The Professional Development of Professional Developers:

Continuing to Learn as Mathematics Teacher Educators

AMTE Pre-Conference Work Session
sponsored by the Center for Proficiency in Teaching Mathematics
University of Georgia and University of Michigan
January 27, 2005

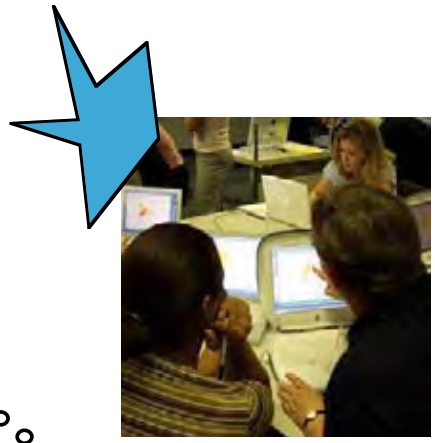
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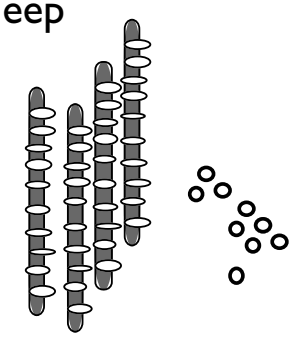
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x 6
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Paola Sztajn
Dave Coffey
Hyman Bass
Alison Castro
Teresa McMahon
Deborah Ball
Bob Allen
Laurie Sleep







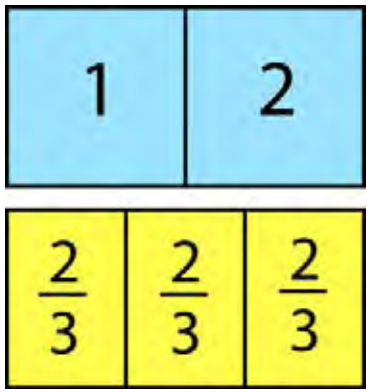
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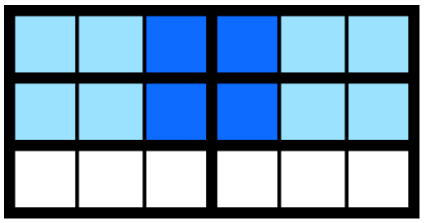
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Which of these could be representations for $2 \div \frac{2}{3}$?

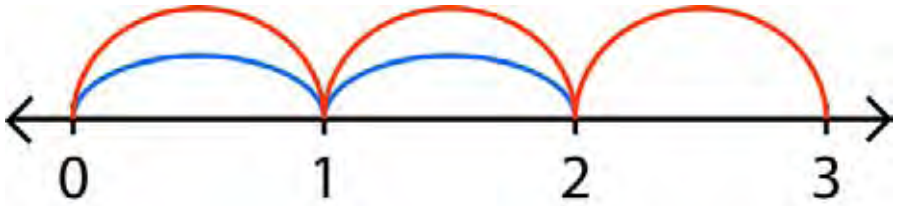
(a)



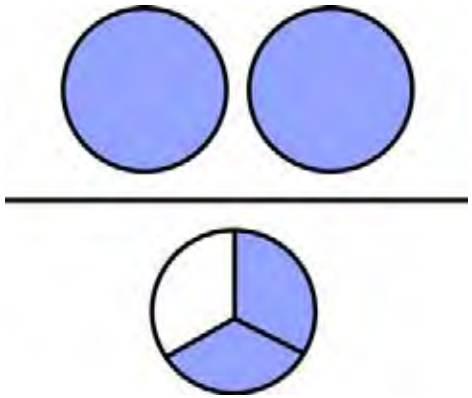
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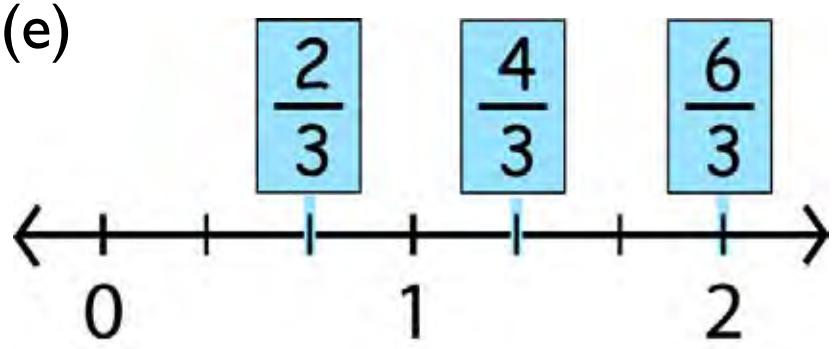
(c)



