Learning mathematics in and for practice:
Using records of practice as (con)texts for learning mathematical knowledge for teaching

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Overview of session

1. Background on MKT and the mod4 project
2. Orientation to the mod4 materials
3. Sampling two activities
4. Discussion: reactions to the materials
5. How to get involved: Participating in our upcoming pilot

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Background on mathematical knowledge for teaching and the mod4 project

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What is the "problem"?

The quality of mathematics teaching and learning

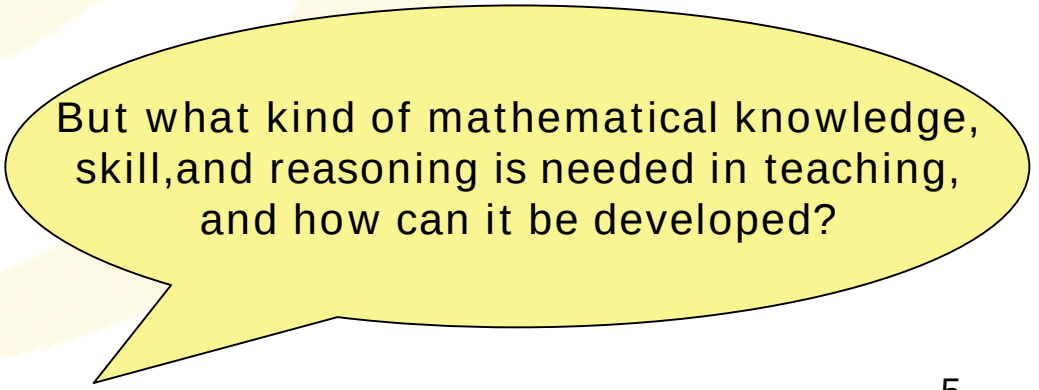
Teachers' knowledge of mathematics and their ability to use it in practice

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Some approaches to solving the problem, and the unresolved question

- Require more mathematics for certification
  - more mathematics courses
  - a major (or minor) in mathematics
  - mathematics test
- Recruit mathematically trained people into teaching
  - engineers, accountants, mathematicians, …
- Fund mathematically focused professional development



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Mathematical knowledge for teaching (MKT)

What do we mean when we use this term, "mathematical knowledge for teaching"?

- mathematical knowledge, skill, habits of mind that are entailed by the work of teaching

What do we mean by the "work of teaching"?

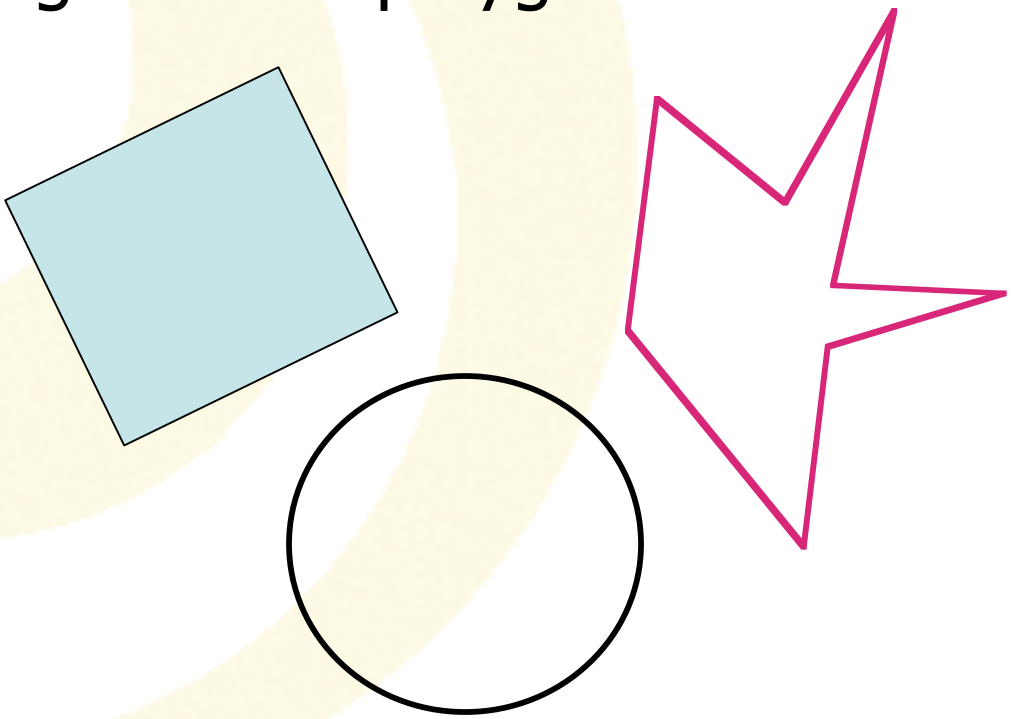
- the tasks in which teachers engage, and the responsibilities they have, to teach mathematics, both inside and outside of the classroom

