What Mathematics Knowledge is Needed for Teaching Mathematics?

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Multiply: What is the correct answer?

$$\begin{array}{r} 35 \\ \times\ 25 \\ \hline \end{array}$$

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What is Necessary Mathematics Knowledge for Teaching?

1. Examine teaching, and identify the mathematical work that teaching entails
2. Analyze what mathematical knowledge — topics and skills — is needed to do that work
3. Analyze also the qualities of that knowledge — how it must be understood and known to be serviceable for the work

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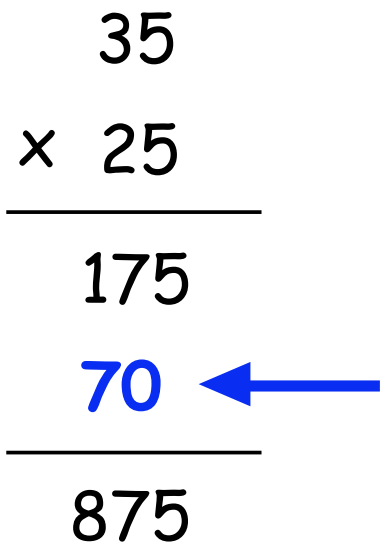
Evaluating and interpreting common incorrect student answers

$$\begin{array}{r} 35 \\ \times\ 25 \\ \hline 175 \\ 70 \\ \hline 245 \end{array}$$

What is the misunderstanding here?

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Explaining mathematical concepts and procedures



Why do we "move over" before writing 70?

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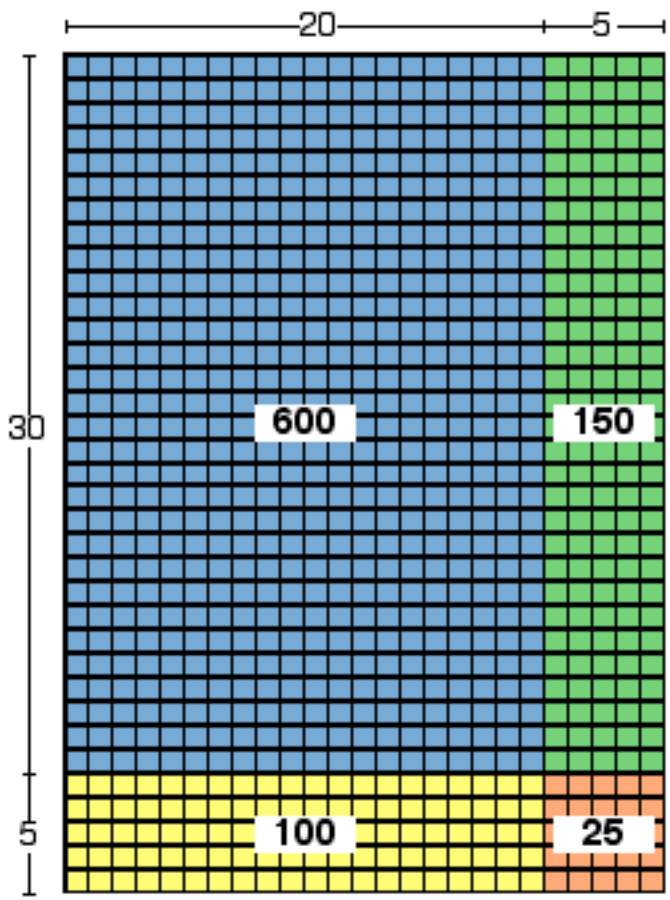
Evaluating and appraising different methods

Student A	Student B	Student C
35 x25 125 +75 875	35 x25 175 +700 875	35 x25 25 150 100 +600 875

Which of these students is using a method that could be used to multiply any two whole numbers?

