How Does Mathematical Language Figure in the Work of Teaching?

How Does this Shape Mathematical Knowledge for Teaching (MKT) and Teacher Education?

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Overview of session

1. Introduction: Mathematical knowledge for teaching, and why focus on mathematical language?
2. Investigate the use of mathematical language in practice: Examples from two classrooms
3. Consider tasks for prospective elementary teachers
4. Conclusions

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1. Introduction: Mathematical knowledge for teaching and why focus on mathematical language?

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Toward a practice-based theory of mathematical knowledge for teaching

1. Study instruction, and identify the mathematical work of teaching
2. Analyze what mathematical knowledge is needed to do that work effectively, and how it must be understood to be useful for the work
3. Develop and evaluate approaches to helping teachers learn mathematical knowledge for teaching
4. Develop, test, and refine measures of mathematical knowledge for teaching using multiple methods as a means to evaluate professional education, investigate effects on students' learning, and improve theory

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Mathematical knowledge for teaching (MKT)

- What do we mean when we use this term, "mathematical knowledge for teaching"?
  - Mathematical knowledge, skill, habits of mind that are entailed by the work of teaching
- What do we mean by the "work of teaching"?
  - The tasks in which teachers engage, and the responsibilities they have, to teach mathematics, both inside and outside of the classroom

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Examples of work of teaching

- Examining the mathematical quality of a textbook treatment
- Using and defining terms
- Producing and evaluating mathematical explanations
- Using notation
- Interpreting and evaluating alternative solutions and thinking
- Choosing contexts with care for mathematical integrity, diversity, and transparency for learning
- Explaining goals and mathematical purposes to others
- Designing homework and quizzes; Selecting and modifying mathematics problems
- Choosing and using representations
- Building correspondences between a model and a concept or procedure
- Evaluating students' work and statements (often quickly)
- Posing questions
- Choosing examples

mathematical language

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What is important about language in mathematics?

- Necessary for mathematical communication
- Foundational to mathematical reasoning

An odd number plus an odd number equals an even number.

- Precision matters

What is an even number?

- Denotative, not inferential

~~**A number that can be divided in two equal parts with nothing left over is even.**~~

Is 7 even?

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What do we mean by "mathematical language"?

- Focus on four aspects of mathematical language:
  - Mathematical vocabulary
  - Mathematical notation
  - Metaphors and jargon
  - Grammar and syntax
- Selected because of prevalence in the literature and their impact on mathematics learning and teaching

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Mathematical vocabulary

- Mathematics uses words also used in other settings. This can be an advantage, but also presents challenges. (Halliday, 1978; Pimm, 1987)

  Examples:
  - Big, odd, even, prime, rational, radical, square
  - Right, similar, circle, regular, acute, obtuse
- The mathematical uses of these words are specialized.
- Definitions are used to specify technical uses of terms within mathematics.

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Mathematical notation

- Symbols compress ideas in useful ways, making them conceptually and linguistically smaller to work with.

  Example:

  It is easier and more precise to write:

  $$x^2 - y^2 = (x + y)(x - y)$$

  than: The difference of the squares of any two numbers is the product of their sum with their difference.

- The same symbol can have different meanings depending on its position, use, etc. (Carpenter, Franke, & Levi, 2003; Woodrow, 1982)

  $1 \quad 2 \quad 3 \ldots \quad 212 \quad a^2 \quad a_2 \quad 1001_2$

8 + 4 = __ + 5

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Metaphors and jargon

- Metaphors can be powerful tools for explaining and understanding new mathematical concepts.
- Although beneficial, not without cost (Nolder, 1991; Pimm, 1987)
  - Many of the confusions between the everyday and mathematical uses of words mentioned above have metaphorical roots (e.g., multiply)
  - Sometimes a metaphor is appropriate when a concept is first introduced, but doesn't hold up as students learn more mathematics
  - Irrelevant features of the metaphor can cause distraction
- Linguistic mnemonics often used in efforts to help students learn new ideas or procedures:
  - "You can't subtract a bigger number from a smaller number."

