new challenges. Records of practice, for example, may enable new insights into mathematics teaching and learning (Lampert & Ball, 1998, 1999) and can offer material for vivid communication, but researchers must exercise new cautions, for primary records such as videotapes of lessons can be interpreted in multiple ways, and their very potency may also mislead. How, for example, does a researcher responsibly select a representative sample from the data set, and what does it mean to be 'representative'? If the data being shown are not representative, how might researchers provide a useful context for the specific examples being examined? The availability of the potent media of videotapes and other records of practice means that there are new demands on scholars and therefore on beginning researchers.

The practice of making research knowledge accessible, interesting, and useful includes a number of constitutive elements that, like many other aspects of the research process we have considered, are largely invisible to new researchers. We have considered three aspects of the communication process in this section by way of illustration and contend that greater opportunities for learning these practices will serve well the development of mathematics education researchers, as well as the broader education community.

LEARNING IN AND FOR THE PRACTICE OF RESEARCH

Oddly, although research is something researchers *do*, little research preparation seems explicitly designed to develop the complex uses of knowledge, skill, and habits of mind fundamental to the practice of research. In most countries, graduate programs supply the professional training thought to be needed for research (Batanero et al., 1994). The research training that students receive is often made up of two components: (a) an introduction to knowledge of mathematics education and different research methods and approaches; and (b) an

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission

introduction to research, at which time in the pursuit of their own research questions, students receive opportunities to use and apply the knowledge to which they have been introduced. Required to conduct a study to earn the degree, beginning researchers seek to demonstrate competence as they complete what is often their first independent research in the form of a dissertation.

Despite the fact that research entails a complex set of practices, most research preparation programs present knowledge of research methods and traditions, separate from the act of conducting research. Thus, beginning researchers in mathematics education generally take courses on research methods as part of a masters or doctoral degree program and then conduct research within a doctoral research study (Batanero et al., 1994). This model of teaching and learning, in which knowledge that is taught is assumed to be available for later application and use, has been questioned in mathematics education and in teacher education, yet it prevails in the corridors of higher education with surprising tenacity.

The absence of opportunities to develop understanding and skill as well as flexibility and reasoning with core practices of research means that new researchers often begin major research studies ill-prepared for the complex demands of the work that they will face. Reading research, doing exercises and assignments in research methods texts, and writing short papers cannot by themselves prepare beginners for the complex practice of disciplined inquiry. A more explicit, purposeful focus on research practices, and the knowledge and habits of mind they require, could provide students with opportunities to learn about research in a deeper and more substantive way in the early stages of their work.

Following this path would entail identifying and unpacking the central domains of the work of disciplined inquiry, a process that we have started by way of illustration in this chapter. But it would also require the design of a curriculum of research preparation replete with opportunities for novice and experienced practitioners to learn to engage in those domains of work. We have not attempted to design such a curriculum as part of this chapter, because to do so would be overly ambitious and necessarily speculative. But such work is being attempted to varying degrees in different institutions across the world.

Our main aim in this chapter has been to raise the possibility that future years may profitably witness ways in which research institutions may offer new researchers experiences, tasks, assignments, and materials that afford opportunities to learn in, from, and for practice. For example, beginning researchers might shadow experienced researchers, observing and querying the current stage of the work, reading rough drafts of papers, and sitting in on group meetings as instruments and analyses are scrutinized and critiqued. Such experiences could function, if designed, as practica in research, and different practices could be in focus at different times. Records of practice could also be created for close scrutiny and opportunities to engage in 'fragments of inquiry' (Schwab, 1978). Possible examples might be drafts of research proposals with critiques, videotapes of conference presentations and discussants' comments, successive diagrams as an idea develops, videotapes of a research team working on an idea or designing a study, researchers' lab notes or analytic memos, and a complete historical

eproduced with permission of the copyright owner. Further reproduction prohibited without permissi

record of a particular paper. Beginning researchers might engage in small slices of the work: framing alternative research problems and comparing their merits and problems, writing a précis of an argument, making alternative representations of data, and investigating what each form makes visible.

