From yesterday



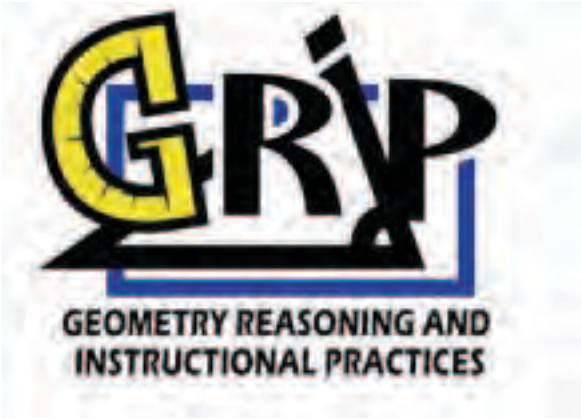
- Good to return to the work on fractions
- Glad to work more on enactment and teaching
- Can we have access to the powerpoints?
- Want to connect the work on fractions and definitions to MKT
- MKT, PCK --??
- Is "usefulness" synonymous with flexible knowledge?
- Definitions for 4th graders vs. for preservice teachers
- Are we understanding fractions better or definitions better?
- How would this play out in urban schools?
- More random distribution of chocolate across tables
- Longer whole group discussions and offer chart paper to groups so that more voices can be heard in the whole group
- Shorter whole group discussions, more time in small groups

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Announcement

Special session from GRIP project at lunch today: see flyers on tables for explanation of this UM project



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Overview of Thursday

Morning: Designing mathematical knowledge for teaching tasks

- What is Mathematical Knowledge for Teaching?
- What are features of tasks that can be used to develop MKT?
- Sharing tasks from your own work

Afternoon: Enacting mathematical knowledge for teaching tasks

- Overview of the Mathematical Task Framework
- What are the analogous issues faced by teacher educators when enacting MKT tasks?

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Block 3:
Designing tasks that develop mathematical knowledge for teaching

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Toward a practice-based theory of mathematical knowledge for teaching

1. Study instruction, and identify the mathematical work of teaching
2. Analyze what mathematical knowledge is needed to do that work effectively, and how it must be understood to be useful for the work
3. Develop and evaluate approaches to helping teachers learn mathematical knowledge for teaching
4. Develop, test, and refine measures of mathematical knowledge for teaching using multiple methods as a means to evaluate professional education, investigate effects on students' learning, and improve theory

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Mathematical knowledge for teaching

What do we mean when we use this term, "mathematical knowledge for teaching"?

- Mathematical knowledge, skill, habits of mind that are entailed by the work of teaching

What do we mean by the "work of teaching"?

- The tasks in which teachers engage, and the responsibilities they have, to teach mathematics, both inside and outside of the classroom

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Knowing multiplication

Multiply:

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline \end{array}$$

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Knowing multiplication for teaching: Analyzing student errors

What mathematical steps produced each of these answers?

(a)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 405 \\ 108 \\ \hline 1485 \end{array}$$

(b)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 225 \\ 100 \\ \hline 325 \end{array}$$

(c)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 1250 \\ 25 \\ \hline 1275 \end{array}$$

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Knowing radical expressions

Simplify:

$$\sqrt{150}$$

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Knowing radical expressions for teaching:
Choosing numerical examples

Which of the following is best for setting up a discussion about different solution paths for simplifying radical expressions?

(a) $\sqrt{54}$

(b) $\sqrt{156}$

(c) $\sqrt{128}$

(d) These examples all work equally well.

