Developing Measures of Mathematical Knowledge for Teaching

Deborah Loewenberg Ball, Heather Hill, Hyman Bass, Stephen Schilling

University of Michigan

Learning Mathematics for Teaching/Study of Instructional Improvement

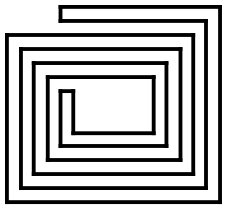
Teachers Development Group Leadership Seminar

Mathematics Professional Development

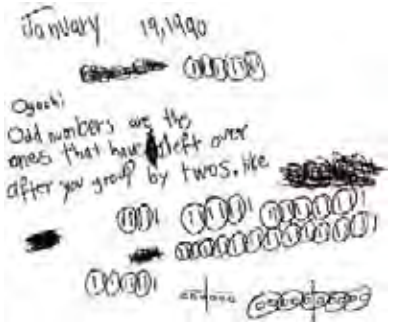
February 18, 2005

Length of side of main square

Possible sized squares	1	2	3	4	5	6	7	10	50
1x1	1	4	9	16	25	36	49		
2x2		1	4	9	16	25	36		
3x3			1	4	9	16	25		
4x4				1	4	9	16		
5x5					1	4	9		
6x6						1	4		
7x7							1		
8x8									
9x9									
10x10									
Total	1	5	14	30	55	91	140		







B 1 2 35
x 25
255
80
1055



www.sii.soe.umich.edu
www.soe.umich.edu/lmt

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Overview

1. "Mathematical knowledge for teaching"
2. Efforts to "measure" teachers' mathematical knowledge: Purposes, history
3. Our measures development approach
4. Using data to test and improve theory: factor analysis, reliability, validity
5. Trying your hand at writing items



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Hypotheses About Knowledge of Mathematics for Teaching

PREVALENT HYPOTHESES

1. Teachers need knowledge of the curriculum, plus N levels more knowledge
2. Pedagogical content knowledge, curricular knowledge

