Using Mathematical Knowledge for Teaching: Putting Equity into the Equation



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Cleveland, OH

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$$\begin{array}{r} 28 \\ \times\ 6 \\ \hline 368 \end{array}$$



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Overview of Session

1. Foundations: Major problems of mathematics education, and the need to focus on instruction
2. Mathematical knowledge for teaching: A theory in progress, and recent developments
3. Using mathematical knowledge for teaching in attending to equity

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1. Foundations: Major Problems of Mathematics Education, the Need to Focus on Instruction

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What are Central Problems of Mathematics Education?

- Ineffective curriculum: Too many students not learning mathematics well enough for
  - Continued mathematics study
  - Use in everyday settings
  - Workplace
- Pervasive inequality: Unequal distribution of mathematical success by race, social class
- Lack of capacity for improvement
  - Public understanding, support, investment
  - Teacher shortages, teacher development and support
  - Teacher educator development and resources

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Common Remedies for Problems of Mathematics Education

- Raise teachers' salaries
- Increase schools' performance accountability
- Build better curriculum
- Import successful practice
- Recruit smarter people into teaching

. . . but -- these may not improve equitable access to high quality learning of mathematics for every student

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A Necessary Strategy for Improvement: Focus on Teaching Practice and the Resources Needed to Produce Effective Instruction

Develop, test, implement, and study:

- Effective instructional approaches
- Resources needed for implementation in a variety of settings, especially urban and poor schools
- First-rate professional education for all teachers

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One Component: Teachers' Knowledge of Mathematics

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Common Framing of the Problem
→ Failure to Solve the Problem

- Broad criticisms of teachers for not knowing enough mathematics (evidence and causes vary)
- Concentration of under-qualified teachers in urban and high poverty schools

- Increase number and rigor of mathematics requirements for teacher preparation
- Recruit mathematically-trained people into teaching

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Clarifying the Problem

The quality of mathematics teaching and learning

Teachers' knowledge of mathematics

and their ability to use it in teaching

What mathematical knowledge do teachers need in order to teach mathematics all students effectively?

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2. Mathematical Knowledge for Teaching: A Theory in Progress, and Recent Developments

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Mathematical Knowledge for Teaching

What do we mean when we use this term, "mathematical knowledge for teaching"?

- Mathematical knowledge, skill, habits of mind that are entailed by the work of teaching

What do we mean by the "work of teaching"?

- The tasks in which teachers engage, and the responsibilities they have, to teach mathematics, both inside and outside of the classroom

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Mathematical Knowledge for Teaching

- A practice-based approach to asking about mathematical knowledge for teaching:
  - Working "backwards" by studying the work of teaching and scrutinizing the mathematical demands of that work
- Produces the realization that issues of **equity** and **mathematical knowledge for teaching** are fundamentally intertwined

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What *is* "Mathematical Knowledge for Teaching"? An Example from Division with Fractions

Divide:

$$2 \div \frac{2}{3}$$

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Analyzing Incorrect Answers for $2 \div \frac{2}{3}$

(a) $2 \div \frac{2}{3} = 1\frac{1}{3}$

(b) $2 \div \frac{2}{3} = \frac{2}{6}$

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Analyzing Correct Answers for $2 \div \frac{2}{3}$

(a)

"We don't need to invert and multiply!"

$$2 \div \frac{2}{3} = \frac{2}{1} \div \frac{2}{3} = \frac{1}{1/3} = 3$$

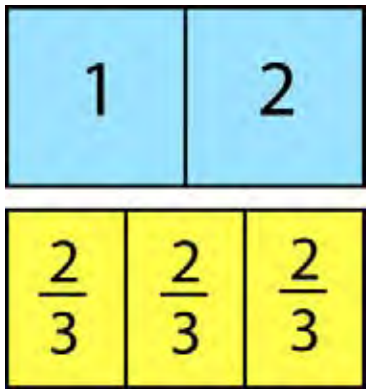
(b)

$$\text{"}2 \div \frac{2}{3} = 3 \text{ because 2 is two-thirds of 3."}$$

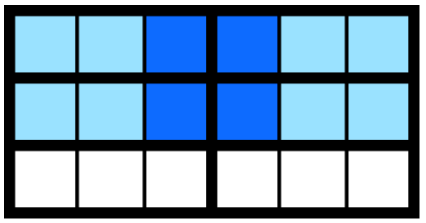
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Using Representations for $2 \div \frac{2}{3}$

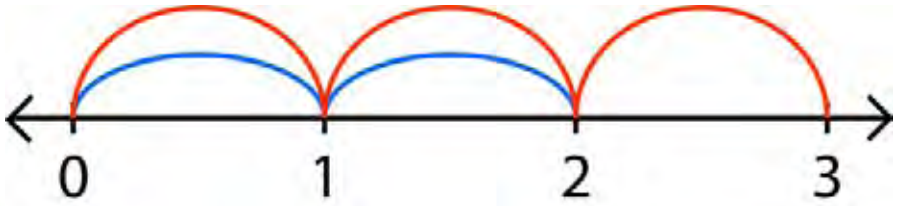
(a)



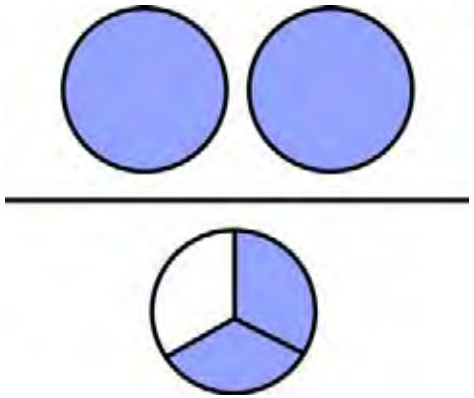
(b)



