Knowing and Using Mathematics in Teaching

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University of Michigan

Learning Network Conference

Teacher Quality, Quantity, and Diversity

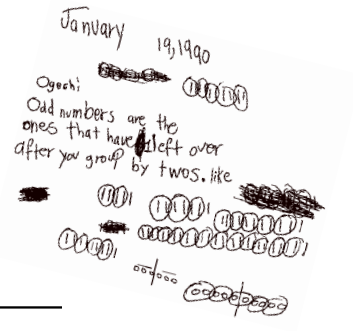
January 30 - 31, 2006

Washington, DC

Length of side of main square

Possible sized squares	1	2	3	4	5	6	7	10	50
1x1	1	4	9	16	25	36	49		
2x2		1	4	9	16	25	36		
3x3			1	4	9	16	25		
4x4				1	4	9	16		
5x5					1	4	9		
6x6						1	4		
7x7							1		
8x8									
9x9									
10x10									
Total	1	5	14	30	55	91	140		





B

$$\begin{array}{r} {}^{1}2\,35 \\ \times\ 25 \\ \hline 255 \\ 80 \\ \hline 1055 \end{array}$$

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cptm lmt

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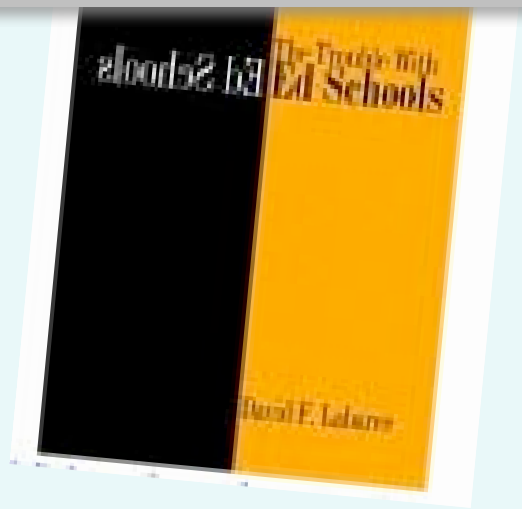
Overview of Session

1. Setting the policy context
2. Mathematical knowledge for teaching: A theory in progress
3. Developing and using measures of mathematical knowledge for teaching
4. How might this work inform work on content knowledge for teaching other subjects?

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U.S. DEPARTMENT OF EDUCATION
Secretary's No Child Left Behind

I. Setting the Policy Context

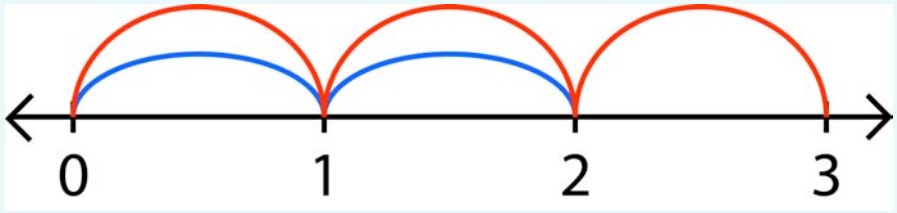


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Challenges to the Education Profession