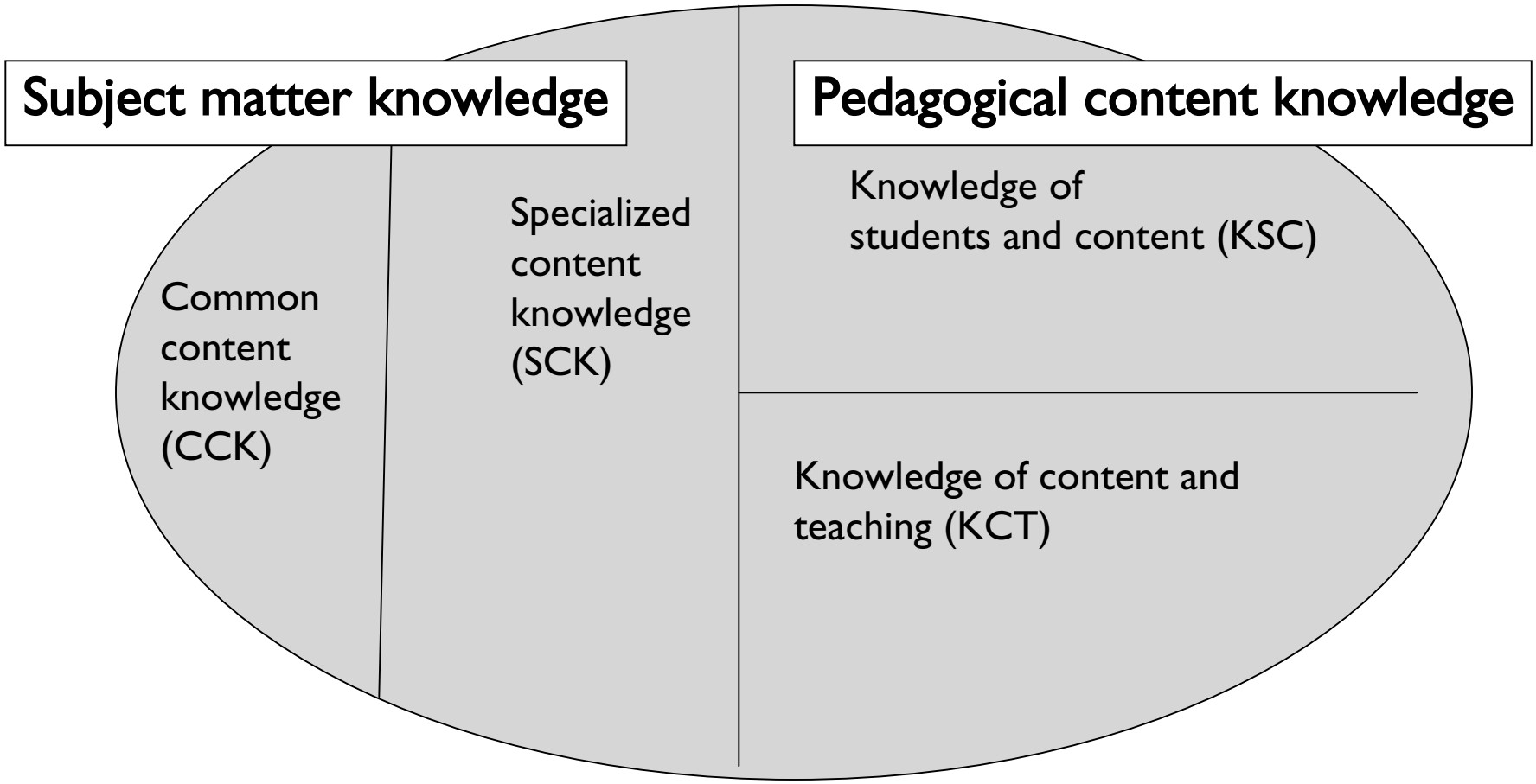
OUR CURRENT HYPOTHESES

- **Common** content knowledge
- **Specialized** content knowledge
- Knowledge of content and **students**
- Knowledge of content and **teaching and curriculum**

What does this have to do with "pedagogical content knowledge"?

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Shulman's Original Category Scheme (1985) Compared with Ours



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Why Would We Want to "Measure" Teachers' Content Knowledge for Teaching?

- To understand role of teachers' content knowledge in students' performance
- To study and compare outcomes of professional development and teacher education
- To inform design of teachers' opportunities to learn content knowledge

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Study of Instructional Improvement

- Study of three Comprehensive School Reforms; teacher knowledge a key variable
- Instrument development goals:
  - Develop measures of content knowledge teachers *use* in teaching – not just *what* they teach
  - Develop measures that discriminate among teachers (not criterion referenced)
  - Non-partisan

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Problems As We Began This Work

- No way to measure teachers' content knowledge for teaching on a large scale
  - Small number of items, many written by Ball, Post, others appeared on every instrument
  - Nothing known about the statistical qualities of those items (difficulty, reliability)
  - Studies relying on single items -- single items unlikely valid or reliable measures of teacher knowledge



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Early Decisions and Activity

- Survey-based measure of content knowledge for teaching mathematics
  - 3000 teachers participating in SII
  - Multiple choice
- Specified domain map
- 5 people + 5 lbs cheese + 5 weeks = 150 items (May 2001)
- Large-scale piloting, summer 2001



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Original Sampling Frame

	Types of knowledge		
Mathematical content		Common and specialized content knowledge	Using knowledge of students and content
	Number		
	Operations		
	Patterns, functions, and algebra		



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Sample Items
K-5 Number and Operations

- Common knowledge

  What number is halfway between 1.1 and 1.11?

- Specialized knowledge
  1. Representing mathematical ideas and operations
  2. Providing explanations for mathematical ideas and procedures
  3. Evaluating mathematical methods, claims, or solutions

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Definitions

- Instrument
- Stem
- Item
- Distractor
- Measure (*n.)*, scale



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Representing Number Concepts

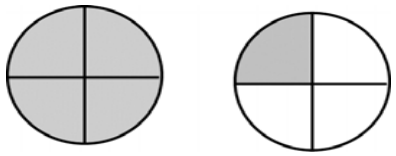
Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or ten students, or a single rectangle. On one particular day, she uses as the whole a picture of two pizzas. What fraction of the two pizzas is she illustrating below? (Mark ONE answer.)

a) 5/4

b) 5/3

c) 5/8

d) 1/4



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Providing Mathematical Explanations: Divisibility Rules

Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4? (Mark ONE answer.)

a) Four is an even number, and odd numbers are not divisible by even numbers.

b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.).

c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.

d) It only works when the sum of the last two digits is an even number.

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Evaluating Solution Approaches

Student A	Student B	Student C
35 x25 125 +75 875	35 x25 175 +700 875	35 x25 25 150 100 +600 875

Which of these students is using a method that could be used to multiply any two whole numbers?