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Grammar and syntax

- Issues of precision and clarity often related to grammar and syntax
- Some syntactic constructions common in mathematics make for dense and complex sentences
- Pronouns often have vague referents (Back, 2000; Rowland, 1999)
- Not understanding the logical structure of mathematical statements hinders students' abilities to reason about and prove mathematical claims (Epp, 1999; Selden & Selden, 1995)

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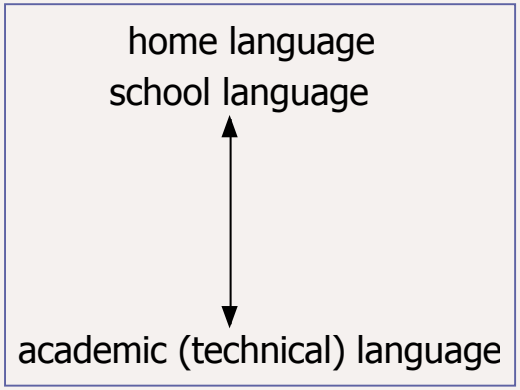
Importance of mathematical language

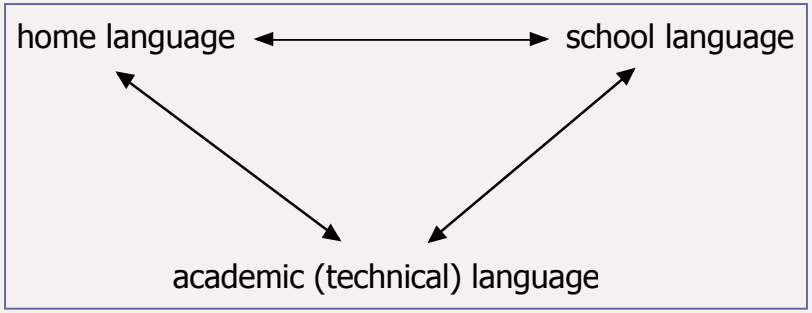
- Mathematical knowledge and reasoning depends on and is supported by mathematical language.
- Teaching and learning mathematics depends on and is supported by language.
- Mathematical language is both mathematical content to be learned and medium for learning mathematical content.

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Issues with mathematical language: Mathematical vs. "everyday" language

- Mathematics often uses and specializes everyday language rather than coining a separate technical vocabulary -- both enabling and complicating entry to its register.
- In learning mathematics, students talk informally and imprecisely
- Equity issue: Additional dimension for students who must navigate between home, school, and mathematical languages





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Measuring the quality of mathematical language in instruction

Video codes about language:

- Conventional notation
- Technical language
- General language
- Explicitness about language

Found a significant correlation between errors in language and scores on paper and pencil measure of MKT (-.708)

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Implications for teacher education

- Need to provide opportunities for prospective teachers to learn to carefully attend to and use mathematical language in practice
- Requires a better understanding and articulation of the work of using mathematical language in teaching
- Requires the design of tasks that support prospective teachers' learning to do this work

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Goals for session

- Uncover other mathematical language issues that arise in teaching: What do teachers have to do with mathematical language in their work?
- Explore the mathematical knowledge demands and practices of carefully attending to and using mathematical language in teaching
- Consider tasks that can be used in teacher education

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2. Investigating the use of mathematical language in practice: Examples from two classrooms

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Video #1

- Beginning of a 5th grade lesson
- Two warm-up problems: Division practice

$$96 \div 8 \text{ and } 32 \div 7$$

- Not in their regular classroom (teachers' lounge)

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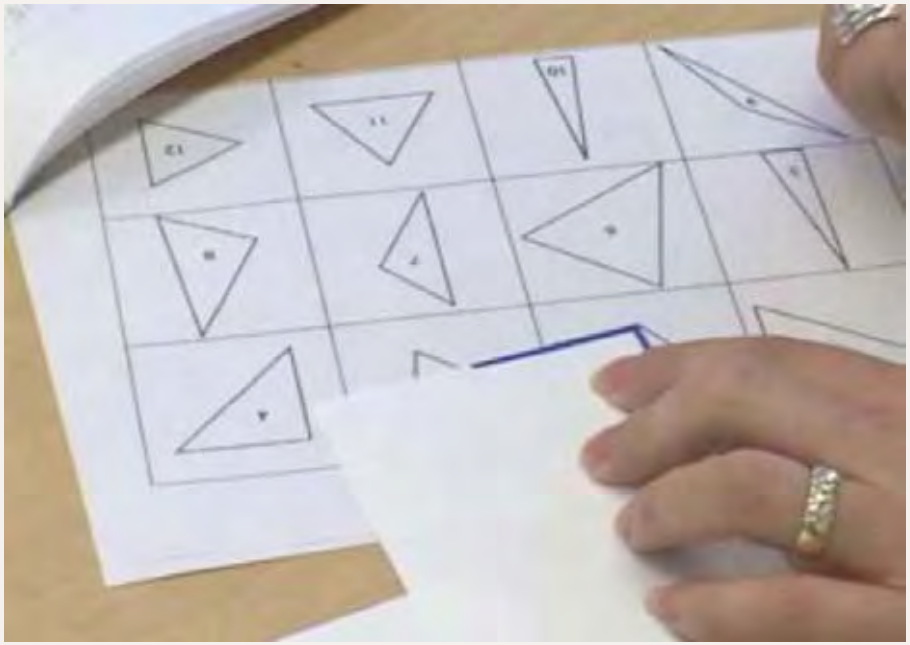
Focus questions