Many more activities organically connected to the core practices of research could be developed. In future years, a research curriculum could be centred more directly in the critical activities of the work and include opportunities to develop the practices central to that work, including the knowledge and dispositions that undergird skilled design, analysis, interpretation, and communication in disciplined inquiry. Learning any complex practice requires opportunities to unpack its components in order to see what underlies competent performance. In this chapter, we have unpacked some of the practices of mathematics education research in the hope that our writing will inform future dialogue, work, and teaching of the practices of mathematics education research.

NOTE

[1] We propose a parallel here between current research on the mathematics knowledge required for teaching (Ball & Bass, 2000) and the mathematical preparation of mathematics education researchers.

REFERENCES

Artigue, M. (1999). The teaching and learning of mathematics at the university level: Crucial questions for contemporary research in education. *Notices of the AMS, 46*(11), 1377–1385.

Balacheff, N. (1990). Towards a *problematique* for research on mathematics teaching. *Journal for Research in Mathematics Education, 21*, 258–272.

Ball, D. L. (1988). *Knowledge and reasoning in mathematical pedagogy: Examining what prospective teachers bring to teacher education.* Unpublished doctoral dissertation, Michigan State University, East Lansing.

Ball, D. L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *Elementary School Journal, 90*, 449–466.

Ball, D. L. (1991). Research on teaching mathematics: Making subject matter part of the equation. In J. Brophy (Ed.), *Advances in research on teaching* (Vol. 2, pp. 1–48). Greenwich, CT: JAI Press.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on the teaching and learning of mathematics* (pp. 83–104). Westport, CT: Ablex.

Ball, D. L., & Cohen, D. (1999). Developing practice, developing practitioners. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (1st ed.). San Francisco, CA: Jossey Bass.

Ball, S. J. (1995). Intellectuals or technicians? The urgent role of theory in educational studies. *British Journal of Educational Studies, 43*(3), 255–271.

Batanero, M. C., Godino, J. D., Steiner, H. G., & Wenzelburger, E. (1994). The training of researchers in mathematics education: Results from an international survey. *Educational Studies in Mathematics, 26*, 95–102.

Bernstein, B. (1966). Sources of consensus and disaffection in education. *Journal of the Association of Assistant Mistresses, 17*(1), 4–11.

Bernstein, B. (1996). *Pedagogy, symbolic control and identity: Theory, research, critique.* London: Taylor & Francis.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission

Black, P. J., & Wiliam, D. (1998a). Assessment and classroom learning. *Assessment in Education, 5*(1), 7–74.

Black, P., & Wiliam, D. (1998b, October). Inside the black box: Raising standards through classroom assessment. *Phi Delta Kappan*, 139–148.

Boaler, J. (2000). Intricacies of knowledge, practice, and theory. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 1–18). Westport, CT: Ablex.

Boaler, J. (2002a). Learning from teaching: Exploring the relationship between 'reform' curriculum and equity. *Journal for Research in Mathematics Education, 33*(4), 239–258.

Boaler, J. (2002b). *Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student learning*. Mahwah, NJ: Erlbaum.

Brousseau, G. (1984). The crucial role of the didactical contract in the analysis and construction of situations in teaching and learning mathematics. In H. G. Steiner (Ed.), *Theory of mathematics education* (pp. 110–119). Bielefeld Germany: Institut für Didactik der Mathematik der Universität Bielefeld.

Brown, A., & Dowling, P. (1998). *Doing research/reading research: A mode of interrogation for education*. London: Falmer.

Carlsen, W. S. (in press). Domains of teacher knowledge. In J. Gess-Newsome & N. G. Lederman (Eds.), *Examining pedagogical content knowledge: The construct and its implications for science education*. Dordrecht: Kluwer.

Cheung, K. C. (1988). Outcomes of schooling: Mathematics achievement and attitudes towards mathematics learning in Hong Kong. *Educational Studies in Mathematics, 19*, 209–219.

Cobb, P., Wood, T., Yackel, E., & McNeal, B. (1992). Characteristics of classroom mathematics traditions: An interactional analysis. *American Educational Research Journal, 29*(3), 573–604.

Cobb, P. (2000). The importance of a situated view of learning to the design of research and instruction. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 45–82). Westport, CT: Ablex.

Cronbach, L. J., & Suppes, P. (Eds.). (1969). *Research for tomorrow's schools: Disciplined inquiry for education*. New York: Macmillan.

