Lesson 12: Division of Rational Numbers, Part I

Initial PST Thinking: Many PSTs will hold the misconception that division always makes smaller, have difficulty writing story problems for division with fractions and decimals, and have difficulties with representing division of fractions with diagrams and interpreting the remainder.

Learning Goals: By the end of the lesson, PSTs will…

- Write simple story problems for division with fractions and decimals.
- Represent division-of-fraction problems with diagrams that show copies of the divisor fitting into the dividend.
- Know how to interpret and represent the remainder in division-of-fractions/decimals problems.

Equipment:

- Transparencies 1, 2, 3, & 4
- Copies of Handout 1
- Copies of the Homework

Associated Files:
L12_Division I_LP_S06.doc (this lesson plan)
L12_Division I_T_S06.doc (Transparencies 1 – 4)
L12_Division I_H_S06.doc (Handout 1)
L12_Division I_HW_S06.doc (Homework)

Associated Text: Section 5.3, Understanding Operations with Fractions, p. 281 – 301

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0-25 min

Activity 1: Going over the Lesson 11 Homework

Go over the Lesson 11 homework. (This part of the lesson plan is omitted here because it deals with the previous mathematical topic.)

25-45 min

Activity 2: Division of Rational Numbers – Writing Story Problems and Drawing Diagrams

Individual Work: Give each PST the following task to do on their own:

> *Write a realistic story problem that can be represented by 6 ½ ÷ ½. Then show how you would illustrate the division to a child.*

Select some PSTs to present their story problems and diagrams. If possible, instructor should select PSTs who have written number sentences that correspond to 6 ½ ÷2 (both partitioning and measurement versions) and ½ x 6 ½. If no PST makes these "mistakes", then instructors should present the following Transparencies to the class for discussion:

> Transparency 1: *Mary and Jim had a pizza party and there were 6 ½ 8" round pizzas left over. They want to split the leftovers between the two of them equally. How much pizza will each one get?* [Partitioning story for 6½ ÷ 2]
>
> Transparency 2: *Jim has 6 ½ gallons of water in a container in the refrigerator. Each day, his household drinks 2 gallons of water from the container. How many days will it take to empty the container?* [Measurement story for 6½÷2]
>
> Transparency 3: *Mica has 6 ½ quarts of Bolda Cola. If she drinks half of it, how much does she drink?* [A story for ½ x 6½]

Once the problems are up, the instructor should discuss the differences between partitioning ("equal sharing") and measurement

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("repeated subtraction") models, as review. Instructors should say something like the following:

> *As you remember from Math251, division can be thought of in two ways. One way is using a partitioning or "equal sharing" model in which you have a quantity to be partitioned into a certain number of equal-sized groups. (For example, in 8÷2, you could have 8 objects and want to partition them into two equal-sized groups. Or, you have 8 items that you want to share evenly between two people.) Another way is using a measurement or "repeated-subtraction" model in which you have a quantity and you want to see how many times you can subtract or remove a certain amount from that original quantity. (For example, in 8÷2, you could have 8 objects, and you take away 2 of the objects, and see how many times you can do that.) In this class, we will be using all of these terms interchangeably, not to confuse you, but so that you gain familiarity with all of them in the future.*
>
> *For future consideration: Maybe we should also include a missing factor interpretation of division. A good problem would be a story problem (see the N. Mack JRME article) such as, "A rectangle has area 6 ½ square meters. If one of its sides has length ½ meter, then what is the length of the other side of the rectangle?" This might provide a useful contrast to the other problems and help the PSTs sort out the critical features of these problems.*

Instructors should remind PSTs that they could interpret the measurement interpretation as asking, "How many copies/groups of the divisor fit into the dividend?" This has helped some PSTs connect what they learned previously with this situation.

As instructors go through each story problem on the board (and possibly the transparencies), they should ask whether the problem uses a partitioning or measurement model of division. Keeping track of this distinction has been found to help PSTs sort out conceptual differences among later fraction division situations.

PST Responses	Instructor Responses
Some PSTs may have difficulty with the story problem and confuse it with division by 2. Others may take ½ of 6 ½ rather than dividing 6 ½ by ½.	Instructor should try to get PSTs to write the number sentences, and even work out the problems either through the context or through the number sentence (whether correct or not).