- What issues of mathematical language arise in teaching?
- What do teachers have to do with mathematical language in their work?
- What are the mathematical knowledge demands and practices of carefully attending to and using mathematical language in teaching?

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Video #2

- 3rd grade geometry lesson
- Students have been introduced to polygons and to different types of angles (right, greater than, less than)
- Introduction to lesson on measuring angles in triangles



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Focus questions

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Discussion questions

- What issues of mathematical language arise in teaching?
- What are the mathematical knowledge demands and practices of using mathematical language in teaching?
- What might be the implications for teacher education? What might we want to teach prospective elementary teachers to do with mathematical language?

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3. Exploring tasks for prospective elementary teachers

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Examples of teacher education tasks about mathematical language

- Analyze and write mathematical definitions
- Identify and investigate words that have different everyday and mathematical meanings
- Analyze and modify textbook lessons
- Analyze assessment items for their mathematical language demands

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What makes a "good" definition?

- Mathematically precise — correctly identifies the kind of object, process, property
- Usable by user community — based on already-defined and understood term

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What is a mathematically precise and usable definition of "even number" for third graders?

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Definitions: Even numbers

a) An even number is a number that can be divided into two equal parts.

b) An even number is any multiple of 2.

c) An even number is any integer multiple of 2.

d) An even number is any number whose unit digit is 0, 2, 4, 6, or 8.

e) An even number is a whole number with zero remainder when divided by 2.

f) A whole number is even if it is the sum of a whole number with itself.

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Definitions of even numbers

a) An even number is a number that can be divided into two equal parts.
b) An even number is any multiple of 2.
of a) and b): All numbers, for example 7, 3/5, $\sqrt{2}$, π, are even!

c) An even number is any integer multiple of 2.
of c): This is a correct definition of even number.

d) An even number is any number whose unit digit is 0, 2, 4, 6, or 8.
of d): In this case, 36.7 is an even number!

e) An even number is a whole number with zero remainder when divided by 2.
of e): According to this, -6 is not even.

f) A whole number is even if it is the sum of a whole number with itself.
of f): This is a correct definition of evenness for whole numbers, and is consistent with the general definition for integers that will arrive later.

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Analyze and modify a textbook lesson

Develop a general understanding of lesson:

- Mathematical content and purpose
- Students as learners of this content

Examine specific features of the lesson in relation to instructional goals:

- Tasks and their progression
- Examples
- Context
- Representations and tools
- **Language**

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Analyzing the **language** of a textbook lesson

Mathematical integrity	All students' learning

What issues around mathematical language do we want preservice teachers to consider when planning lessons?

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Analyzing the **language** of a textbook lesson

Mathematical Integrity	All Students' Learning
Is learning vocabulary or symbols part of the goals of the lesson? Are some more important and other less so?	Are there words that have different mathematical and everyday meanings?
Is there language in the lesson (or things you might be tempted to say) that portrays the math in ways that might conflict with what students learn later on?	What terms or symbols do students need to understand to be able to do the work of this lesson?
Is their language you want to deliberately use when you describe or explain procedures or concepts?	Are students familiar with the vocabulary/symbols in the problems? Can they read them?
What language is used around representations and tools? Is there language you want to make sure you use consistently?	Are there terms that might pose special problems for ELLs?
	Is there confusing language or unnecessary words that might hinder some students from being able to engage in the work?

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Conclusions

1. Language is the medium of teaching and learning.
2. Language is foundational to mathematical proficiency.
3. Developing fluency and skills with mathematical language is a crucial part of mathematical knowledge for teaching.
4. Tasks designed to engage teachers in aspects of the work of teaching that involve language can help to develop that fluency.

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