Dossey, J., & Lappan, G. (2000). The mathematical education of mathematics educators in doctoral programs in mathematics education. In R. E. Reys & J. Kilpatrick (Eds.), *One field, many paths: U.S. doctoral programs in mathematics education* (CBMS Issues in Mathematics Education, Vol. 9, pp. 67–72). Providence, RI: American Mathematical Society.

Doyle, W. (1983). Academic work. *Review of Educational Research, 53*(2), 159–199.

Doyle, W. (1988). Work in mathematics classes: The context of students' thinking during instruction. *Educational Psychologist, 23*, 167–180.

Ely, M., Vinz, R., Anzul, M., & Downing, M. (1997). *On writing qualitative research: Living by words*. London: Falmer.

Ensor, P. (2001). From preservice mathematics teacher education to beginning teaching: A study in recontextualizing. *Journal for Research in Mathematics Education, 32*(3), 296–320.

Erikson, F. (1977). Some approaches to inquiry in school community ethnography. *Anthropology and Education, 8*(2), 58–62.

Grossman, P. (1990). *The making of a teacher*. New York: Teachers College Press.

Hashweh, M. Z. (1986). An exploratory study of teacher knowledge and teaching: The effects of science teachers' knowledge of subject matter and their conceptions of learning on their teaching (Doctoral dissertation, Stanford University, 1985). *Dissertation Abstracts International, 46*, 3672A.

Hashweh, M. (1987). Effects of subject matter knowledge in the teaching of biology and physics. *Teaching and Teacher Education, 3*(2), 109–120.

Hiebert, J., & Wearne, D. (1993). Interactional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal, 30*(2), 393–425.

Krieger, S. (1991). *Social science and the self: Personal essays on an art form*. New Brunswick, NJ: Rutgers University Press.

Laborde, L. (1989). Audacity and reason: French research in mathematics education. *For the Learning of Mathematics, 9*(3), 31–36.

roduced with permission of the copyright owner. Further reproduction prohibited without permiss

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*, 29–63.

Lampert, M. (1992). Practices and problems in teaching authentic mathematics. In F. Oser, D. Andreas & J. Patry (Eds.), *Effective and responsible teaching: The new synthesis* (pp. 295–314). San Francisco: Jossey Bass.

Lampert, M. (2001). *Teaching problems and the problems of teaching*. Ann Arbor, CT: Yale University Press.

Lampert, M., & Ball, D. (1998). *Teaching, multimedia, and mathematics: Investigations of real practice*. New York: Teachers College Press.

Lampert, M., & Ball, D. (1999). Aligning teacher education with contemporary K-12 reform visions. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 33–53). San Francisco: Jossey-Bass.

Leinhardt, G., & Smith, D. (1985). Expertise in mathematics instruction: Subject matter knowledge. *Journal of Educational Psychology, 77*, 247–271.

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 19–44). Westport, CT: Ablex.

Lester, F. K., Jr., & Carpenter, T. (2001). The research preparation of doctoral students in mathematics education. In R. E. Reys & J. Kilpatrick (Eds.), *One field, many paths: U.S. doctoral programs in mathematics education* (CBMS Issues in Mathematics Education, Vol. 9, pp. 63–66). Providence, RI: American Mathematical Society.

Lester, F. K., Jr., & Lambdin, D. V. (in press). From amateur to professional: The emergence and maturation of the U.S. mathematics education research community. In G. M. A. Stanic & J. Kilpatrick (Eds.). *A history of school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Lester, F. K., Jr., & Wiliam, D. (2002). On the purpose of mathematics education research: Making productive contributions to policy and practice. In L. D. English (Ed.), *Handbook of international research in mathematics education* (pp. 489–506). Mahwah, NJ: Erlbaum.

Lingefjärd, T., & Dahland, G. (Eds.). (1998). *Research in mathematics education: A report from a follow-up conference after PME 1997*. Gothenburg, Sweden: Gothenburg University.

Lubienski, S. (2000). Problem solving as a means towards mathematics for all: An exploratory look through the class lens. *Journal for Research in Mathematics Education, 31*(4), 454–482.