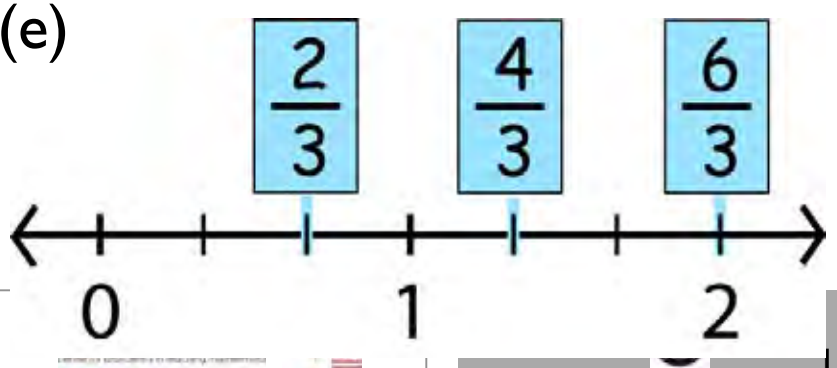
(c)



(d)



(e)



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Developing and Using Valid Measures of Mathematical Knowledge of Teaching

- Interdisciplinary item development
- Piloting and analysis
  - Discriminating ***common content knowledge*** from ***specialized content knowledge***
- Cognitive tracing interviews with teachers, laypeople, mathematicians
  - Solving mathematical problems involving specialized content knowledge not always obvious or easy for mathematicians
- Content validity checks
- Examining videotapes of classroom teaching
- Using measure scores to predict student achievement

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Linking Teacher Knowledge and Student Achievement: New Results

- Questionnaire consisting of 30 items
- Model: Student Terra Nova gains predicted by:
  - Student descriptors (family SES, absence rate)
  - Teacher characteristics (math methods/content, content knowledge)
- Teacher content knowledge significant
  - Small effect (< 1/10 standard deviation): 2 - 3 weeks of instruction
  - But student SES is also about the same size effect on achievement
    - (Hill,Rowan, and Ball, in press)

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3. Using Mathematical Knowledge for Teaching in Attending to Equity

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In Class:
The Mathematical Work of Teaching

- Third grade (ages 8 - 10)
- Mathematics: birth of fractions from division problems
- Teaching: Scaffolding students' mathematical work
- Diverse class (language, race, culture)

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Mathematics Problem: Fractions and Division

PROBLEM 2:

(A) I have one dozen small raisin cookies. If I want to share them equally with my family at supper tonight, how many cookies would each person get? How do you know?

(B) How would this work out in YOUR family—how many cookies would person get if YOU had a dozen cookies to share with them? How do you know?

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Video segment from third grade class

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Probing the Teacher's Work in Attending to the Mathematics and to Considerations of Equity

- What are the central mathematical tasks in which the teacher is engaged that matter for the integrity of the mathematics and for issues of equity?
  1. Getting everyone on the "same page" with the mathematical work
  2. Choosing and using problems that use students' experience to support crucial mathematical work
  3. Attending to and using mathematical language, representations, and notation
  4. Expecting students to engage in the individual and collective work of mathematical reasoning
  5. Others? Dilemmas?

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Tasks of Teaching Involved in Attending to the Mathematics and to Considerations of Equity

1. Getting all students on the same page with the mathematical work
2. Choosing and using problems that use students' experience to stage crucial mathematical work
3. Attending to and developing mathematical language, representations, and notation
4. Expecting students to engage in the individual and collective work of mathematical reasoning

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Tasks of Teaching Involved in Attending to the Mathematics and to Considerations of Equity

- Getting all students on the same page with the mathematical work
  - Deciding what is mathematically central to highlight
  - Creating varied opportunities for a range of mathematical contributions
  - Requesting and providing restatements of students' contributions
- Choosing and using problems that use students' experience to stage crucial mathematical work
  - Choosing problem context that can support the mathematical development into fractions
  - Managing the translation between everyday experience and mathematics; negotiating mathematization, ambiguity

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- Attending to and developing mathematical language, representations, and notation
  - Rectangular regions rather than circles
  - Mapping correspondences among word problem, drawings, tables
  - Attaching language of "fifths" to describe parts
  - Licensing use of initials
- Expecting students to engage in the individual and collective work of mathematical reasoning
  - Explicit scaffolding to enable individual students to reason mathematically about their work
  - Supporting the staging of the practice of public presentation of solutions and reasons: rehearsal, creating tools, assisting with the enactment of a solution
  - Building and supporting classroom culture of respect and mathematical practice
  - Explicit direction to students to attend to, make sense of, and evaluate others' mathematical solutions and reasons

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Conclusions

1. Mathematical knowledge for teaching involves knowledge of topics and skills in the school curriculum, but also beyond it.
2. Mathematical knowledge for teaching is the knowledge needed for the work of teaching. That work entails specialized content knowledge distinct from that required by other mathematically-intensive professions.
3. The work of teaching demands effective attention to equity.
   - This depends on equitable access, high expectations, and explicit attention to what it takes to be mathematically proficient.
   - Such work depends crucially on teachers' skillful use of specialized mathematical knowledge.

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Slides will be available at

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