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Knowing polygons

Which of these figures is a polygon?

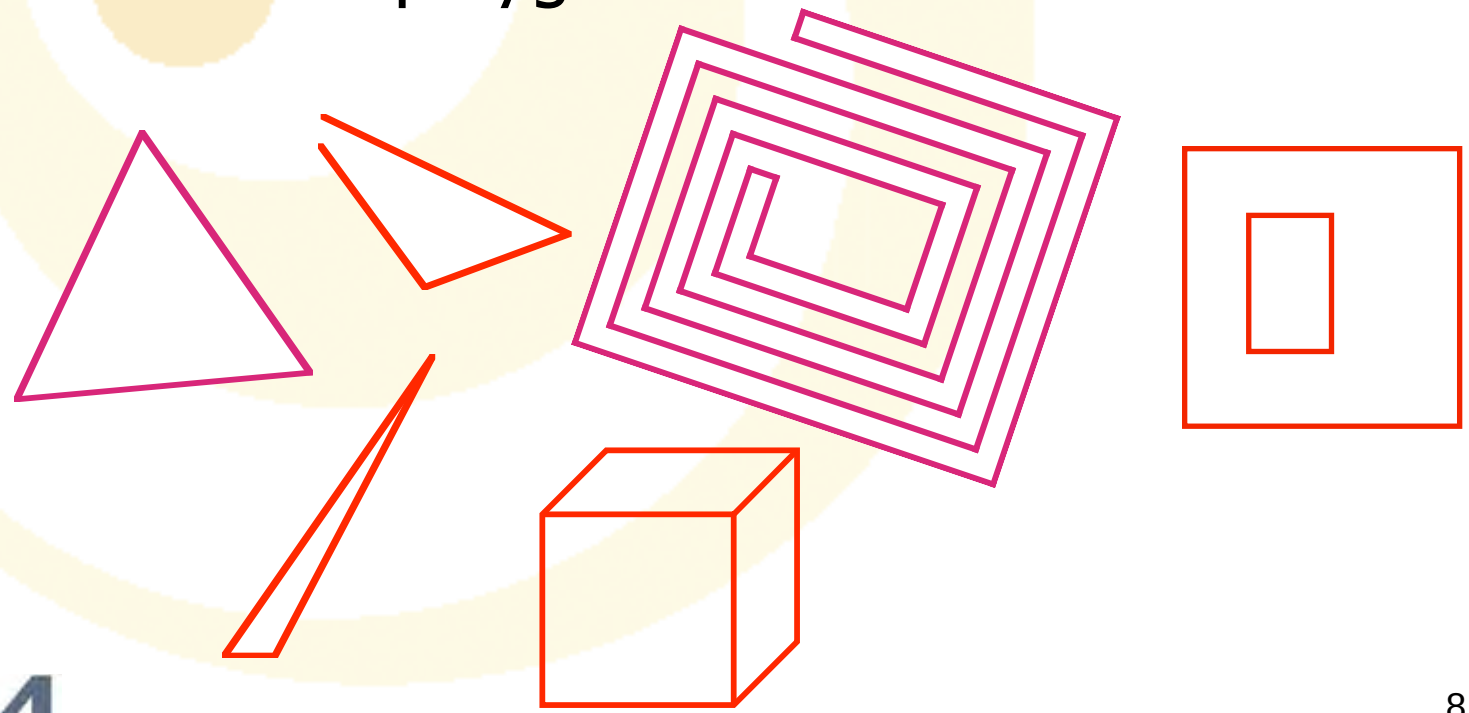


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Knowing polygons for teaching

In what ways could each of these shapes be used to precipitate a discussion of the definition of polygon?



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Knowing long division

Divide: $3\overline{)416}$

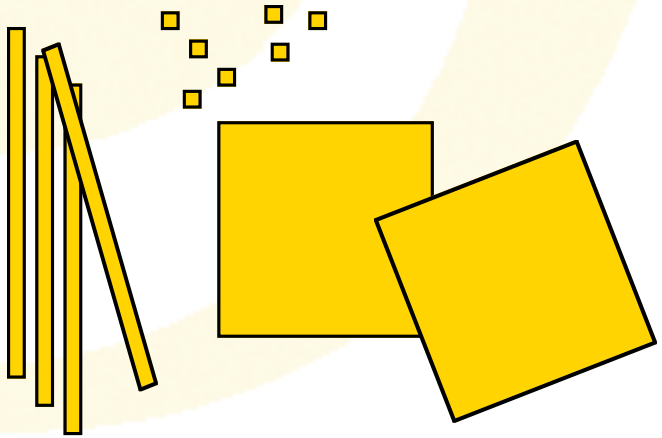
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Knowing long division for teaching

When modeling long division with base ten blocks, what are the differences among the following numerical examples?

$7\overline{)568}$ $13\overline{)247}$ $4\overline{)913}$



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Other tasks of teaching mathematics

- Unpacking and decomposing mathematical ideas
- Sequencing ideas
- Choosing and using representations and examples
- Explaining and guiding explanation
- Using mathematical language and notation
- Analyzing errors
- Interpreting and evaluating alternative solutions and thinking
- Analyzing mathematical treatments in textbooks
- Making mathematical practices explicit
- Attending to issues of equity (e.g., language, contexts, mathematical practices)

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Elements of our "practice-based" approach

1. Study instruction, and identify the mathematical work of teaching
2. Analyze what mathematical knowledge is needed to do that work effectively, and how it must be understood to be useful for the work
3. Develop and evaluate approaches to helping teachers learn mathematical knowledge for teaching
4. Develop, test, and refine measures of MKT using multiple methods as a means to evaluate professional education, investigate effects on students' learning, and improve theory

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A new problem: How to create opportunities for teachers to learn MKT

- Learning in most mathematics courses is not geared for this kind of knowledge, skill, and problem solving
- Much professional development is content-free
- Lack of materials for learning this sort of math for these applications

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Teaching mathematics for teaching: Goals and challenges

- Finding/developing tasks that create opportunities for learning mathematical knowledge for teaching
- Situating teachers' opportunities to learn in the contexts of use
- Providing opportunities to practice the mathematical thinking and reasoning used in teaching
- Enacting tasks in ways that maintain the focus on developing MKT and the ability to use it in teaching
- Assessing actual mathematical practices and ways of reasoning as they are used in teaching

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Orientation to the mod4 materials

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Components of the mod4 materials