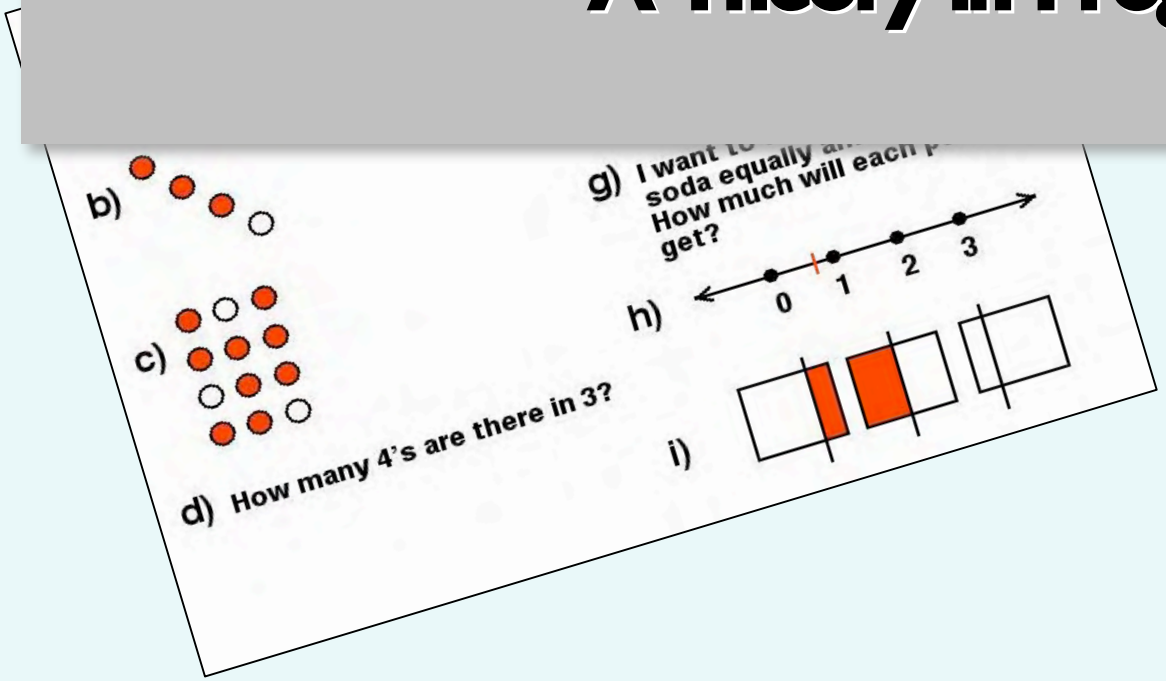
1. Articulate characteristics of "high quality" teaching, and the knowledge and skill demands for teachers
2. Seek evidence on effects
3. Develop rigorously and relevantly warranted professional knowledge that is both credible and useful to
   - education professionals (teachers, teacher educators, researchers)
   - other stakeholders (policymakers, public)

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$$2 \div \frac{2}{3}$$

2. Mathematical Knowledge for Teaching: A Theory in Progress



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Common Framing of the Problem → Failure to Solve the Problem

- Teachers do not know mathematics well enough (evidence and causes vary)
- Concentration of under-qualified teachers in urban and high poverty schools

- Increase number and rigor of mathematics requirements for teacher certification
- Recruit mathematically-trained people into teaching

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Clarifying the Problem

The quality of mathematics teaching and learning

Teachers' knowledge of mathematics

and their ability to use it in teaching

Not: How MUCH mathematics do teachers need to know?

But: What mathematical knowledge and skill does high quality teaching require?

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A Practice-Based Approach to Answering the Question

1. **Study instruction**, and identify the mathematical work of teaching
2. Analyze **what mathematical knowledge** is needed to do that work effectively, and **how** it must be understood in order to be useful for the work
3. Develop, test, and refine **approaches to helping teachers** develop and use mathematical knowledge for teaching, study results

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Mathematical Knowledge for Teaching

What do we mean when we use this term, "**mathematical knowledge for teaching**"?

- Mathematical knowledge, skill, habits of mind that are entailed by the work of teaching

What do we mean by the "**work of teaching**"?

- The tasks in which teachers engage, and the responsibilities they have, to teach mathematics, both inside and outside of the classroom

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Knowing Multiplication

Multiply:

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline \end{array}$$

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Knowing Multiplication for Teaching: Analyzing Student Errors

How was each answer produced?

What might lead a student to make these errors?