McIntosh, J. A., & Crosswhite, F. J. (1973). *A survey of doctoral programs in mathematics education*. Columbus, OH: ERIC Information Analysis Center for Science, Mathematics and Environmental Education.

Marks, R. (1990). Pedagogical content knowledge: From a mathematical case to a modified conception. *Journal of Teacher Education, 41*(3), 3–11.

Metz, M. (2001). Intellectual border crossing in graduate school. *Educational Researcher, 30*(5), 12–18.

Miles, M., & Huberman, M. (1994). *Qualitative data analysis: An expanded sourcebook*. Thousand Oaks, CA: Sage.

National Academy of Education. (1999). *Recommendations regarding research priorities: An advisory report to the National Educational Research Policy and Priorities Board*. New York: Author.

Page, R. (2001). Reshaping graduate preparation in educational research methods: One school's experience. *Educational Researcher, 30*(5), 19–25.

Pallas, A. (2001). Preparing education doctoral students for epistemological diversity. *Educational Researcher, 30*(5), 6–11.

Pea, R. (1999). New Media Communications Forums for Improving Education Research and Practice. In E. C. Lagemann & L. S. Shulman (Eds.). Issues in Education Research: Problems and Possibilities. Jossey Bass: San Francisco. (pp. 336–370).

Pickering, A. (1995). *The mangle of practice: Time, agency, and science*. Chicago: University of Chicago Press.

Reys, R., & Kilpatrick, J. (Eds.) (2000). *One field, many paths: U.S. doctoral programs in mathematics education* (CBMS Issues in Mathematics Education, Vol. 9). Providence, RI: American Mathematical Society.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission

Ruthven, K. (1990). The influence of graphic calculator use on translation from graphic to symbolic form. *Educational Studies in Mathematics, 21*, 431–450.

Schoenfeld, A. (1999). The core, the canon, and the development of research skills: Issues in the preparation of education researchers. In E. Lagemann & L. S. Shulman (Eds.), *Issues in education research: Problems and possibilities* (pp. 166–202). San Francisco, CA: Jossey-Bass.

Schwab, J. J. (1958). Inquiry and the reading process. *Journal of General Education, 11*, 72–82.

Schwab, J. J. (1978). Education and the structure of the disciplines. In I. Westbury & N. J. Wilkof (Eds.), *Science, curriculum, and liberal education: Selected essays of Joseph J. Schwab* (pp. 229–272). Chicago: University of Chicago Press. (Original work published 1961)

Shulman, L. S. (1997). Disciplines of inquiry in education: A new overview. In R. M. Jaeger (Ed.), *Complementary methods for research in education* (pp. 3–19). Washington DC: American Education Research Association.

Shulman, L. S. (1999). Professing educational scholarship. In E. C. Lagemann & L. S. Shulman (Eds.), *Issues in education research: problems and possibilities* (pp. 159–165). San Francisco: Jossey-Bass.

Sierpinska, A., & Kilpatrick, J. (Eds.) (1998). *Mathematics education as a research domain: A search for identity. An ICMI Study*. Dordrecht, The Netherlands: Kluwer.

Stein, M. K., Grover, B., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*, 455–488.

Steinberg, R., Haymore, J., & Marks, R. (1985, April). *Teachers' knowledge and content structuring in mathematics*. Paper presented at the annual meeting of the American Educational Research Association, Chicago.

Stigler, J., & Hiebert, J. (1997, September). Understanding and improving mathematics instruction. *Phi Delta Kappan*, 14–21.

Stigler, J., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York: Free Press.

Stodolsky, S. S. (1988). *The subject matters: Classroom activity in math and social studies*. Chicago: University of Chicago Press.

Strauss, A. L. (1987). *Qualitative analysis for social scientists*. Cambridge: Cambridge University Press.

Wilson, S. M., & Wineburg, S. S. (1988). Peering at history through different lenses: The role of disciplinary perspectives in teaching history. *Teachers College Record, 89*, 525–539.

Zeichner, K. M., & Tabachnik, B. R. (1981). Are the effects of university teacher education 'washed out' by school experience? *Journal of Teacher Education, 32*(3), 7–11.

roduced with permission of the copyright owner. Further reproduction prohibited without permiss