Center for Proficiency in Teaching Mathematics
SUMMER INSTITUTE, Part II
(Winter Version)

University of Georgia

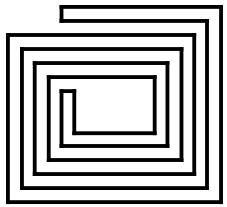


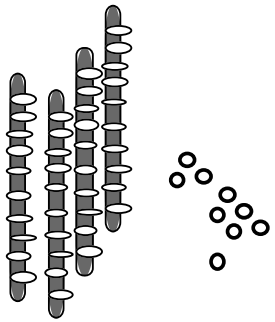
University of Michigan

$$\begin{array}{r} 28 \\ \times\ 6 \\ \hline 368 \end{array}$$



$$\frac{3}{4}$$





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Overview

Welcome, introductions, and overview

Block 1: How have we been using what we learned from the 2004 Summer Institute?

- Goals of CPTM and the 2004 Summer Institute
- Revisit our efforts to define "fraction"
- Defining fractions with 4th graders

Block 2: Posters & presentations

Block 3: Designing tasks focused on mathematical knowledge for teaching?

- What is Mathematical Knowledge for Teaching (MKT)?
- What are features of tasks that can be used to develop MKT?

Block 4: Enacting tasks focused on mathematical knowledge for teaching

- Overview of the Mathematical Tasks Framework
- What are the analogous issues faced by teacher educators when enacting MKT tasks?

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Introduction

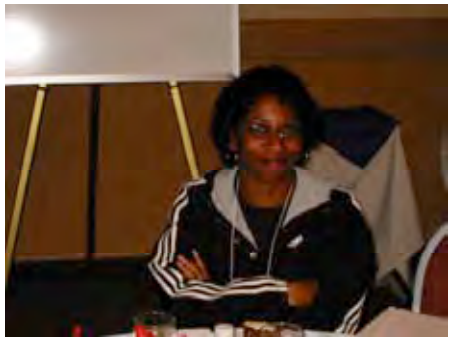
- Welcome and introductions
- Introduction
  - What is the Center for Proficiency in Teaching Mathematics? What are its goals?
  - The Summer Institute, and what we did
  - Focus on mathematical knowledge for teaching

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What is the
Center for Proficiency in Teaching Mathematics
and what is its central focus?

The professional development of mathematics teacher developers



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CPTM goals

Make visible two keys to improving the quality of teachers' learning

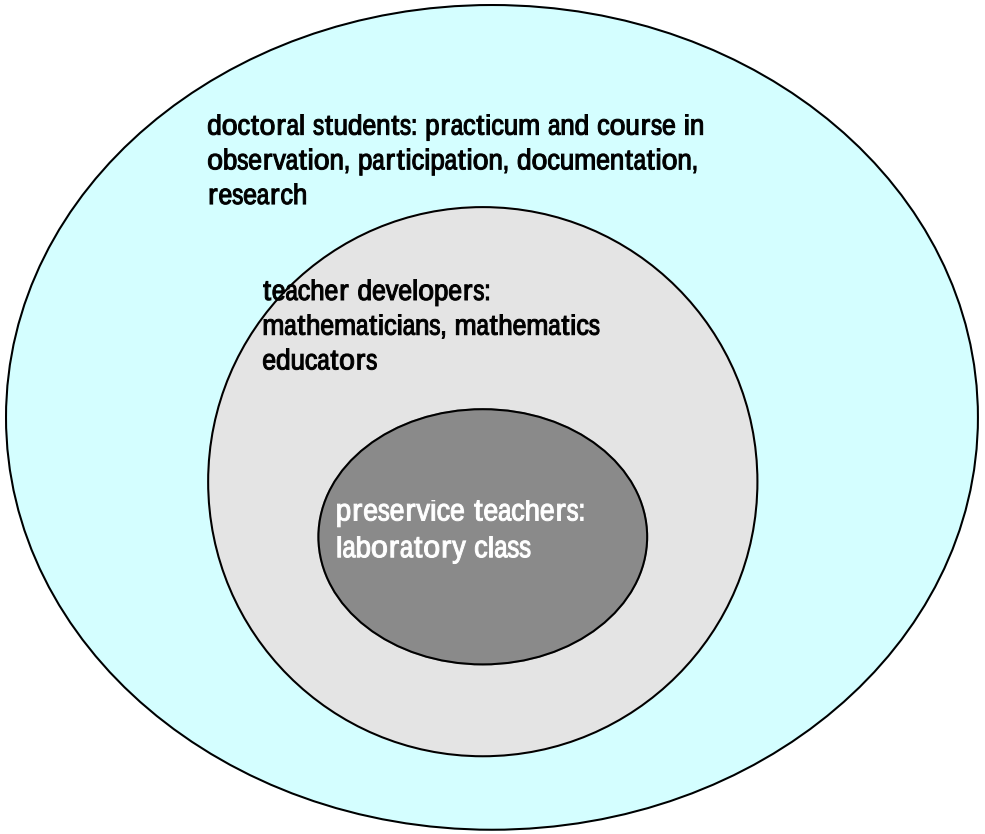
- Awareness of teacher developers' role and the nature of their work
- The need for professional development for teacher developers

1. Develop a professional community of teacher developers
2. Build teacher development into doctoral programs
3. Create portfolio of approaches to professional development of teacher developers
4. Learn from involvement with community of teacher developers

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How was the summer institute designed, and why?



- What mathematical knowledge and practices play a central role in the everyday work of teaching?
- What are promising approaches for helping teachers learn mathematics for teaching and learn to use it in their work?

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Mathematical knowledge for teaching (MKT)

- A practice-based theory of the knowledge needed by teachers
  - More on this and the progress of the theory tomorrow morning; today, just a brief reminder:
- What do we mean when we use this term, "mathematical knowledge for teaching"?
  - Mathematical knowledge, skill, habits of mind that are entailed by the work of teaching
- What do we mean by the "work of teaching"?
  - The tasks in which teachers engage, and the responsibilities they have, to teach mathematics, both inside and outside of the classroom

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Block 1:
How have we been using what we learned from the Summer Institute?

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Revisiting a task from the Summer Institute:

Defining "fraction"

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Definitions as a crucial component of MKT

Is 0 even or odd?

An even number is an integer multiple of 2.

What is 7 divided by 0?

For any a and b (real numbers), a/b = c if and only if cb = a.

A polygon is a simple closed plane curve formed by straight line segments.

An exterior angle is the angle formed between the side of a polygon and the extension of an adjacent side.

What is the sum of the exterior angles around a polygon?

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Task:
Write a definition for "fraction."

- Mathematically precise — correctly identifies the kind of object, process, property

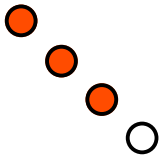
- Usable by user community — based on already-defined and understood terms

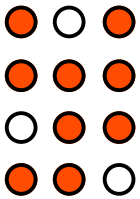
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Representations of three-fourths

a) 

b) 

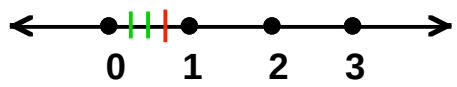
c) 

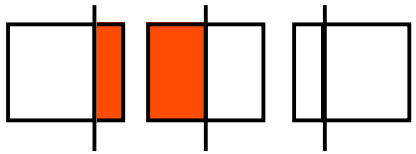
d) **How many 4's are there in 3?**

e) **18 crayons out of a box of 24**

f) **.75**

g) **I want to share 3 bottles of soda equally among 4 people. How much will each person get?**

h) 

i) 

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Video clip from our work in 2004

- Watch clip from our discussion that ends with Hy saying that maybe fraction isn't something that can be defined for elementary school
- What stands out to you from our discussion at that time?

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Elementary Mathematics Laboratory PCMI 2006

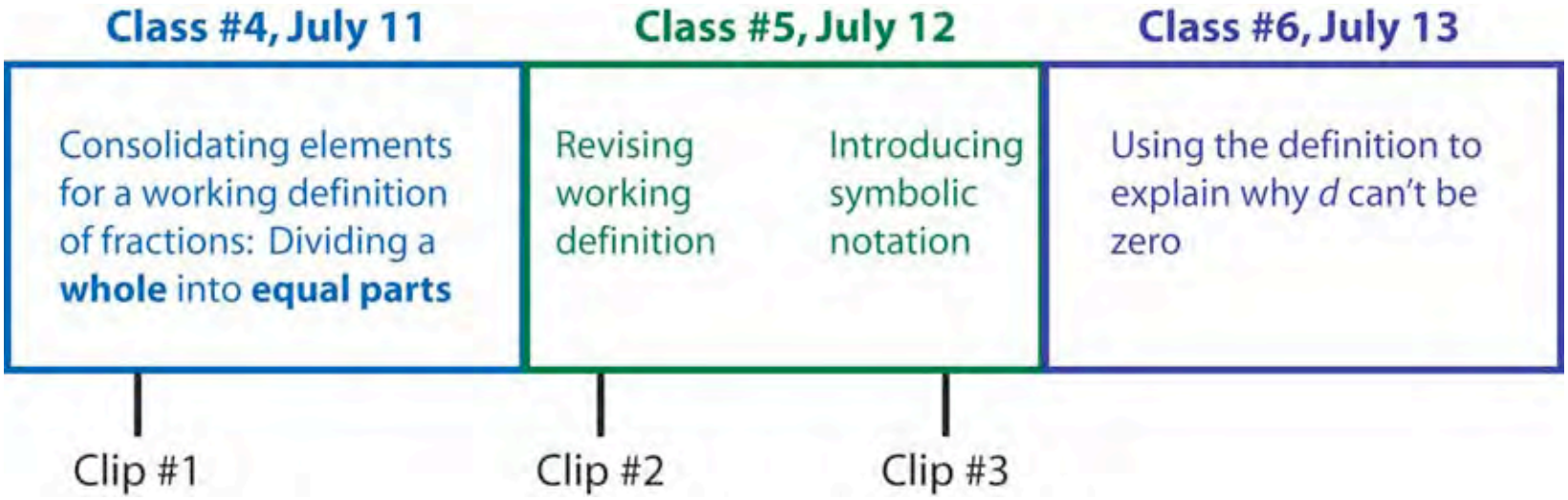
- Summer class for 21 incoming fifth graders (part of the Park City Mathematics Institute)
- Wide range of mathematical achievement and dispositions
- 6 days of instruction (2.25 hours per day)
- The working definition of fractions was developed over days 4-6

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Development of fraction work in fourth grade class

In classes #2 and #3, the pupils solved fraction problems that surfaced central fractions concepts, such as the importance of the unit, or whole, and equal parts.



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Definition of fraction

We will write $\frac{1}{d}$, where some whole is divided into d equal parts. If a part is $\frac{1}{d}$ of a whole, then it takes d copies of $\frac{1}{d}$ to make the whole. $\frac{n}{d}$ is then defined as n • ($\frac{1}{d}$), or n copies of $\frac{1}{d}$.

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Assessing the definition and preparing to watch the video

1. The mathematical quality of the definition: Is it correct? Is it mathematically precise?
2. The usability of the definition: Does this seem usable by fourth graders? What problems might it present?
3. What might it take to introduce this in a class, and make it usable with children?

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Using mathematics in teaching:
What is involved in getting a complex mathematical definition up and running in a classroom?

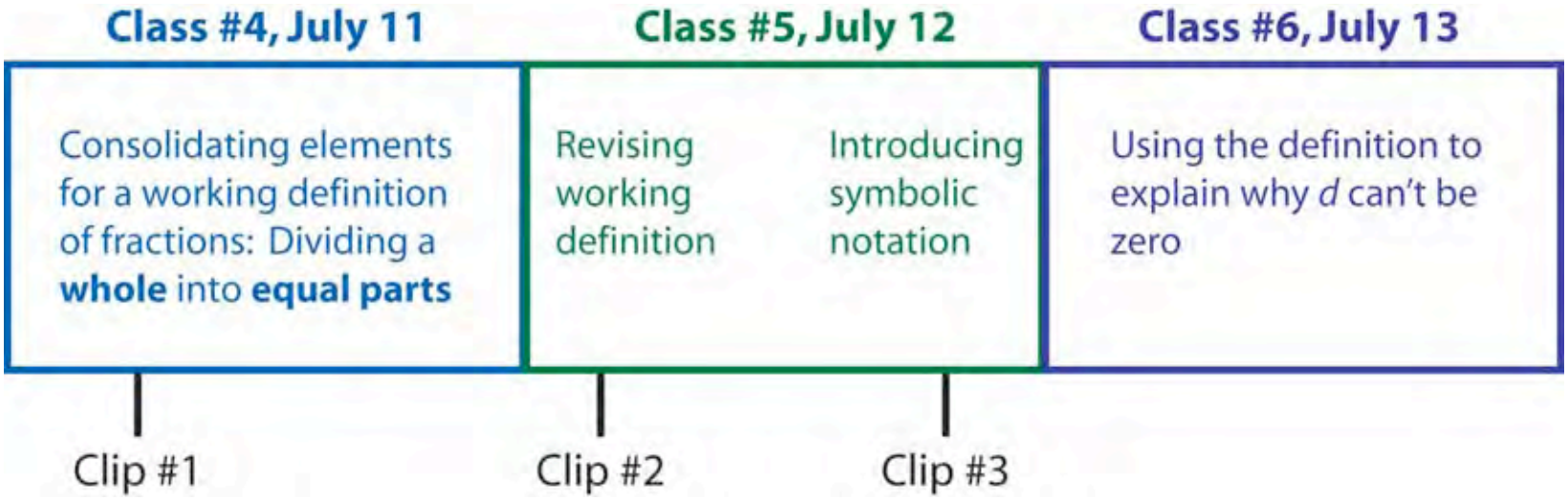
1. What specific issues of mathematical language arise?
2. Where does the teacher slow down and spend time, and why might that be?
3. What issues are not emphasized or taken up, and why might that be?
4. What issues related to children's understanding arise in the development of the definition of fraction? How are these managed?

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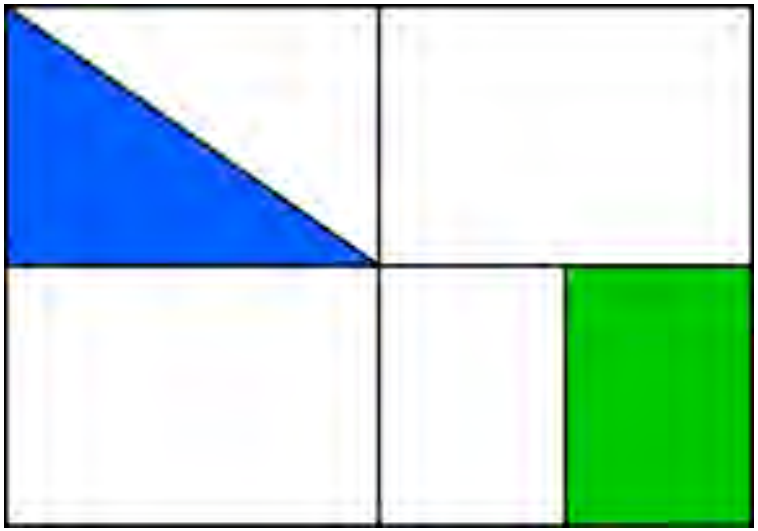
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What fraction of the big rectangle is the blue region?

What fraction of the big rectangle is the green region?

Explain your answers.

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Definition of fractions as part of any whole

- Divide the whole into d equal parts.
- Write $\frac{1}{d}$ for one of these equal parts.
- If you have d copies of $\frac{1}{d}$, you have the whole.

We call d the denominator.

- If you have n copies of $\frac{1}{d}$, you write $\frac{n}{d}$

We call n the numerator.

d and n are whole numbers; $d \neq 0$

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Reasoning with the definition of fractions: Why can't *d* be equal to 0?

1. Use the definition of fractions to build a mathematical explanation that makes sense to you.

2. Construct an explanation that is usable in fourth grade.

   What is the mathematical work of making a explanation that is usable with children?

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Comment cards

- What question or comment do you have about what we worked on today?
- Is there something special you want to do tomorrow? (no guarantees, but you can always request or wish!)

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Block 2:
Posters & presentations

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Poster session

Group 1: 6:15 - 6:45

Group 2: 6:45 - 7:15

Group 3: 7:15 - 7:45

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