(a)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 405 \\ 108 \\ \hline 1485 \end{array}$$

(b)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 225 \\ 100 \\ \hline 325 \end{array}$$

(c)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 1250 \\ 25 \\ \hline 1275 \end{array}$$

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Knowing Division with Fractions

Divide:

$$2 \div \frac{2}{3}$$

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Knowing Division with Fractions for Teaching:
Analyzing Alternative Methods

(a)

"We don't need to invert and multiply!"

$$2 \div \frac{2}{3} = \frac{2}{1} \div \frac{2}{3} = \frac{1}{1/3} = 3$$

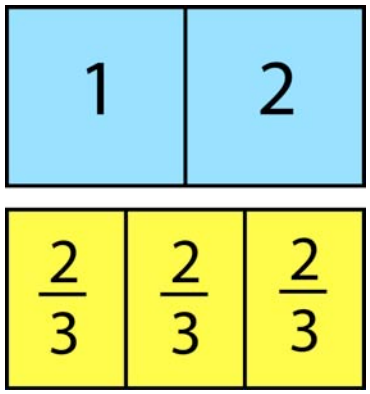
(b) "$2 \div \frac{2}{3} = 3$ because 2 is two-thirds of 3."

Which of these methods would work for dividing any two fractions?

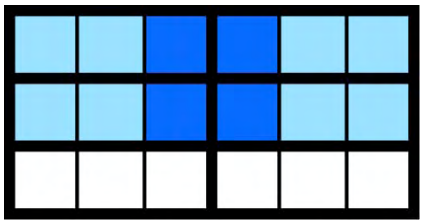
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Knowing Division with Fractions for Teaching:
Choosing and Using Representations

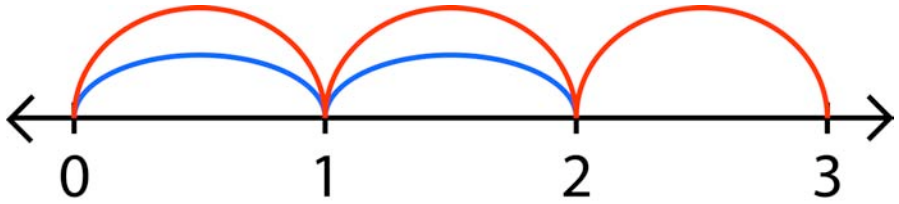
(a)



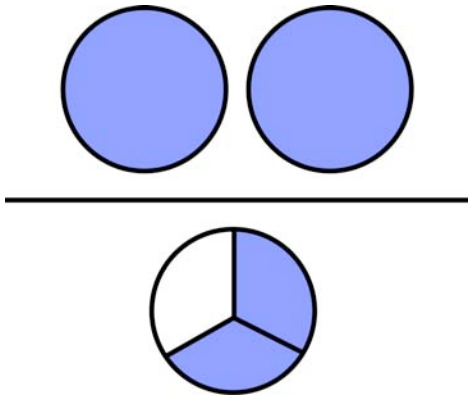
(b)



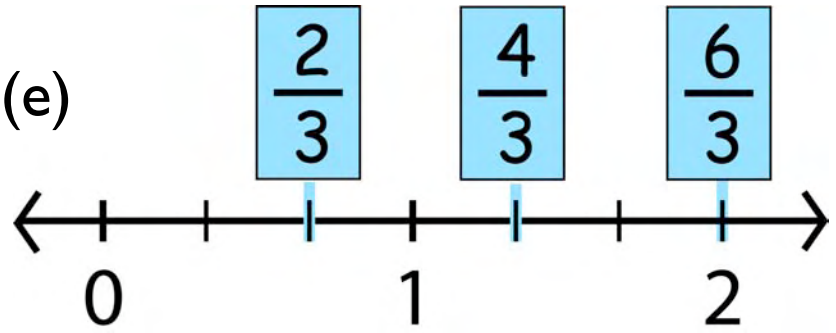
(c)



(d)



(e)



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Knowing Radical Expressions

Simplify:

$$\sqrt{150}$$

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Knowing Radical Expressions for Teaching: Choosing Numerical Examples

Which of the following is <u>best</u> for setting up a discussion about different solution paths for simplifying radical expressions?

(a) $\sqrt{54}$

(b) $\sqrt{156}$

(c) $\sqrt{128}$

(d) These examples all work equally well.

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Other Work of Teaching

- Attending to issues of equity (e.g., language, contexts, mathematical practices)
- Using and defining terms
- Using notation
- Explaining and guiding explanation
- Listening to (and hearing) others' solutions and thinking
- Choosing contexts with care for mathematical integrity, diversity, and transparency for learning

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Hypotheses About Knowledge of Mathematics for Teaching

PREVALENT HYPOTHESES

1. Teachers need knowledge of the curriculum, plus N levels more knowledge
2. "Pedagogical content knowledge" (— but what is it?)

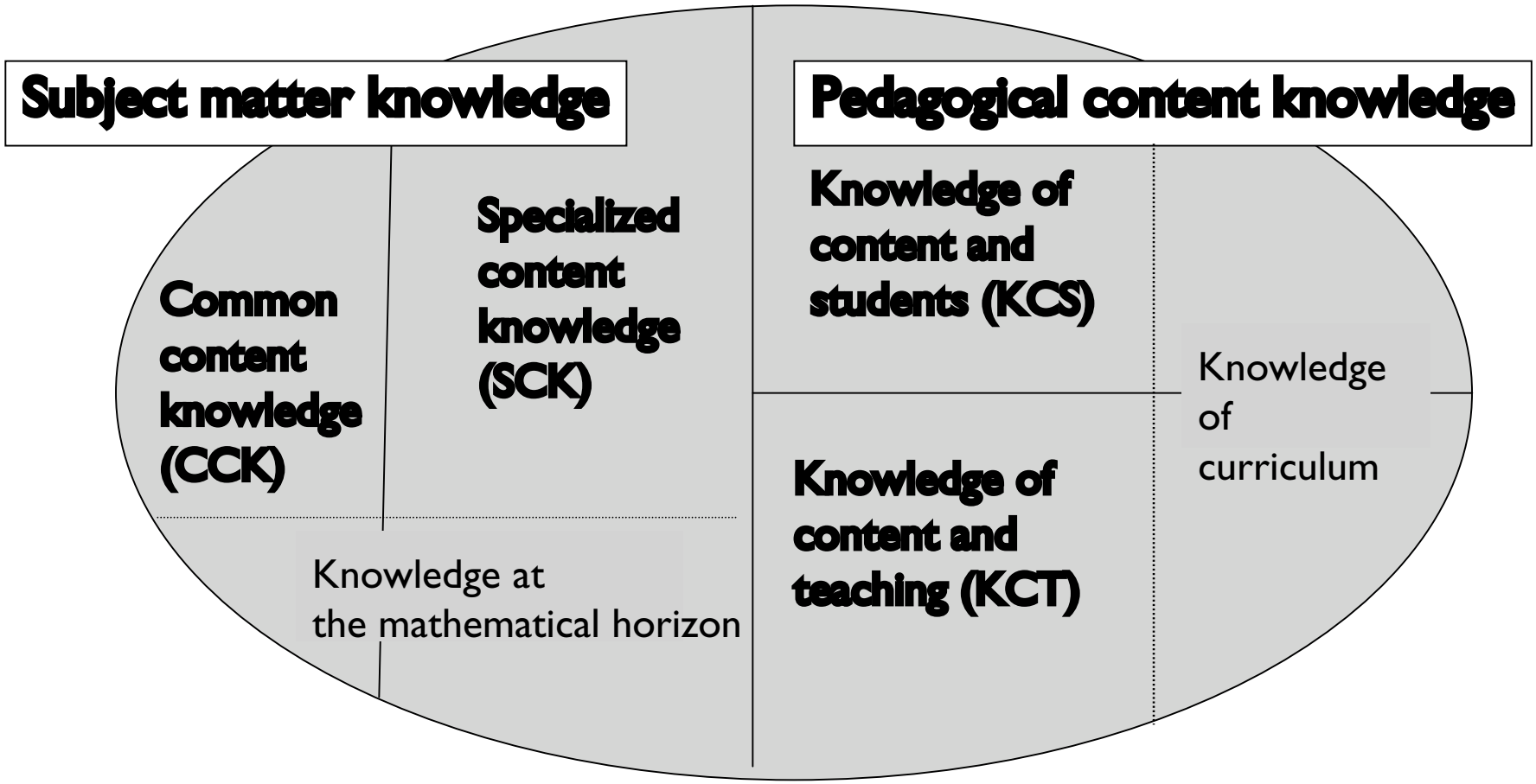
OUR CURRENT HYPOTHESES

- **Common** content knowledge
- **Specialized** content knowledge
- Knowledge of content and **students**
- Knowledge of content and **teaching**

What does this have to do with "pedagogical content knowledge"?

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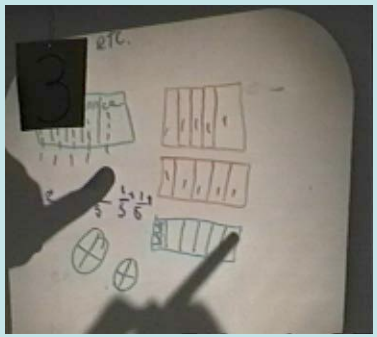
Shulman's Original Category Scheme (1985) Compared with Ours



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3. Developing and Using Measures of Mathematical Knowledge for Teaching





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Opportunity: Study of Instructional Improvement

- Study of three Comprehensive School Reforms; teacher knowledge a key variable
- Instrument development goals:
  - Develop measures of content knowledge teachers *use* in teaching – not just *what* they teach
  - Develop measures that discriminate among teachers (not criterion referenced)
  - Non-partisan

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Overarching Findings: Factor Analyses

- Multidimensionality of mathematical knowledge for teaching
  - Knowledge of students and content different from "pure" content knowledge
  - Knowledge differentiable by topic domain (e.g., algebra, geometry)
  - Number and and operations mathematical knowledge for teaching
    - Specialized
    - Common
- Tentative conclusion: existence of a kind of "professional" knowledge for teaching

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Validating Our Measures

How do we interpret teachers' performance on our questions?

1. Their score reflects their mathematical thinking
   - **Cognitive interviews**
2. Higher scores mean higher-quality mathematics instruction
   - **Videotape validation study**
3. Scores reflect common and specialized knowledge of content
   - **Mathematician and non-teacher interviews**
4. Higher scores related to improved student learning
   - **Study of Instructional Improvement student gains analysis**

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Studying Links Between Teachers' Knowledge of Mathematics for Teaching and Student Achievement

- Question: Do students learn more when teachers perform better on our measures?
  - SII data and analysis – measured CKT on questionnaire
  - SII measures highly reliable, from test construction perspective
- Model: Student Terra Nova gains predicted by:
  - Student descriptors (family SES, absence rate)
  - Teacher characteristics (math methods/content, content knowledge)
- Teacher content knowledge significant
  - Small effect (< 1/10 standard deviation)
  - But student SES is also on same order of magnitude

(Hill, Rowan, & Ball, AERJ, 2005)

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Studying Teachers' Gains in Mathematics for Teaching from Professional Development

- Items piloted in California's Mathematics Professional Development Institutes (MPDI)
  - Instructors: Mathematicians and mathematics educators;
  - 40-120 hours of professional development
  - Focus was squarely on mathematics content
  - Summer 2001
  - Pre/post assessment format (parallel forms)

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Findings from MPDI Evaluation(2001)

- Length of institute predicts teacher gains
  - 120-hour institutes most effective, on average
  - But some 40-hour institutes very effective
- Focus on mathematical analysis, proof, and communication leads to higher gains
- Many questions remain
  - Effects of content (e.g., mathematics vs. student thinking)
  - Treatment of content: "packed"/"unpacked"
  - Effects of teacher motivation

(Hill & Ball, JRME, 2004)

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Who Knows Content Well Enough to Teach It, and How Do We Make the Argument?

- Content knowledge for teaching is the knowledge needed for the work of teaching. That work entails specialized content knowledge distinct from that required by other professions.
- Figuring out what teachers need to know requires multiple kinds of work, including research-based evidence. It is possible to write questions that reliably and validly measure content knowledge for teaching. The work of producing usable items requires multiple kinds of expertise, a good conceptual frame, and many cycles of revision and improvement.
- Such questions are more likely to seem credible to teachers.
- Work on these questions is central to the future of the profession, and to the improvement of education.

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Slides will be available at

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