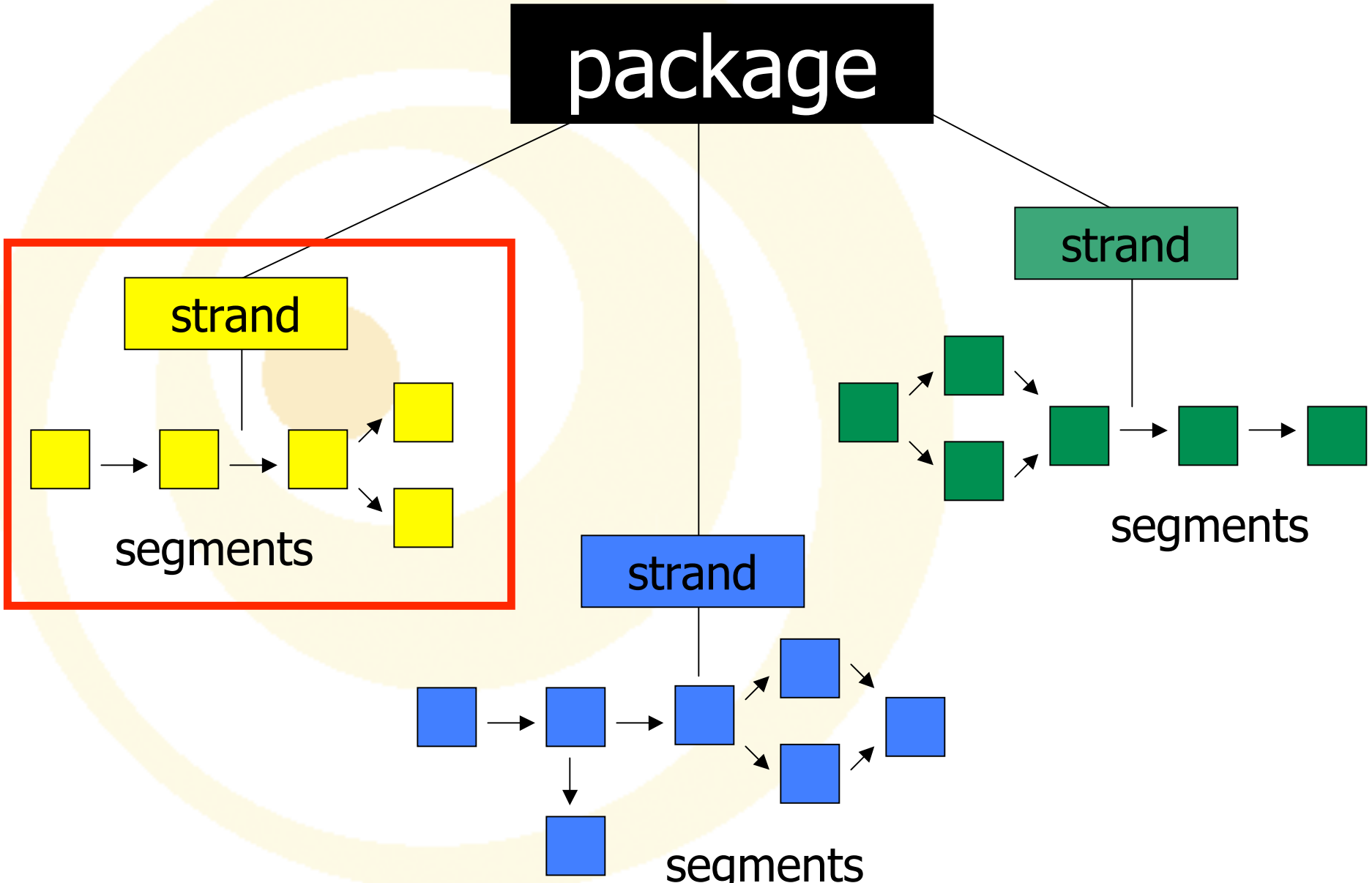
Package -- collection of instructional materials based on a particular set of records of practice

Strand -- sequence of work focused on developing a particular aspect of MKT

Segment -- instructional activities that make up a strand; each segment provides detailed descriptions of activities focusing on a particular part of the strand's theme