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If no PSTs had issues such as these, the three story problems from the transparencies can elicit these issues.	In addition, the instructor can always ask PSTs if the answer makes sense if you actually did this.
Most PSTs who succeed in writing a correct story problem for *6 ½ ÷ ½* will probably use the measurement interpretation for division.	If no one suggests a correct division story for 6 ½ ÷ ½, present Transparency 4: "Sara was planning a 6½-mile race for charity and wanted to place water stands every ½ mile. How many water stands will she have to set up?"

Remind PSTs of the terms "dividend", "divisor", and "quotient". Also, remind PSTs of the terms "whole" (corresponding to dividend), "number of groups", and "size of groups" from Math 251.

- In a partitioning problem, you know the whole (dividend) and the number of groups (divisor), and you want to find the size of the groups that can be made (quotient).
- In a measurement problem, you know the whole (dividend) and the size of the groups (divisor), and you want to find the number of groups that can be made (quotient).

Point out that 6 ½ divided by 2 (as exemplified in either the incorrect PST responses or in Transparencies 1 & 2) is an example of "division making smaller." However, 6 ½ divided by 1/2 is an example of "division making bigger". A division problem "makes smaller" when it produces a quotient that is smaller than the dividend. A division problem "makes bigger" when it produces a quotient that is bigger than the dividend.

Rationale

This activity was selected because we have learned that the activity helps PSTs make explicit for themselves the difference between dividing by a natural number and dividing by a positive number less than one. The learning principle of this activity is that confronting one's misconceptions is a good way to increase one's understanding of the key ideas. Solving a problem that will likely invoke a common misconception and then making the misconception explicit is an effective activity for many PSTs. (By the way, division by ½ is especially problematic, given everyday language like "divide in half.") Once the PSTs are reminded of the ways of interpreting division problems with partitioning and measurement, we want to extend this to division of fractions and decimals. They should see that a partitioning model can be difficult to apply when the divisor is a non-integer. However, the measurement interpretation can easily be

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applied to any division problem. We also use this activity because it leads to the notion that division does not always "make smaller."

45-75 min

Activity 3: "Susie Q"

Group Work: Distribute Handout 1 and give PSTs about 15 minutes to work on the problems with their group members.

Susie Q has $4\frac{2}{3}$ *yards of ribbon. She wants to make prize badges that each requires* $\frac{3}{4}$ *yards of ribbon.*

For future consideration: We might want to change the numbers so that the remainder isn't so awkward for this first problem. Some instructors have reported that this awkwardness might interfere with PSTs focusing on the essential elements of this situation.

1) How many whole prize badges can Ms. Q make? (Answer: 6 whole badges)
2) How much ribbon is left over?(Answer:1/6 yard of ribbon)
3) How much of another prize badge can Ms. Q make?(Answer: 2/9 of another badge)

PST Responses	Instructor Responses
Most PSTs will correctly respond that Ms. Q can make 6 whole badges. Some will determine this by using the standard "invert and multiply" algorithm for dividing fractions to divide 4 2/3 by ¾. Others will repeatedly subtract ¾ from 4 2/3. Others may draw fraction-strip diagrams to represent and solve the problem.	
PSTs who used the standard algorithm for dividing fractions will confuse the non-whole part of the answer (or, the remainder) with the amount of ribbon leftover.	

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In light of question #2, some PSTs will still be confused and unsure of their responses.	If PSTs are unsure of what this question is asking, instructors might say something like the following: *In question #2, you told me how much ribbon Ms. Q would have left over. Using that leftover ribbon, she might not be able to make another whole badge, but she could at least begin a new badge. How much, or what fraction, of another badge could she make with the leftover ribbon?*
Rationale: This activity is designed to improve PSTs' ability to interpret and represent the remainder in fraction division problems. It will be a necessary component to the next activity in the next day of making more detailed diagrams of division problems. Responses to this task provide a measure for how well PSTs are achieving the learning goal.	
Homework: • Lesson 12 Homework	
Rationale The homework is similar to what was discussed during this lesson about remainders. In addition to providing a useful assessment, it will provide the PSTs with another opportunity to figure out the meaning of the remainder in a fraction situation. In addition, the last problem on the homework will serve as practice of what was covered in this lesson and set the stage for drawing diagrams in Lesson 13.	

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