Qualities of Mathematics Knowledge Useful for the Work of Teaching

- Respectful of the integrity of the discipline
- Able to be extended and opened up for learners — "unpacked"
- Justified, reasoned
- Connected within and across domains, building on earlier ideas and anticipating more advanced topics
- Organized psychologically as well as logically

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Overview of Session

1. Introductions
2. Overview and framing of the day: Checking in with you?
3. Cookie jar problem: Noticing mathematical practices
4. Investigating mathematical practices in third grade: student learning and teacher practices
5. Mathematical practice #1: Teaching mathematical explanation in a mathematics course for teachers
6. Mathematical practice #2: Defining mathematical terms
7. Wrap up and homework for Workshop II

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Investigating "Mathematical Practices"

What do we mean by "mathematical practices"?

The tools, skills, habits of mind and actions that form the basis of learning, doing and using mathematics proficiently. (RAND, 2001)

- For students: an important part of mathematical proficiency; a key to equity in mathematics achievement

- For teachers:
  - Helping ***students*** develop mathematical practices
  - The ***work of teaching*** itself involves using mathematical practices

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Cookie Jar Problem

There was a jar of cookies on the table. Jerry was hungry because he hadn't had breakfast, so he ate half the cookies. Then Maria came along and noticed the cookies. She thought they looked good, so she ate a third of what was left in the jar. Julie came by and decided to take a fourth of the remaining cookies with her to her next class. Then Sister Rita came dashing up and took a cookie to munch on. When Hillary looked at the cookie jar, she saw that there were two cookies left. "How many cookies were there in the jar to begin with?" she asked Joel.

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Discuss Cookie Jar Problem

- What is the content of the problem in its written form? In its enacted form?

- What mathematical practices were used in our work on the Cookie Jar Problem?

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Studying a Classroom Lesson Segment: Mathematical Practices in Students' Learning, in the Work of Teaching

- Third grade class, students aged 8 - 10
- Multicultural, multilingual
- Addition and subtraction computation, using various models (number line, beansticks)

Joshua ate 16 peas on Monday and 32 peas on Tuesday. How many more peas did he eat on Tuesday than he did on Monday?

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Viewing Focus for Video

What mathematical practices are being used by the students?

What mathematical practices are being used by the teacher?

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Focus: Two Mathematical Practices

1. Giving, interpreting, and evaluating mathematical explanations
2. Defining mathematical terms

WHY THESE TWO PRACTICES?

- Central to sense-making
- Fundamental for mathematical reasoning
- Key for mathematical communication

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The Lab Class: Context

- A two-credit mathematics content course for elementary education students
- 18 preservice elementary teachers:
  - 7 master's level certification program students; 11 undergraduate certification program students
- Second day of class
- Content course (focused on mathematical knowledge for teaching):
  - fractions
  - mathematical practices: using representations, making and evaluating mathematical explanations
- Laboratory for working on the teaching and learning of MKT

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Clip #1: Students' Initial Explanations

1. What do you notice about what the students are doing, about what teacher is doing, and about the content?
2. Is this an explanation? What does it have, or lack?
3. What would you count as an explanation in this situation?

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Clip #2: Explanations Elaborated

1. What do you notice about what the students are doing, about what teacher is doing, and about the content?
2. Is this an explanation? What does it have, or lack?
3. What would you count as an explanation in this situation?

Pay particular attention to the moves of the teacher.

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Clip #3: Launching Explicit Work on Mathematical Explanation

STUDENTS' WORK IN CLASS

- Are you convinced that the only solution to the Cookie Jar Problem is 12? If so, what has convinced you? If not, what is leaving you skeptical?

- Are there things you have noticed that you would call "features of good mathematical explanations"?

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Investigating Student Work

- What do the preservice teachers seem to be doing and learning with respect to mathematical explanation? Find specific examples.

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Features of "Good" Mathematical Explanations for the Cookie Jar Problem

- Makes clear at the outset what is being explained, and why you start there, and carefully connects the explanation to the question or idea being explained
- Starts from the beginning, and traces the logical flow of the reasoning
- Should be logical and complete, makes conclusion clear and links back to original question or claim or problem
- Might number the steps if appropriate, or label parts of a diagram
- Strives to be as simple and clear as possible
- Defines terms as needed, uses available definitions as needed
- Uses representation(s) accurately (algebraic, geometric, etc.), and combining representations
- Links the language and diagrams clearly to the steps of the argument
- Shows what something means or why is true, and is convincing to the person to whom you are explaining
- Is calibrated to the context (considers the person to whom you are explaining, and what is already established as true and does not need more explanation)

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Tell Us How This Went . . .

- What was interesting or useful for you?
- What worked well?
- What was less interesting or useful?
- What did not work well?
- Other suggestions?

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Slides will be available at

http://www-personal.umich.edu/~dball/

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