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Using representations to model and reason about procedures



A

35
25
125
+75
875

B

35
25
175
+700
875

C

35
x 25
25
100
150
+ 600
875

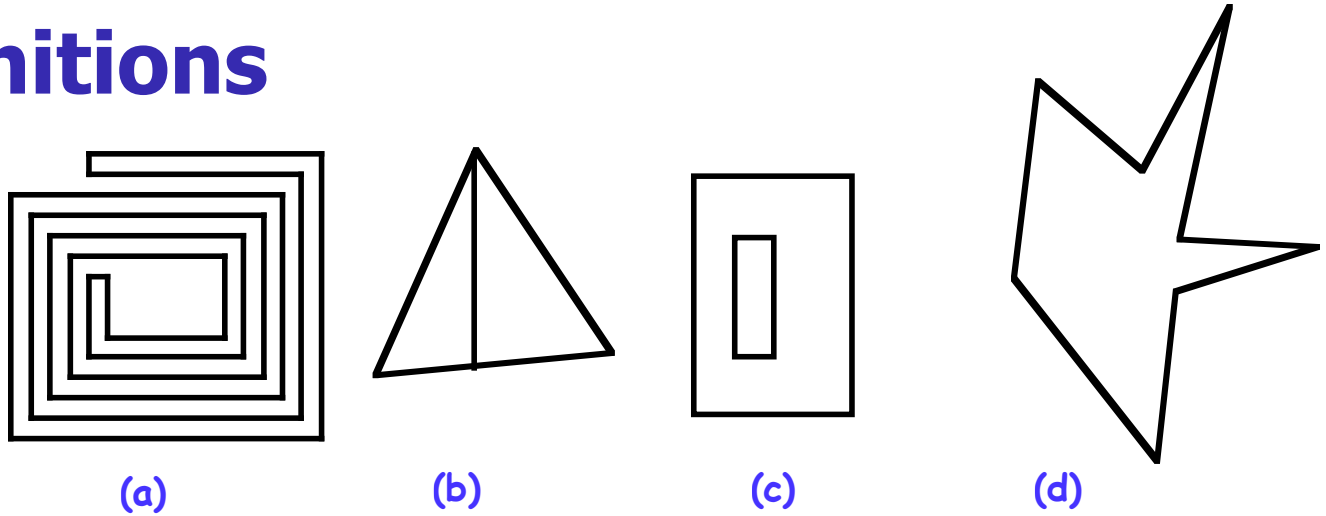
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Qualities of Mathematics Knowledge for Effective Instruction

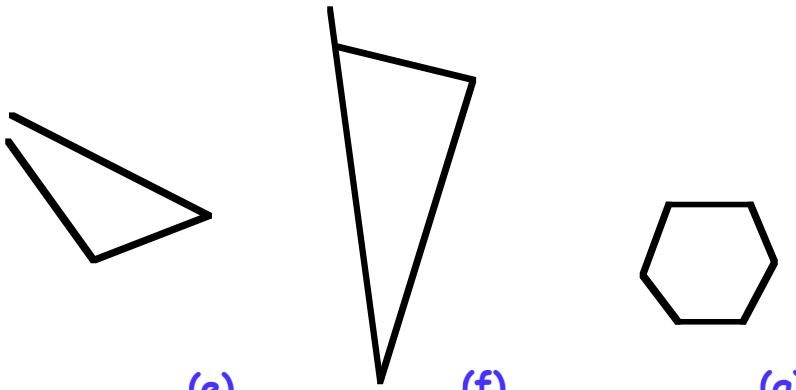
- Respectful of the integrity of the discipline
- Able to be extended and opened up for learners — "unpacked"
- Justified, reasoned
- Connected within and across domains, building on earlier ideas and anticipating more advanced topics
- Organized psychologically as well as logically

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Using definitions



(a) (b) (c) (d)



(e) (f) (g)

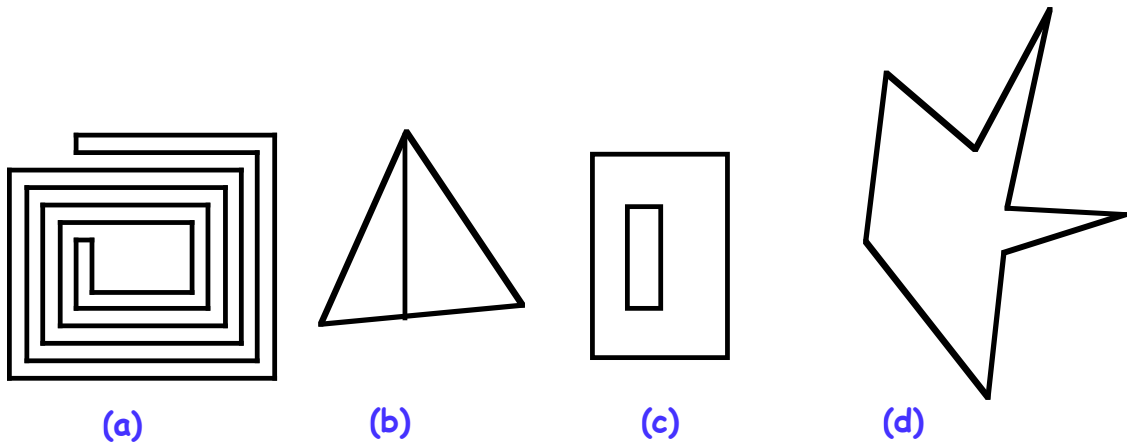
Which shapes are polygons?

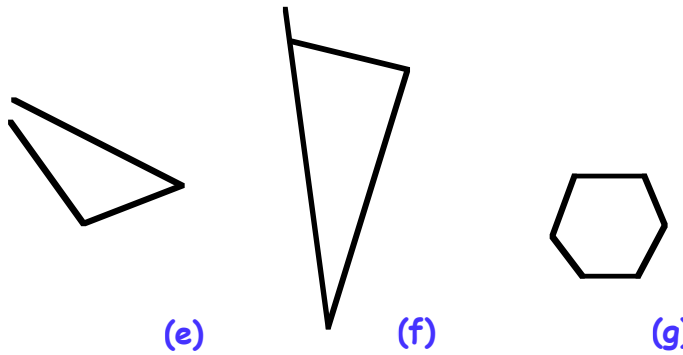
Textbook definition: A closed flat two-dimensional shape whose sides are formed by line segments.

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Definition: A simple closed plane curve formed by straight line segments.

But — what's a mathematically acceptable definition of "polygon" that is usable by fifth graders?





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Looking Back: What Mathematics Do Teachers Have to Do?

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Looking Back: What Mathematics Do Teachers Have to **Do**?

- Design mathematically accurate explanations that are comprehensible and useful for students
- Use mathematically appropriate and comprehensible definitions
- Represent ideas carefully, mapping between a physical or graphical model, the symbolic notation, and the operation or process
- Interpret and make mathematical and pedagogical judgments about students' questions, solutions, problems,and insights (both predictable and unusual)
- Be able to respond productively to students' mathematical questions and curiosities
- Make judgments about the mathematical quality of instructional materials and modify as necessary
- Be able to pose good mathematical questions and problems that are productive for students' learning
- Assess students' mathematics learning and take next steps

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Looking Back: What Mathematics Do Teachers Need to **Know** to Do Those Things?

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Looking Back: What Mathematics Do Teachers Need to **Know** to Do Those Things?

In general:

- Topics and ideas that are fundamental to the school curriculum— and beyond
- Tools and skills for reasoning about mathematical claims, ideas, representations, and solutions; and sensibility about what constitutes adequate proof
- Fluency and care with mathematical language and notation
- Familiarity with applications of mathematics

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Looking Back: What Mathematics Do (Elementary) Teachers Need to Know?

What provides mathematical leverage?

- Concepts of number and place value notation
- Operations
- Number theory and number systems
- Common algorithms and how and why they work
- Concepts and tools of algebra
- Geometric concepts and reasoning
- Concepts and tools of statistics and probability
- Representing and connecting representations (e.g., symbols, graphs, geometric models)
- Mathematical language and definitions
- Mathematical reasoning and justification
- Good sense about mathematical precision
- Mathematical curiosity and interest

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Conclusion:
Knowing Mathematics for Teaching — Three Core Principles

1. Teachers need to know the same things and be able to do the same things as any adult, and much more.

2. Knowledge needed for teaching is different from what is needed for other occupations or professions where mathematics is used (e.g., physics, mathematics, carpentry, tailoring, business).

3. Knowledge needed for teaching must be usable for the mathematical work that teachers have to do.

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