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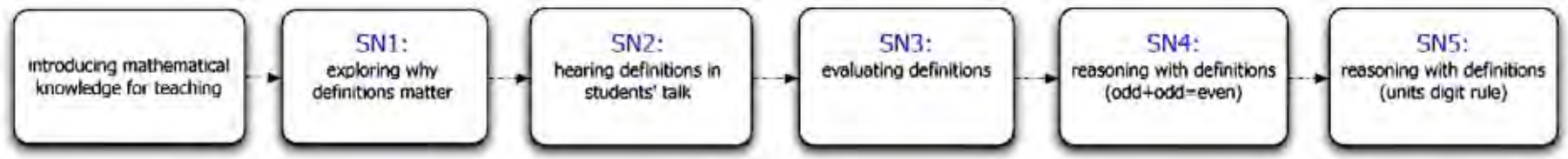


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Prototype strand:

Using definitions in mathematics learning and teaching



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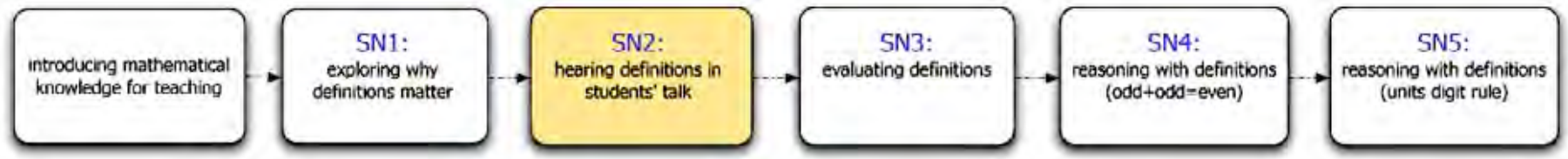
Sampling two activities from the materials

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Activity 1

From segment #2:
Hearing definitions in students' talk



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Viewing mathematics teaching

Context

- third graders
- multi-cultural and multi-lingual class
- lesson takes place in January

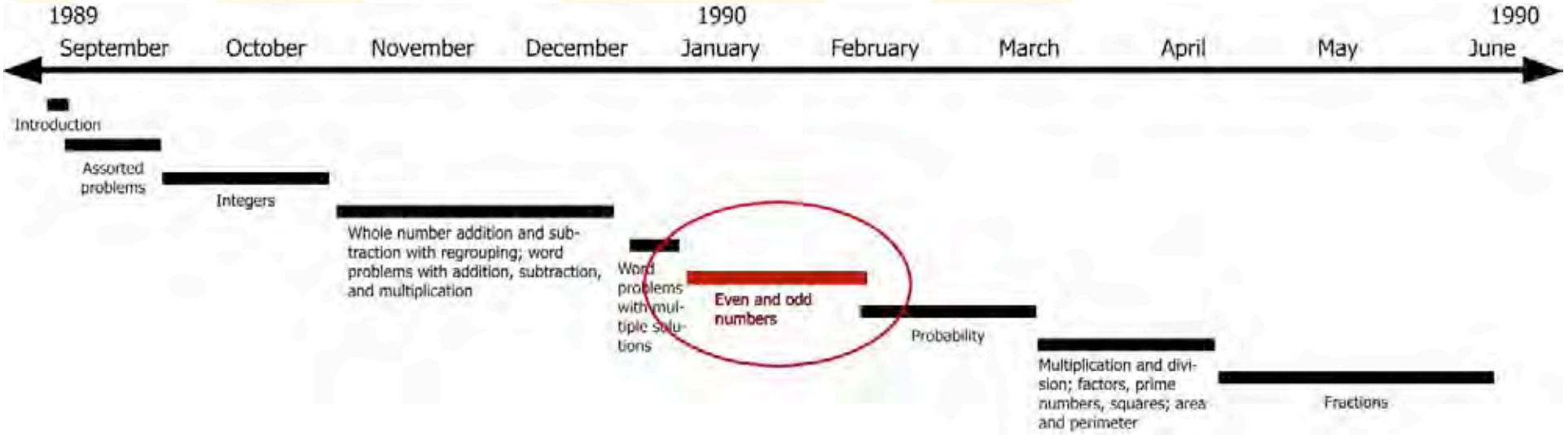
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Viewing mathematics teaching