(d)



(e)



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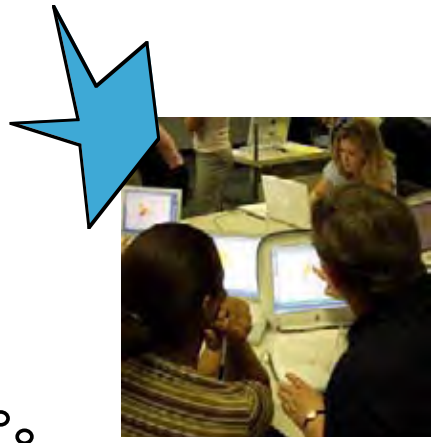
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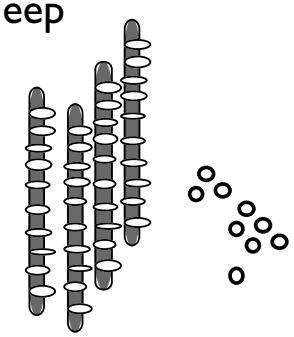
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Overview of Session

- Welcome and introductions

BLOCK 1

- CPTM and its goals
- What do we mean by "mathematical knowledge for teaching"?

BLOCK 2

- The 2004 CPTM Summer Institute as a case of professional development for teacher educators

BLOCK 3

- Making connections to your own work

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BLOCK 1

CPTM and Its Goals

What Do We Mean By "MKT"?

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What is CPTM and What Are Its Goals?

The professional development of teacher developers



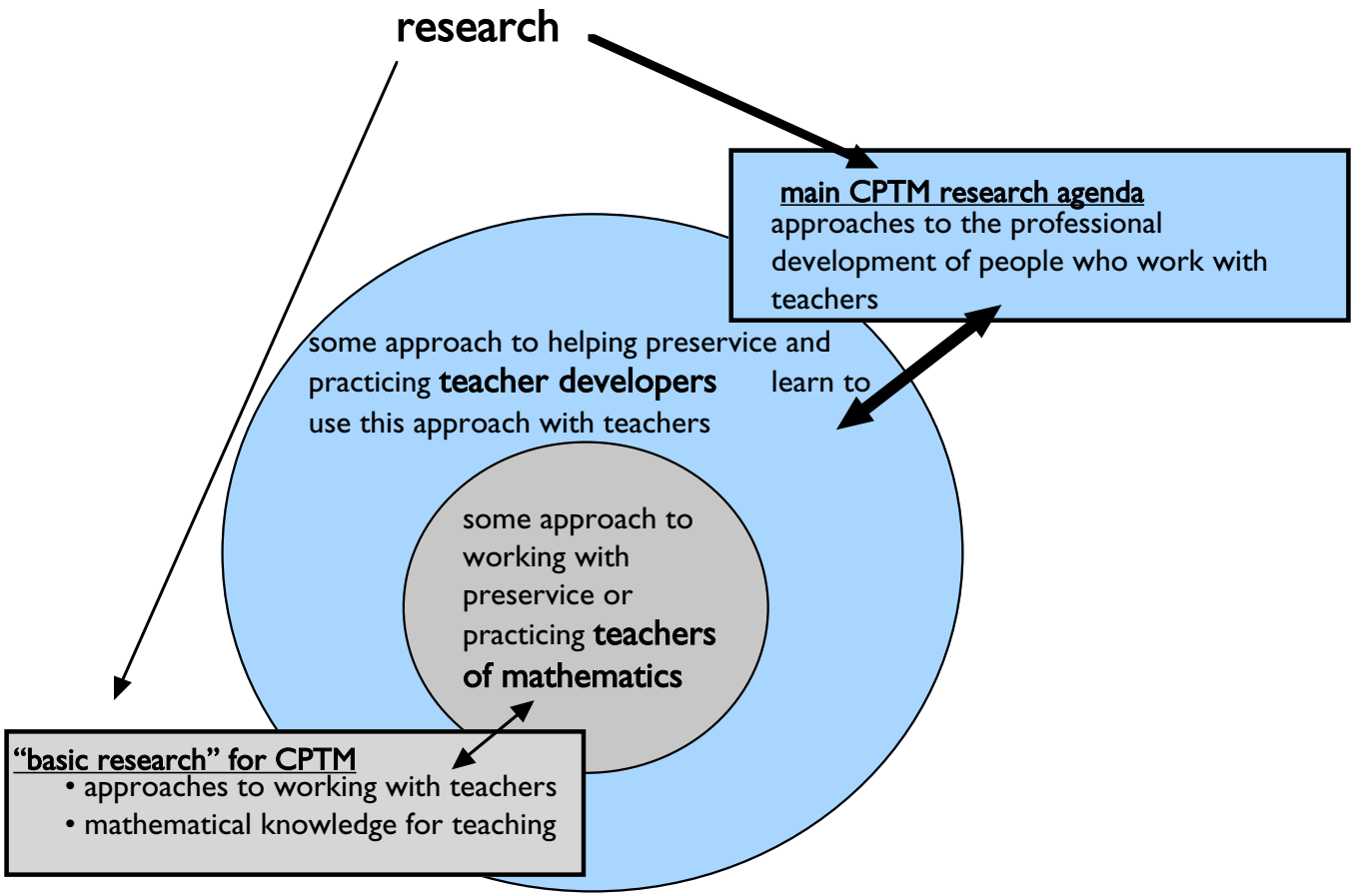
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CPTM's Focusing Question

What promising models and methods can support and increase the effectiveness of the many different kinds of people who teach and work with teachers of mathematics?

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How We Approach Our Focus



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CPTM Goals

Make Visible Two Keys to Improving the Quality of Teachers' Learning

- Awareness of teacher developers' role and the nature of their work
- The need for professional development for teacher developers

1. Develop a professional community of teacher developers
2. Build teacher development into doctoral programs
3. Create portfolio of approaches to professional development of teacher developers
4. Learn from involvement with community of teacher developers

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What Do we Mean By "Mathematical Knowledge for Teaching"?

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Common Framing of the Problem → Failure to Solve the Problem of Teachers' Content Knowledge

- Broad criticisms of teachers for not knowing enough mathematics
- Concentration of under-qualified teachers in urban and high poverty schools
- Solutions: increase number and rigor of mathematics requirements for teacher preparation
- Recruit mathematically-trained people into teaching

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Clarifying the Problem

The quality of mathematics teaching and learning

Teachers' knowledge of mathematics

and their ability to use it in teaching

What mathematical knowledge do teachers need in order to teach mathematics all students effectively?

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What is Mathematical Knowledge for Teaching? A Practice-Based Approach

1. **Study instruction**, and identify the mathematical work of teaching
2. Analyze **what mathematical knowledge** is needed to do that work effectively, and **how** it must be understood in order to be useful for the work
3. Test and refine theory of MKT
4. Develop, test, and refine **approaches to helping teachers** develop and use mathematical knowledge

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Mathematical Knowledge for Teaching

What do we mean when we use this term, "mathematical knowledge for teaching"?

- Mathematical knowledge, skill, habits of mind that are entailed by the work of teaching

What do we mean by the "work of teaching"?

- The tasks in which teachers engage, and the responsibilities they have, to teach mathematics, both inside and outside of the classroom

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Mathematical Knowledge for Teaching

- A practice-based approach to asking about mathematical knowledge for teaching:
  - Working "backwards" by studying the work of teaching and scrutinizing the mathematical demands of that work
- Shows that there is much mathematics **deep inside** the elementary curriculum as well as **beyond** it, and that attending to **equity** and diversity is fundamentally connected to knowing and **using mathematics** in teaching

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What *is* "Mathematical Knowledge for Teaching"? An Example from Division with Fractions

Divide:

$$2 \div \frac{2}{3}$$

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Analyzing Incorrect Answers for $2 \div \frac{2}{3}$

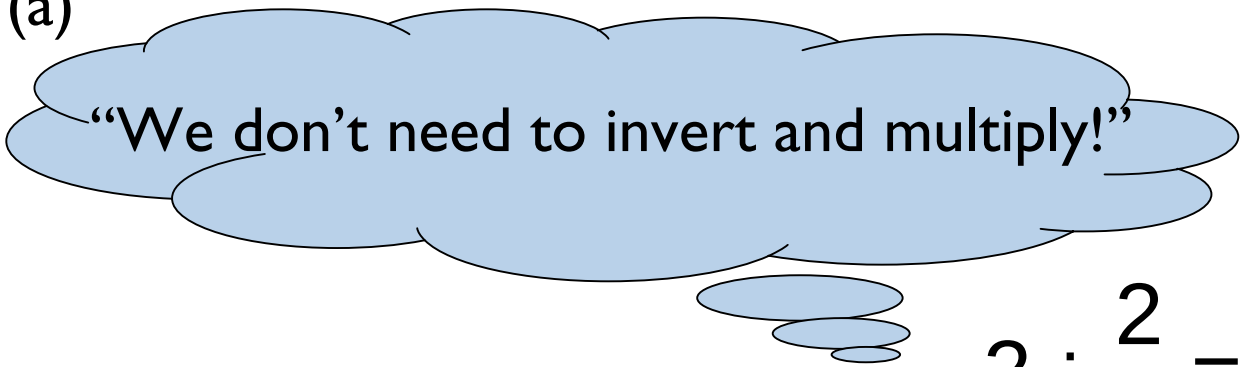
(a) $2 \div \frac{2}{3} = 1\frac{1}{3}$

(b) $2 \div \frac{2}{3} = \frac{2}{6}$

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Analyzing Correct Answers for $2 \div \frac{2}{3}$

(a)



$$2 \div \frac{2}{3} = \frac{2}{1} \div \frac{2}{3} = \frac{1}{1/3} = 3$$

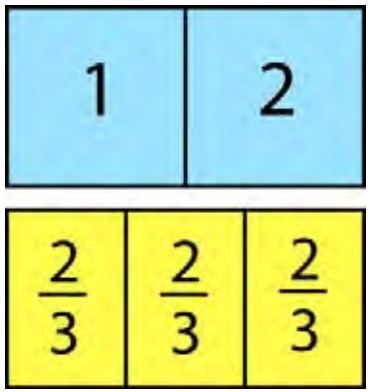
(b) "$2 \div \frac{2}{3} = 3$ because 2 is two-thirds of 3."

Which of these methods would work for dividing any two fractions?

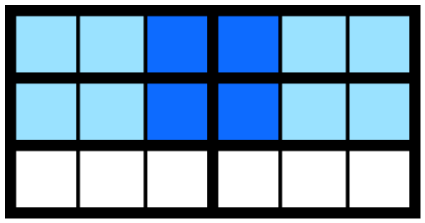
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Using Representations for $2 \div \frac{2}{3}$

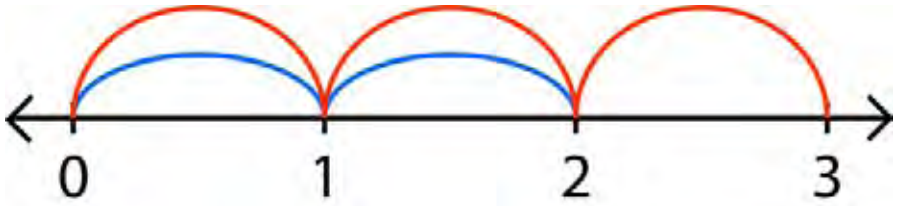
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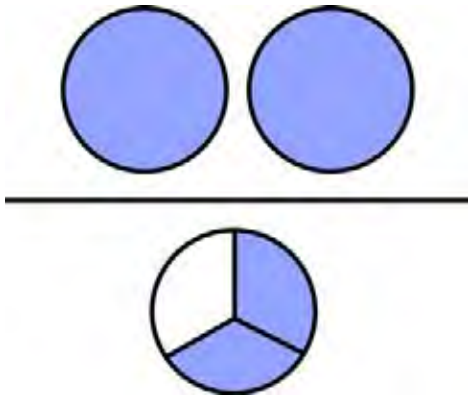
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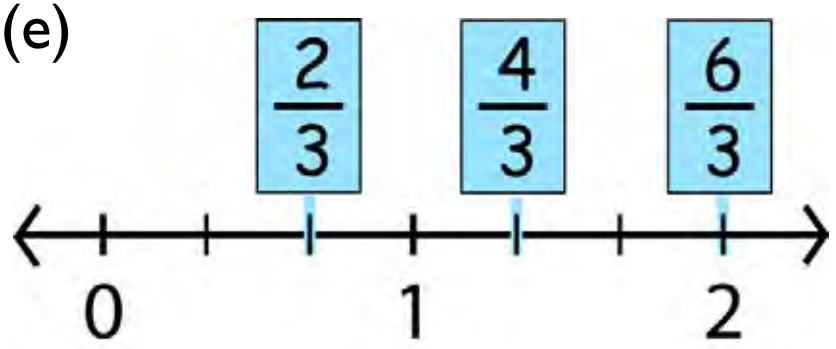
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BLOCK 2

The Summer Institute and Lab Class as a Case of Professional Development for Professional Developers

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CPTM 2004 Summer Institute



68 teacher developers



18 prospective elementary teachers

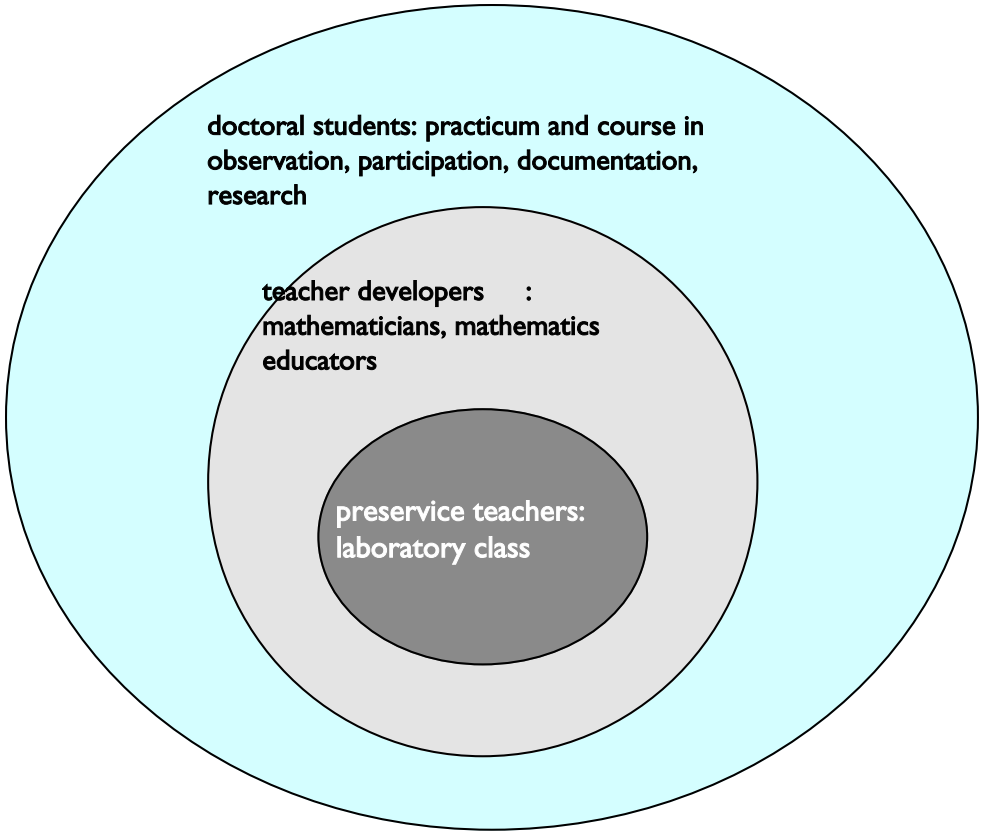
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Summer Institute Focal Questions

- What mathematical knowledge and practices play a central role in the everyday work of teaching?
- What are promising approaches for helping teachers learn mathematics for teaching and learn to use it in their work?

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How was the Summer Institute Designed, and Why?



Layers of learning

68 participants

18 preservice teachers

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A Typical Day

- Plan for lab class (8:00 - 8:45)
- Observe lab class (9:00 - 11:00)
- Analyze what happened in lab class (11:30 - 12:30)
- Lunch (12:30 - 2:00)
- Issues involved in helping teachers develop mathematical knowledge for teaching (2:00 - 3:00)
- Mini-courses (3:30 - 5:30)
- Dinner (6:30 - 8:00)
- Work with colleagues and explore resources (8:00 -)

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Cookie Jar Problem

There was a jar of cookies on the table. Kira was hungry because she hadn't had breakfast, so she ate half the cookies. Then Steve came along and noticed the cookies. He thought they looked good, so he ate a third of what was left in the jar. Niki came by and decided to take a fourth of the remaining cookies with her to her next class. Then Kayla came dashing up and took a cookie to munch on. When Pam looked at the cookie jar, she saw that there were two cookies left. "How many cookies were there in the jar to begin with?" she asked Kira.

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The Lab Class: Context

- A one- or two-credit mathematics content course for elementary education students
- 18 preservice elementary teachers:
  - 7 master's level certification program students; 11 undergraduate certification program students
- Second day of class
- Content course (MKT):
  - fractions
  - mathematical practices: using representations, making and evaluating mathematical explanations
- Laboratory

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Video from Lab Class

- What stands out to you about —
  - the students
  - the mathematics problem as the students work on it
  - the teaching
  - the classroom
  - the class as a "lab" class

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Video clips here

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Student Work from Lab Class

- What stands out to you about the students' work?
- What are they asked to write about? What do they write and draw?
- What did they not record or write about?

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Shifting Our Focus: The Lab Class as an Opportunity for Professional Developers' Learning

- What is available for participants to learn from observing the lab class?
  - *about mathematical knowledge for teaching?*
  - *about preservice teachers as learners?*
  - *about teaching preservice teachers?*

- How do you think participants may have reacted to the lab class? What do you think they will notice and attend to?

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Video from Participants' Discussion

- What strikes you about the participants? What do you notice?

- What are the participants doing in these clips? What do they notice and attend to?

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Participants' Work

- What stands out to you about the participants' work in their notebooks?
- What kinds of things do they attend to, and in what ways? What do they not comment on?

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BLOCK 3
Connecting to Your Own Work

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Questions

- How might you be able to use these design and content principles in developing the professional development you could do?
- What are other design and content principles you would consider important for the education of professional developers?

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Design Principles

- Practice-based (live and records)
- Intensiveness of time spent together
- Shared experience provides common context for learning/discussing
- Allow for and take advantage of diversity of participants
- Participants contribute to the content of the institute
- Provide public time and private time
- Provide formal and informal opportunities for learning
- Make teaching visible
- Mathematical knowledge for teaching
- Developing preservice teachers' MKT
- Alternative images of mathematics courses
- Alter views of preservice teachers

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Tell Us How This Went . . .

- What was interesting or useful for you?
- What worked well?
- What was less interesting or useful?
- What did not work well?
- Other suggestions?

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