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Using Data to Test and Improve Theory

- Factor analyses
- Analyses of reliability
- Analyses of validity



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Factor Analysis

- Enables analyses of the number of underlying factors in a data set
- Can link items to constructs
- Ask similar question of our data:
  - How many factors? What are they named?
  - How do items group?



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Overarching Findings: Factor Analyses

Multidimensionality of CKT

- Number and and operations CK
  - Specialized
  - Common
- Patterns, functions & algebra CK
- Geometry CK
- Knowledge of students and content



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Analyses of Reliability

- What is "reliability"?
  - How well a set of items discriminates among individuals of different ability levels
  - Ratio of "signal" in the data to "signal + noise"
- Item response theory (IRT) enables analyses of:
  - "Difficulty" of an item relative to other items for a particular population
  - Whether items items discriminate well among individuals of different ability

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Reliability Results

	Number of items	1 pl reliability	2 pl reliability	max info at
Number & Operations	18	.76	.79	-.25
Patterns, functions, algebra	15	.84	.87	-1.00
Geometry	43	.91	.92	-.62



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

An Example:
Evaluating Teacher Professional Development



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Example: Tracking Teacher Growth

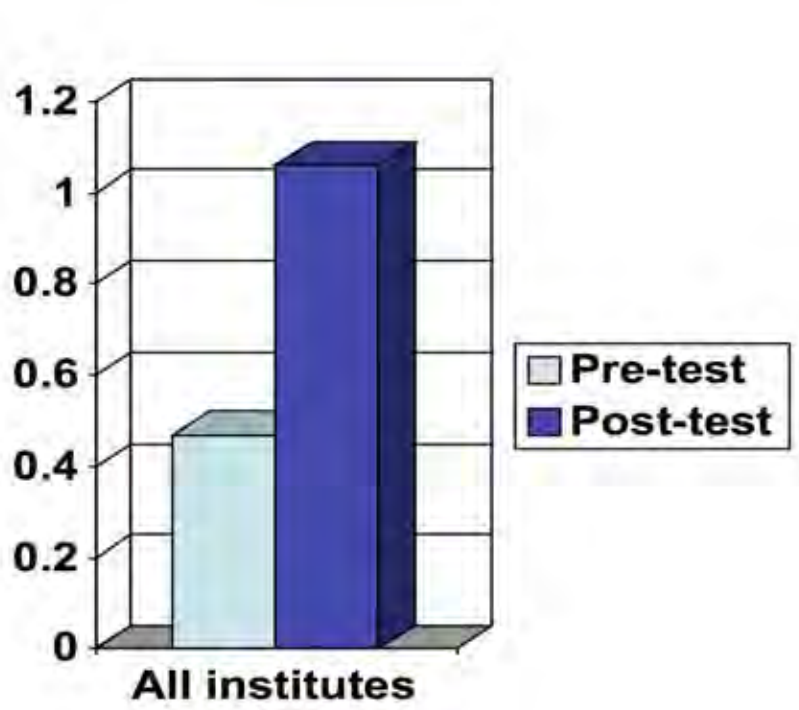
- Items piloted in California's Mathematics Professional Development Institutes (MPDI)
  - Instructors: Mathematicians and mathematics educators
  - 40-120 hours of professional development
  - Focus is squarely on mathematics content
  - Summer 2001
  - Pre/post assessment format (parallel forms)