Content

The class is learning about even and odd numbers



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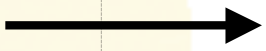
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Viewing mathematics teaching

Student learning trajectory

Start point
(initial understanding)

- Do not yet have a clear, shared, usable definition for even/odd
- Able to correctly label small cases



End point
(goal)

- Have a clear, shared, usable definition for even/odd
- Able to define even/odd numbers and use this definition to reason, make claims, and critique others' ideas

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Viewing focus

- What do students in this class understand about what an even or an odd number is?
  - What definitions do they have for "even number"? (What evidence do you have?)
  - What other observations can you make about their knowledge of even and odd numbers? (What evidence do you have?)

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video

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How does skill with mathematical defining and definitions matter in teaching?

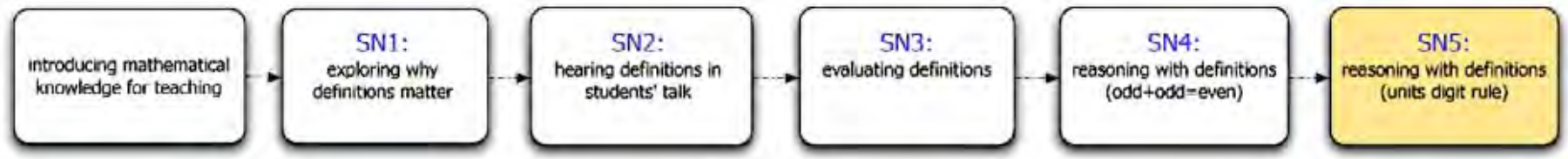
- In class, listening carefully to students' talk, noticing and helping improve ambiguous talk, including mediating everyday uses of language
- Analyzing and managing disagreements for their possible roots in different or imprecise uses of terms
- Recognizing, comparing, and reconciling different definitions
- Considering how to define terms so that they are usable with an eye toward the future
- Defining terms carefully for or with students
- Scrutinizing definitions in curriculum materials and modifying as needed
- Making sure that students can interpret test items

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Activity 2

From segment #5:
Reasoning with definitions (units digit rule)



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Why might it matter to know why the units digit rule works?

- To help students remember the rule clearly
- To help develop what it means to "know" something in mathematics
- As a context for learning how to explain

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Units digit rule

To determine whether a number is even, examine the ones digit. If it is even, then the entire number is even. If it is odd, then the entire number is odd.

Why does it work?

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Discussion:
Reactions to the materials

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Discussion questions

- What opportunities for learning MKT are provided by these segments?
- What challenges might implementing these segments pose for the teacher educator?

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How to get involved: Participating in our upcoming pilot

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$winter_{2007}$ pilot

- what: small exploratory investigation into the use of our prototype strand
- when: february 15 - april 15
- who: teacher educators and professional developers interested in using one or more of the segments in their work with teachers and providing us with feedback

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winter$_{2007}$ pilot

our purposes:

- learn about the materials we've developed so far
- learn about the types of contexts in which the materials will be used
- learn about the types of supports and resources needed to use the materials
- develop and refine effective ways to learn from those who are using our materials

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$winter_{2007}$ pilot

process:

1. sign up (online)
   (or indicate interest on signup sheet)
2. UM friends account
3. online preview
4. notification

•••••

5. materials use information & agreement
6. pilot materials
7. feedback, site visits, & camp

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Other opportunities

- Future pilots and field tests
- Future materials use workshops
- Project news & information
- Other explorations

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slides will be available at

http://www-personal.umich.edu/~dball/
http://sitemaker.umich.edu/mod4/amte2007

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