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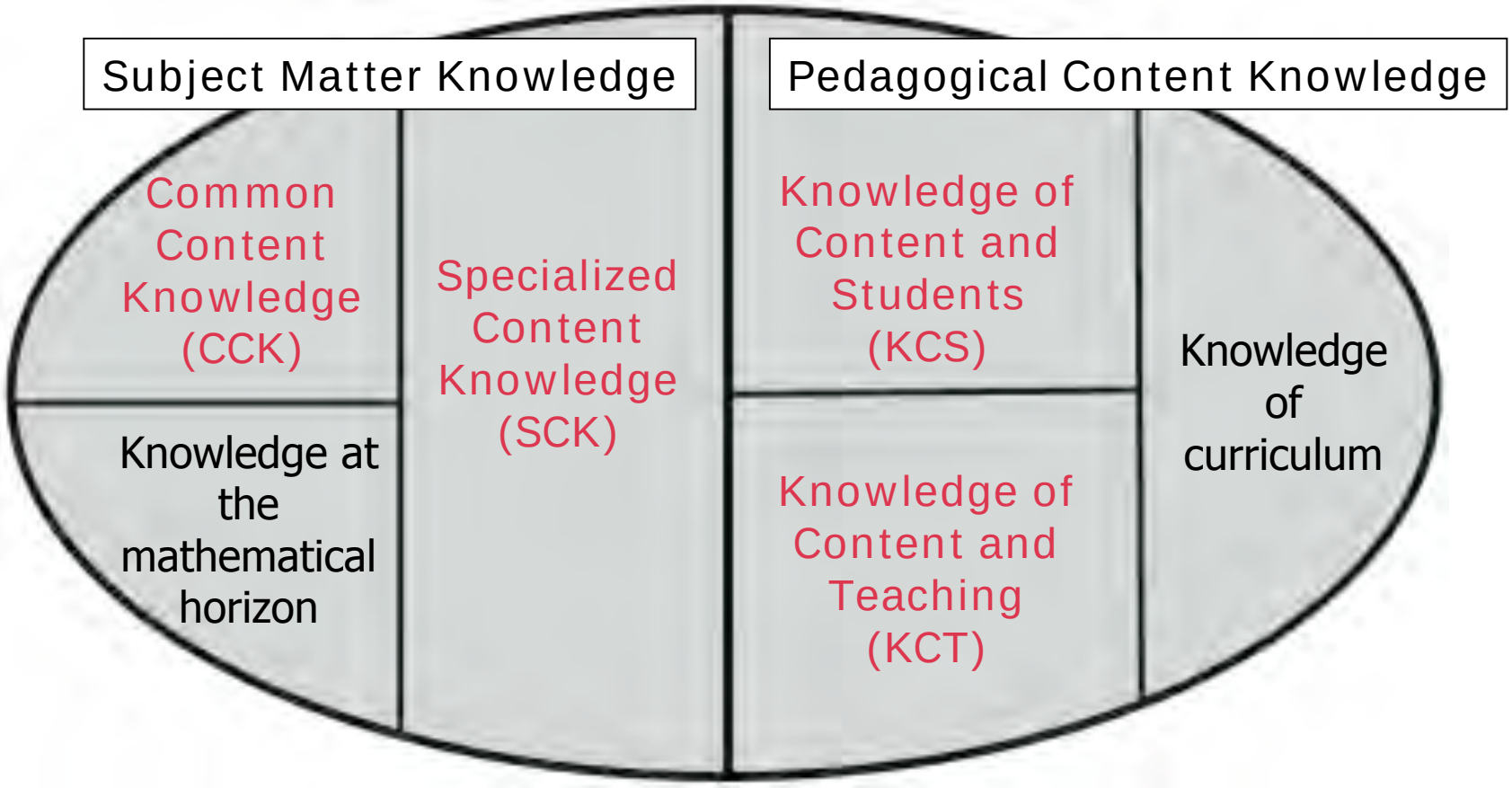
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Tasks of teaching mathematics

- Unpacking and decomposing mathematical ideas
- Sequencing ideas
- Choosing and using representations and examples
- Explaining and guiding explanation
- Using mathematical language and notation
- Analyzing errors
- Interpreting and evaluating alternative solutions and thinking
- Analyzing mathematical treatments in textbooks
- Making mathematical practices explicit
- Attending to issues of equity (e.g., language, contexts, mathematical practices)

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Mathematical knowledge for teaching



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Studying or assessing teachers' mathematical knowledge

- Multiple approaches used in the field
  - Interviews
  - Structured tasks
  - Observations
  - Tests
- No one "best" method; depends on purposes and questions

(Hill, Sleep, Lewis, and Ball, forthcoming in new *Handbook for Research on Mathematics Education*)

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Opportunity: Study of Instructional Improvement

- Study of three Comprehensive School Reforms; teacher knowledge a key variable
- Instrument development goals:
  - Usable with 5000 teachers: survey, multiple choice
  - Measure content knowledge teachers *use* in teaching – not just what they *know*
  - Differentiate among teachers
  - Non-partisan

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Specialized Content Knowledge
(Example Item)

Which of these students is using a method that could be used to multiply any two whole numbers?

Student A	Student B	Student C
35 x 25 125 +75 875	35 x25 175 +700 875	35 x 25 25 150 100 +600 875

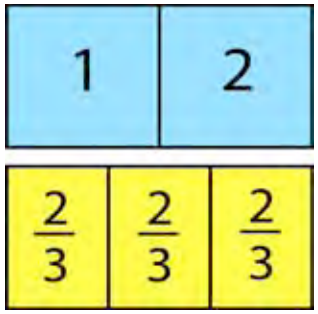
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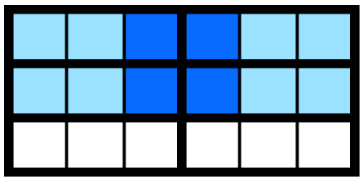
Specialized Content Knowledge
(Example Item)

Which of the following can be used to represent 2 ÷ 2/3?