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

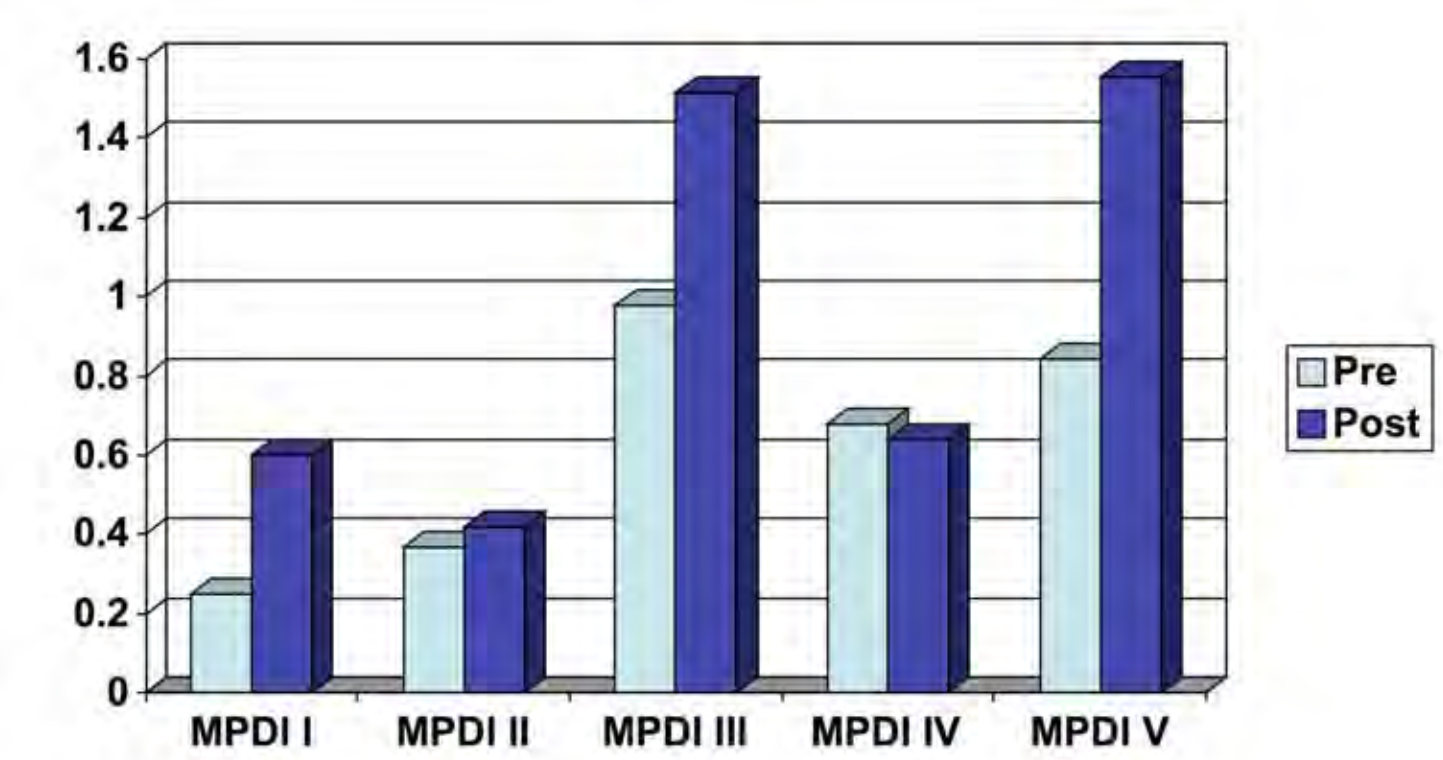
MPDI Teacher Growth

- For all institutes for which we have data, teachers gained .48 logits, or roughly ½ standard deviation
- Translates to 2-3 item increase on assessment
- Considered substantial gain



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Results from Sample Institutes



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

MPDI Evaluation: Other Findings

- Length of institute predicts teacher gains
  - 120-hour institutes most effective, on average
  - But some 40-hour institutes very effective
- Focus on mathematical analysis, proof, and communication leads to higher gains
- Many questions remain
  - Effects of content (e.g., mathematics vs. student thinking)
  - Treatment of content: "packed"/"unpacked"
  - Effects of teacher motivation



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Validating the Measures

1. Validation theory

2. Validation results
   a) Cognitive interviews
   b) Videotape validation study
   c) Linking measures to student achievement
   d) Mathematician interviews

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Assessing Validity: Our Approach

- Constructing a validity argument (Kane)
  - How do we interpret teachers' performance on our assessment?
    - Reflects their mathematical thinking
    - Higher scores mean higher-quality mathematics instruction
    - Higher scores related to improved student learning
    - Scores reflect common and specialized knowledge of content
  - Investigate interpretation with multiple sources of evidence

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Answering Interpretive Questions

- How do we interpret teachers' performance on our assessment?
  - Reflects their mathematical thinking
    - *Cognitive interviews*
  - Higher scores mean higher-quality mathematics instruction
    - *Videotape validation study*
  - Higher scores related to improved student learning
    - *Study of Instructional Improvement student gains analysis*
  - Scores reflect common and specialized knowledge of content
    - *Mathematician and laypeople interviews*



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

1. Cognitive Interviews



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Answering Interpretive Questions

- How do we interpret teachers' performance on our assessment?
  - **Reflects their mathematical thinking**
    - *Cognitive interviews*
  - Higher scores mean higher-quality mathematics instruction
    - *Videotape validation study*
  - Higher scores related to improved student learning
    - *Study of Instructional Improvement student gains analysis*
  - Scores reflect common and specialized knowledge of content
    - *Mathematician and laypeople interviews*

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Cognitive Interviews

- Recruited K-6 teachers via email, flyers
- Teachers completed surveys, then:
  - Selected 23 lowest- and highest-performing teachers for interviews
  - Sent interview items, asked teachers to answer
  - Interviewed around 36 number & operations and geometry items (45-60 minutes)
- Question: What made you choose (answer)? What process did you go through to decide?
- Limited cognitive interview

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Cognitive Interviews

Mr. Lewis asked his students to divide 6/8 by 1/2. Charlie said, "I have an easy method, Mr. Lewis. I just divide numerators and denominators. I get 6/4, which is correct." Mr. Lewis was not surprised by this as he had seen students do this before. What did he know?

a)He knew that Charlie's method was wrong, even though he happened to get the right answer for this problem.

b)He knew that Charlie's answer was actually wrong.

c)He knew that Charlie's method was right, but that for many numbers this would produce a messy answer.

d)He knew that Charlie's method only works for some fractions.

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Answering Charlie

$$\frac{6}{8} \div \frac{1}{2} = \frac{6 \div 1}{8 \div 2} = \frac{6}{4}$$

$$\left(\frac{6}{8} \cdot \frac{2}{1} = \frac{12}{8} = \frac{6}{4} \right)$$

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Answering Charlie

$$\frac{6}{8} \div \frac{1}{6} = \frac{6 \div 1}{8 \div 6} = \frac{6}{\frac{8}{6}} = \frac{6}{1} \cdot \frac{6}{8} = \frac{36}{8} = 4\frac{1}{2}$$

$$\left(\frac{6}{8} \cdot \frac{6}{1} = \frac{36}{8} = 4\frac{1}{2} \right)$$

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Cognitive Interviews

Mr. Lewis asked his students to divide 6/8 by 1/2. Charlie said, "I have an easy method, Mr. Lewis. I just divide numerators and denominators. I get 6/4, which is correct." Mr. Lewis was not surprised by this as he had seen students do this before. What did he know?

a)He knew that Charlie's method was wrong, even though he happened to get the right answer for this problem.

b)He knew that Charlie's answer was actually wrong.

c)He knew that Charlie's method was right, but that for many numbers this would produce a messy answer.

d)He knew that Charlie's method only works for some fractions.

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Cognitive Interviews

Wrong answer, wrong reasoning:

> "For 3 I put A ("he knew that Charlie's method was wrong, even though he happened to get the right answer for this problem"), basically because that's not how you do it, or it's not how I learned how to do it. And he did get the right answer but I thought it was just happenchance."

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Cognitive Interviews

Right answer, right reasoning

"I did a whole bunch of figuring on this one. Let's see...I finally went with answer C ("He knew that Charlie's method was right, but that for many numbers this would produce a messy answer") and I was actually kind of surprised when I started doing it that it worked, and it just kept working and then-but then I said I gotta be doing something. So then I picked numbers where I would get a messy answer. So I ended up with decimals and a fraction."

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Cognitive Interviews

Inconsistent answer/thinking:

(This respondent answered A: He knew that Charlie's method was wrong, even though he happened to get the right answer for this problem) …And I kind of started from the bottom (answer) working my way up. (D) Charlie's method only works for some fractions so then I started thinking of different numbers of trying it in my head and it was working for all of them I was trying in my head, so I didn't circle that one. Then it said (C) his method was right and I thought well it may be right for him, but it's not the one I learned so I'm not going to circle that one. And then (B) answer was wrong, it wasn't that. And then for this one I thought Mr. Lewis being the teacher has probably taught this method where you invert the fraction. So he's probably going to say that the method is wrong (A) even though he got the right answer for the problem.