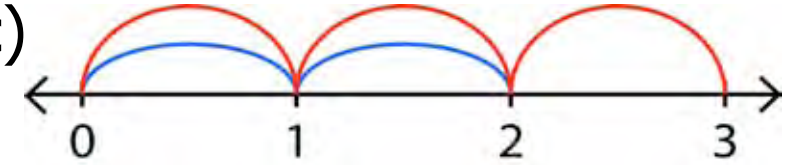
(a)



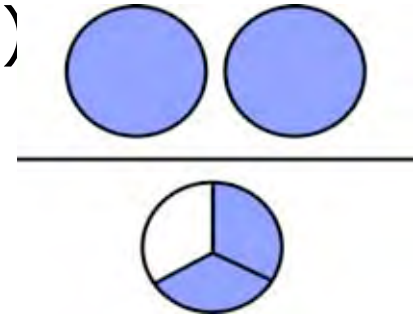
(b)



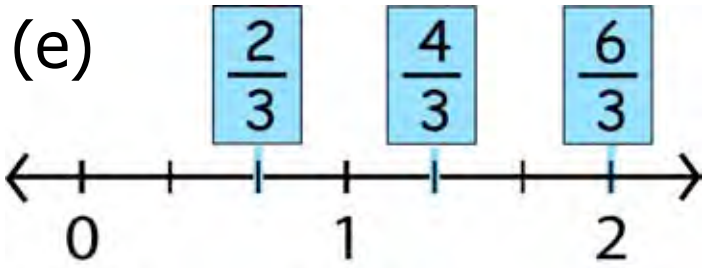
(c)



(d)



(e)



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Knowledge of Content and Students
(Example Item)

Ms. Violeta was looking carefully at her students' papers, and she saw the following responses to the problem:

$$8 + 4 = ___ + 5$$

i) 12 ii) 17 iii) Can't do it iv) 1

Which of the following is the <u>most likely</u> explanation of the difficulty the students are having? (Mark ONE answer.)

a) They do not know their basic addition facts.

b) They cannot do multi-step problems.

c) They do not know that addition is commutative.

d) They do not understand the meaning of the equals sign.

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Knowledge of Content and Teaching (KCT)
(Example Item)

To introduce the idea of grouping by tens and ones with young learners, which of the following materials or tools would be most appropriate? (Choose ONE.)

a. A number line
b. Plastic counting chips
c. Pennies and dimes
d. Straws and rubber bands
e. Any of these would be equally appropriate for introducing the idea of grouping by tens and ones.

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Overarching findings: Factor analyses

- Multidimensionality of mathematical knowledge for teaching
  - Knowledge of students and content different from "pure" content knowledge
  - Knowledge differentiable by topic domain (e.g., algebra, geometry)
  - Number and and operations mathematical knowledge for teaching
    - Specialized
    - Common
- Tentative conclusion: existence of a kind of "professional" knowledge for teaching

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Validating our measures

How do we interpret teachers' performance on our questions?

1. Their score reflects their mathematical thinking
   - Cognitive interviews
2. Higher scores mean higher-quality mathematics instruction
   - Videotape validation study
3. Scores reflect common and specialized knowledge of content
   - Mathematician and non-teacher interviews
4. Higher scores related to improved student learning
   - Study of Instructional Improvement student gains analysis

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Linking teacher knowledge and student achievement

- Questionnaire consisting of 30 items (scale reliability .88)
- Model: Student Terra Nova gains predicted by:
  - Student descriptors (family SES, absence rate)
  - Teacher characteristics (math methods/content, content knowledge)
- Teacher content knowledge significant
  - Small effect (< 1/10 standard deviation): 2 - 3 weeks of instruction
  - But student SES is also about the same size effect on achievement

    (Hill, Rowan, and Ball, AERJ, 2005)

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Designing tasks to develop teachers' mathematical knowledge for teaching

What is the difference between a good mathematics problem or task and one that is good for developing mathematical knowledge for teaching?

Why does this matter?

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Central tasks of teaching mathematics

- Unpacking and decomposing mathematical ideas
- Sequencing ideas
- Choosing and using representations and examples
- Explaining and guiding explanation
- Using mathematical language and notation
- Analyzing errors
- Interpreting and evaluating alternative solutions and thinking
- Analyzing mathematical treatments in textbooks
- Making mathematical practices explicit
- Attending to issues of equity (e.g., language, contexts, mathematical practices)

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Division of Fractions

$$1\frac{3}{4} \div \frac{1}{2}$$

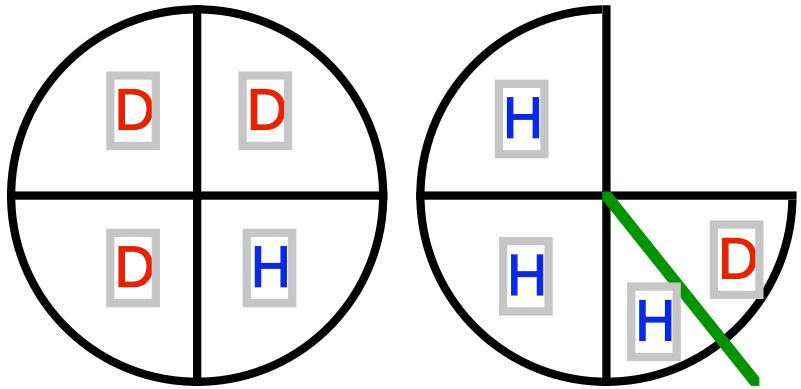
1. Calculate the answer.

2. Write a story problem, or describe a situation, that corresponds to 1 3/4 ÷ 1/2.

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I have two pizzas. My friend eats one quarter of one of the pizzas. I have one and three quarters pizzas left. Then I split it evenly between two of my other friends. Each person gets three and a half pieces of pizza.



1. What is wrong with this?

2. Write a story problem that correctly represents the division.

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Design: What are features of problems or tasks designed to develop MKT?

- Unpacks and develops a flexible understanding of mathematical ideas that are central to understanding the school curriculum
- Provokes a stumble due to a superficial "understanding" of the idea
- Lends itself to alternative/multiple representations and solution methods
- Provides opportunities to engage in mathematical practices that are central to teaching (explaining, representing, using mathematical language, analyzing equivalences, proving, proof analysis)
- Provides opportunities to engage in teaching practices that are central to mathematics teaching (interpreting others' thinking, posing questions, writing math on the board)
- Represents some important aspects of what "doing math" is
- Benefits from collective work; different perspectives/solutions add to an understanding of the problem or concept

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The bagel problem

Becky has 3 dozen bagels that she wants to share equally among 5 people. How many dozen or how much of a dozen can she give to each person?

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Writing division story problems

$38 \div 4$

Write as many different stories as you can that correspond to this division expression and that represent different interpretations of the meaning of division or what it means in specific situations.

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Analyzing tasks designed to develop mathematical knowledge for teaching

What are features of tasks that are designed to develop MKT?

For each task:

- What aspects of MKT is the task designed to develop?
- What features of the task make it possible to use to develop MKT?

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How is the task designed to help teachers to learn mathematics in ways that they need to use it in their work?

- Unpacks and develops a flexible understanding of mathematical ideas that are central to understanding the school curriculum
- Provokes a stumble due to a superficial "understanding" of the idea
- Lends itself to alternative/multiple representations and solution methods
- Provides opportunities to engage in mathematical practices that are central to teaching (explaining, representing, using mathematical language, analyzing equivalences, proving, proof analysis)
- Provides opportunities to engage in teaching practices that are central to mathematics teaching (interpreting others' thinking, posing questions, writing math on the board)
- Represents some important aspects of what "doing math" is
- Benefits from collective work; different perspectives/solutions add to an understanding of the problem or concept

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Analyzing other MKT tasks

How is the task designed to help teachers to learn mathematics in ways that they need to use it in their work?

- Consider a new task in light of our list of features
- Are there other features we should add to our list?

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What have we learned?

What have we learned about how tasks can be designed to help teachers learn mathematics in ways they will use mathematics in their work?

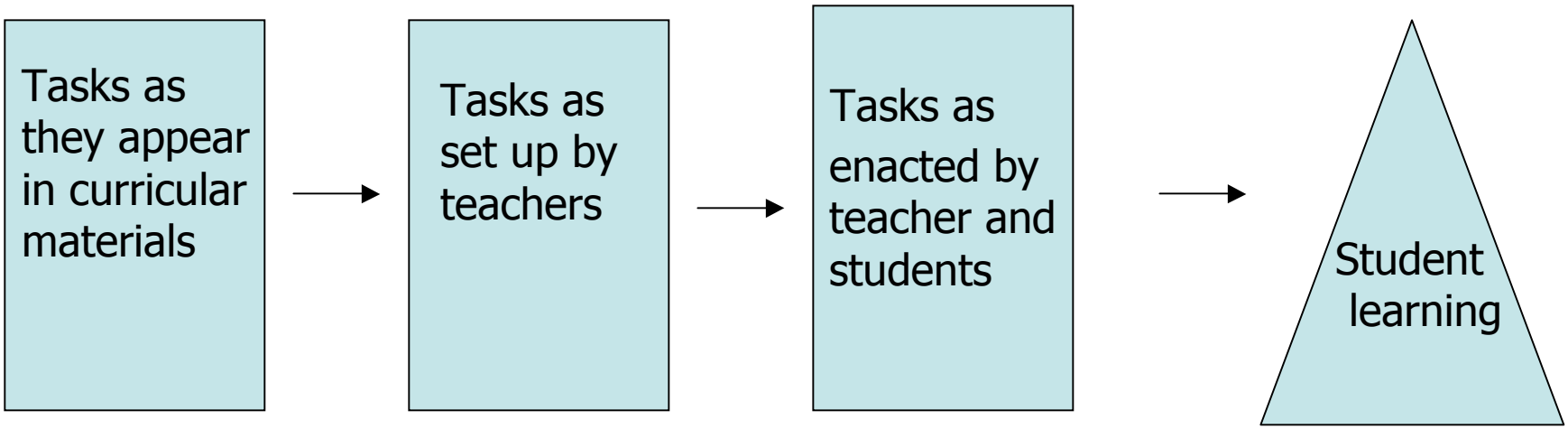
What does it seem like it would take to get the tasks to work that way?

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Block 4:
Enacting mathematical knowledge for teaching tasks

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The Mathematical Task Framework



Stein, Grover & Henningsen (1996)
Smith & Stein (1998)
Stein, Smith, Henningsen & Silver (2000)

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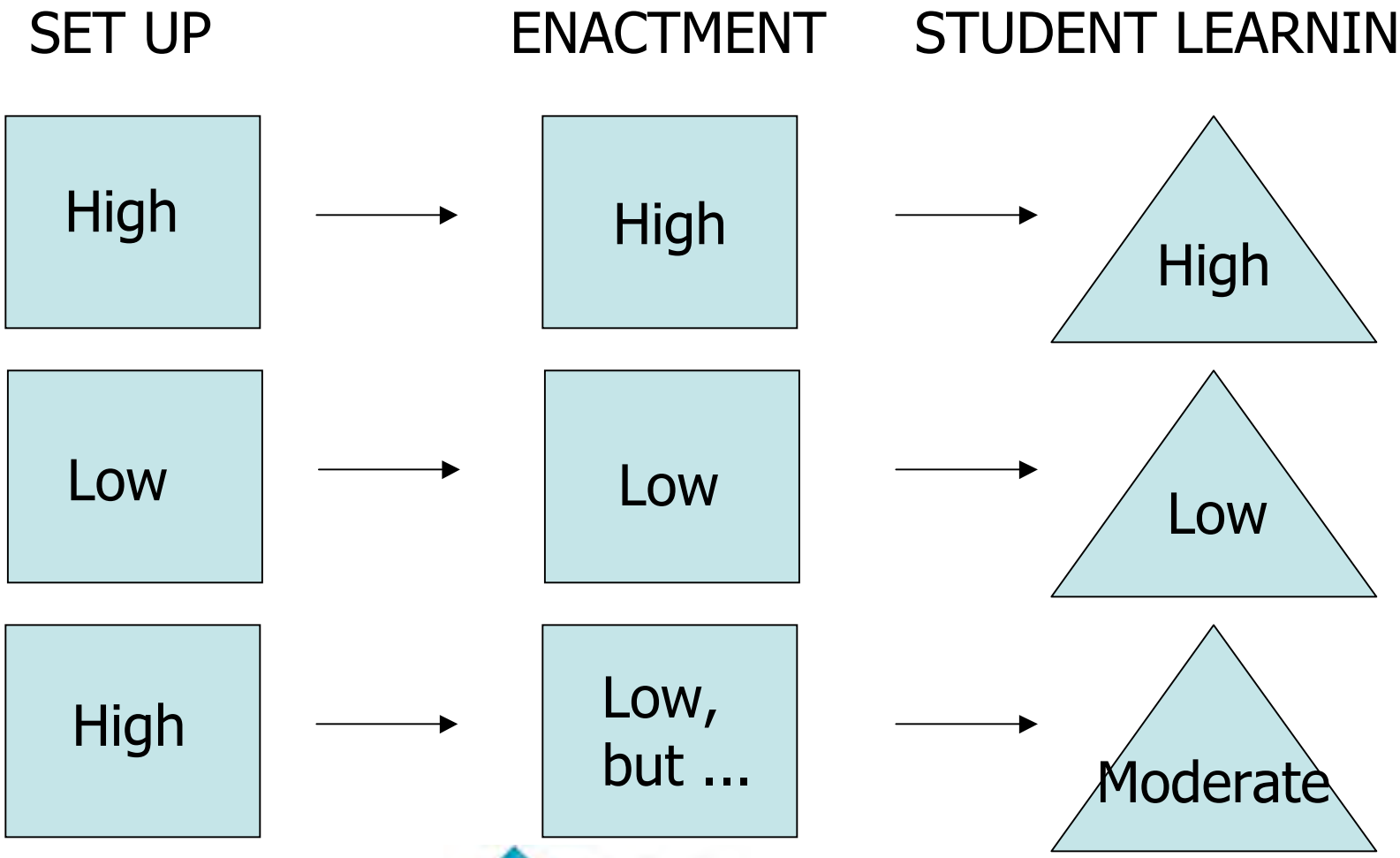
What can become of high-level tasks during a lesson?

- Maintain high-level demands
- Decline into procedures without connection to meaning
- Decline into unsystematic and unproductive exploration
- Decline into absence of mathematical activity

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Enactment affects student learning



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Teacher actions affect task enactment & student learning

- Build on students' prior knowledge
- Scaffold students' thinking
- Provide appropriate amount of time
- Model high-level performance
- Sustain pressure for explanation and meaning

Henningsen & Stein (1997)

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Factors associated with decline of high-level demands

- Challenging aspects are removed or reduced to render the task a non-problem
- Goal shifts attention from process and reasoning to answer only
- Too much or too little time is provided

Henningsen & Stein (1997)

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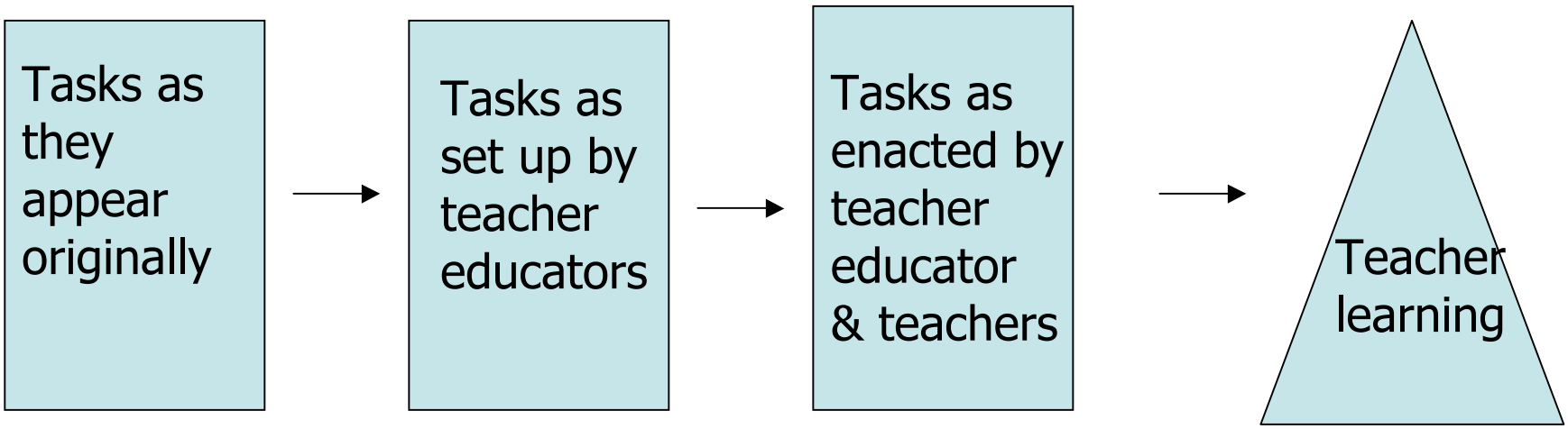
Some MTF-related challenges facing all teachers of mathematics

- Resisting the persistent urge to tell and to direct; allowing time for students to think
- Knowing when/how to ask questions and to provide information to support rather than replace student thinking
- Helping students accept the challenge of solving worthwhile problems and sustaining their engagement at a high level

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The Mathematical Task Framework adapted to teacher education



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What can happen to MKT tasks during a lesson?

- Maintain focus on MKT
- "Morph" into consideration of mathematics without connection to teaching
- "Morph" into consideration of teaching without connection to mathematics
- Devolve into ritualized activity

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Teacher educator actions affect task enactment & teacher learning

- Provide appropriate amount of time
- Model consideration of MKT
- Prompt teachers to make comments and give explanations that focus on MKT
- Scaffold teachers' thinking about MKT to keep MK and T both in play whenever possible

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Enactment: What are key questions and moves that can be used to keep a task focused on developing MKT?

- Asking students to explain their solutions to the class
- Having students explain what is/was confusing them
- Asking students to figure out what might be confusing/difficult for someone else about the problem
- Having students ask questions to become more clear about their classmates' solutions
- Asking students to make correspondences between solutions and/or representations
- Asking students to explain someone else's thinking
- Providing opportunities to "talk mathematics" and write on the board
- Provoking a common error
- Narrating how something a student does/says relates to or is a skill used in teaching

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Looking at practice…

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Focus questions

- What teacher educator actions influence opportunities to learn MKT from a task?
- What moves and questions can teacher educators use to maintain a task's focus on MKT?

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Video context: The summer institute lab class

- A one- or two-credit mathematics content course for elementary education students
- 18 preservice elementary teachers:
  - 7 master's level certification program students; 11 undergraduate certification program students
- Content course (MKT):
  - fractions
  - mathematical practices: using representations, making and evaluating mathematical explanations
- Laboratory
- Third and fourth days of class: The bagel problem

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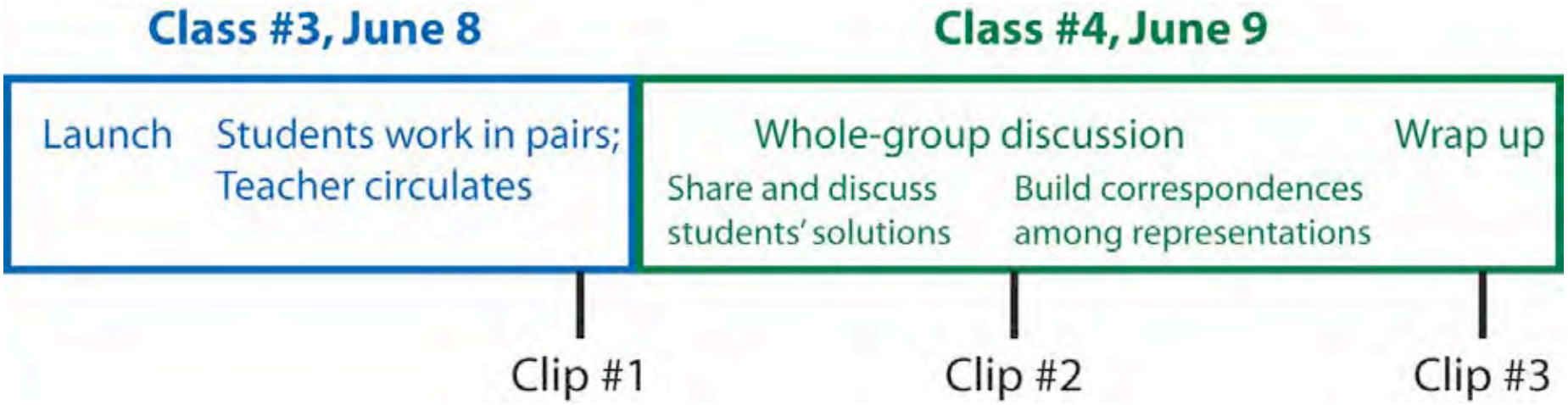
The bagel problem

Becky has 3 dozen bagels that she wants to share equally among 5 people. How many dozen or how much of a dozen can she give to each person?

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Enacting the bagel problem
2004 Summer Institute



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Launching the bagel problem

Teacher: I'm going to put up the Bagel Problem, and I'm going to start the same way that we did before, which is I want you to just read it. It's very short, so I didn't print it for you, and put it in your notebook, and I want you to size it up kind of the way you did with the Cookie Jar problem, and that is: What's the problem seem to be involving? What's your reaction to it? Do you have a sense about what it's going to be like, what kind of solution it might have, and so on.

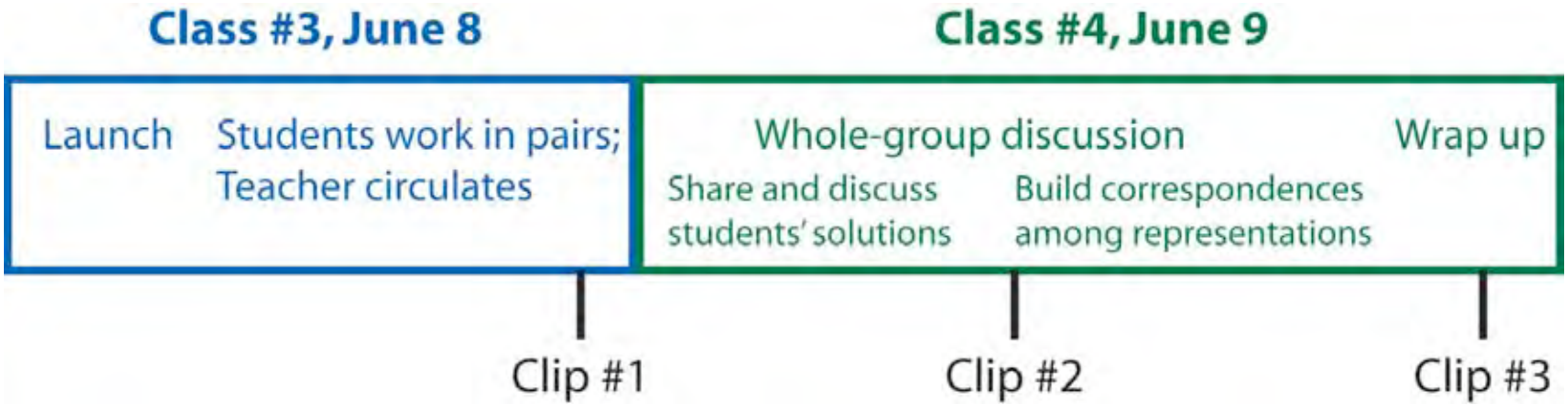
After 3 minutes of individual writing…

Teacher: Does anyone have a question about the problem for the whole group? [*no questions*] Turn to person next to you and make sure the two of you have a shared sense of what the problem is asking.

Students began working on the problem in pairs.

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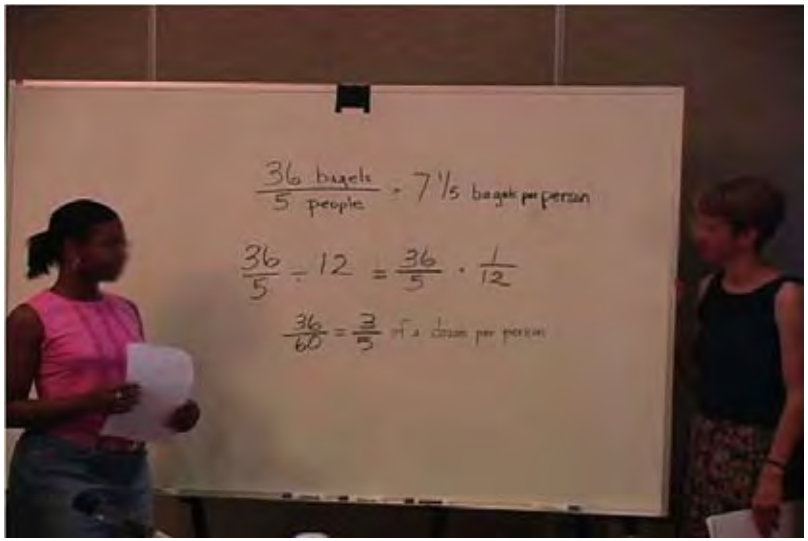
Clip #1
Circulating among students

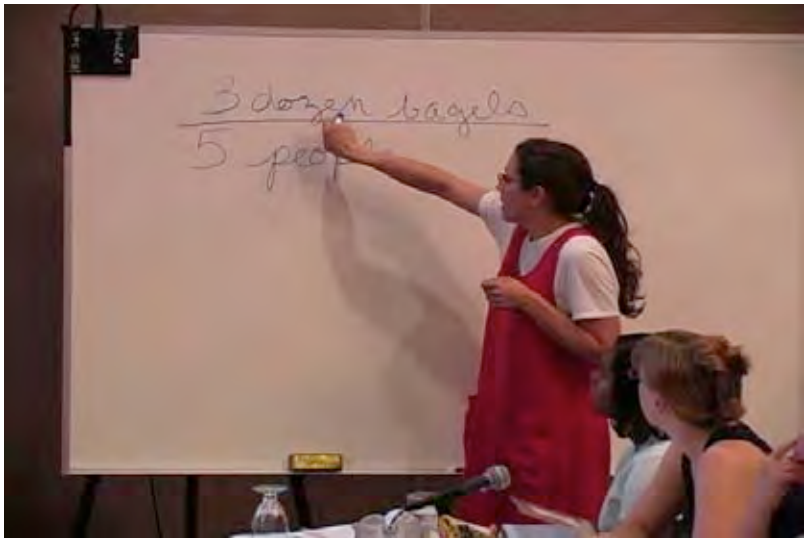


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Whole-group discussion: Students share solutions

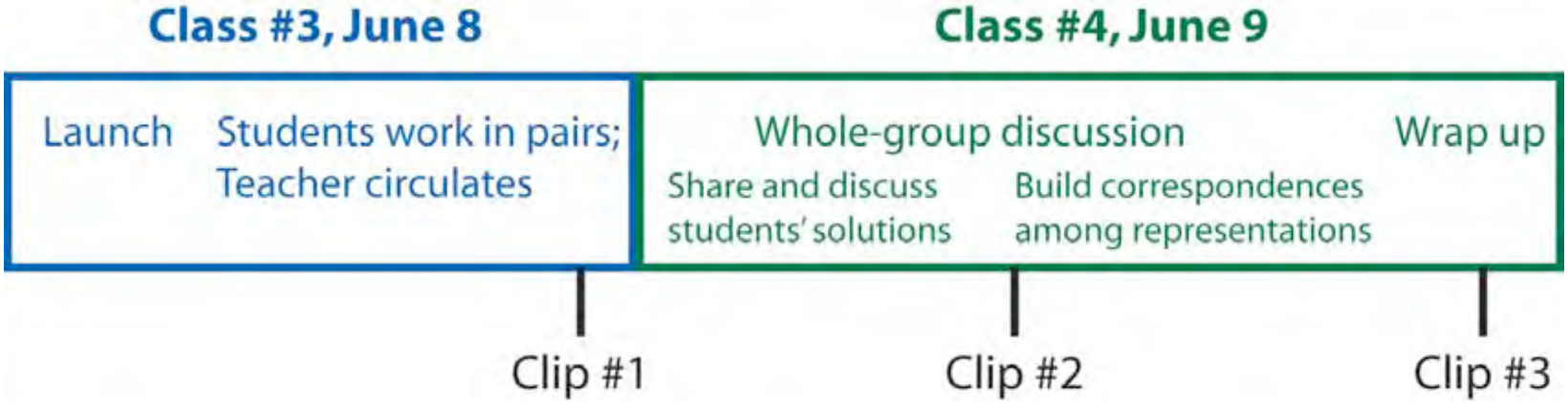




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Clip #2:
Building correspondences



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Clip #3: Concluding the bagel problem



The answer 3/5 of a dozen is equivalent to:

3/5 dozen = ___ of 1 dozen

3/5 dozen = ___ of 3 dozen

If a bagel shop sold bagels by the "1/5 dozen," how many "1/5 dozens" does each person get? ___ "1/5 dozens"

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CPTM goals

Make visible two keys to improving the quality of teachers' learning

- Awareness of teacher developers' role and the nature of their work
- The need for professional development for teacher developers

1. Develop a professional community of teacher developers
2. Build teacher development into doctoral programs
3. Create portfolio of approaches to professional development of teacher developers
4. Learn from involvement with community of teacher developers

Next steps: What would be good things to do to spur our collective work?

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Next steps

- What would be good things to do to spur our collective work?

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Wrap up

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