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Other Cognitive Interview Results for Charlie

- Among teachers, 7% for this item (2/27 responses)
  - One response changed as interviewer probed on answer
  - One stem problem (anticipating what Mr. Lewis will say, rather than assessing method)
- Distractors work well in general
- IRT slope .721; difficulty 1.67

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Overarching Findings: Cognitive Interviews

- Cognitive interviews
  - 1.85% inconsistency rate across items
  - Half of inconsistents are changed incorrect-correct during interviews
  - KSC problems

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

2. Videotape Validation Study



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Answering Interpretive Questions

- How do we interpret teachers' performance on our assessment?
  - Reflects their mathematical thinking
    - *Cognitive interviews*
  - **Higher scores mean higher-quality mathematics instruction**
    - ***Videotape validation study***
  - Higher scores related to improved student learning
    - *Study of Instructional Improvement student gains analysis*
  - Scores reflect common and specialized knowledge of content
    - *Mathematician and laypeople interviews*

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Videotape Validation Study

- Question: Do higher scores correspond with better classroom mathematics instruction?
- Method
  - 10 K-6 teachers took our survey
  - Videotaped each teacher teaching 9 lessons
  - Developing rubric to quantify quality of mathematical instruction delivered



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Videotape Validation Study

- Rubric grades 5-minute segments as appropriate/inappropriate on:
  - Use of mathematical language
  - Representation of mathematical ideas
  - Presence of mathematical explanation or justification
  - Linkages between classroom task (i.e., what kids are doing) and important mathematical ideas
  - Facility in listening to children's mathematical productions
  - Computational or other mathematical errors (always inappropriate)

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Videotape Validation Study

- No findings as of yet
  - Blind to teachers' scores as we code



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

3. Links to Student Achievement



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Answering Interpretive Questions

- How do we interpret teachers' performance on our assessment?
  - Reflects their mathematical thinking
    - *Cognitive interviews*
  - Higher scores mean higher-quality mathematics instruction
    - *Videotape validation study*
  - **Higher scores related to improved student learning**
    - ***Study of Instructional Improvement student gains analysis***
  - Scores reflect common and specialized knowledge of content
    - *Mathematician and laypeople interviews*



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Links to Study of Instructional Improvement Student Achievement Analysis

- SII measure – 38 items, 6 from Comparison Group
  - SII: .89 IRT reliability
  - 6 items overlap with Comparison B
- Model: Student Terra Nova gains predicted by:
  - Student descriptors (family SES, absence rate)
  - Teacher characteristics (math methods/content, content knowledge)
- Teacher content knowledge significant
  - Small effect (LT 1/10 standard deviation)
  - But student SES is also on same order of magnitude

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

4. Mathematician Interviews



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Answering Interpretive Questions

- How do we interpret teachers' performance on our assessment?
  - Reflects their mathematical thinking
    - *Cognitive interviews*
  - Higher scores mean higher-quality mathematics instruction
    - *Videotape validation study*
  - Higher scores related to improved student learning
    - *Study of Instructional Improvement student gains analysis*
  - **Scores reflect common and specialized knowledge of content**
    - ***Mathematician and laypeople interviews***

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Analyzing the Interview Data: What We Hope to Learn

- How do professional mathematicians answer the items? How do they explain and justify their responses?
- Which items do mathematicians seem to find more difficult, and why? Which items take them longer to answer, and why?

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Why Might Mathematicians Have Trouble with SCK Items?

- Compressed knowledge that is characteristic of post-graduate mathematical expertise
- Tendency to make additional assumptions to remove perceived ambiguity
- Lack of familiarity with the school curriculum
- Flaws in the items

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Trying Your Hand at Writing Items

"Item Camp"



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

When Writing Items (1)

- Try to write in following content areas:
  - Number and operations
  - Patterns, functions, and algebra
  - Geometry
- We can use grades K-8 items; write others for your own use



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

When Writing Items (2)

- Items must be unambiguously right or wrong
- Think of clever distractors—what would someone with little knowledge think?
- Write easy, medium, and hard items
- Do not cross content (e.g., items requiring both content knowledge and knowledge of student thinking, or both geometry and algebra)



© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

When Writing Items (3)

- Ignore (or subvert) ideologies about teaching mathematics
- Avoid excess verbiage
- Provide item documentation
  - Author
  - Inspiration (your student teachers; observations of a classroom; curriculum materials; research)
  - Correct answer

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

When Writing Items (4)

- Write items that reflect how mathematics is <u>used in teaching</u>
  - Use your knowledge of classrooms and teachers – what issues arise? What do new teachers struggle with?

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Study of Instructional Improvement

Learning Mathematics for Teaching (LMT) Project

lmt

For further information, please go to:

http://www.sii.soe.umich.edu

http://www.soe.umich.edu/lmt

© 2005 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission