

# Making the Iowa Mathematics Core Curriculum Learnable By All


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**Iowa Council of Teachers of Mathematics Annual Meeting • Des Moines, IA**

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SCHOOL OF EDUCATION UNIVERSITY OF MICHIGAN


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# The urgency

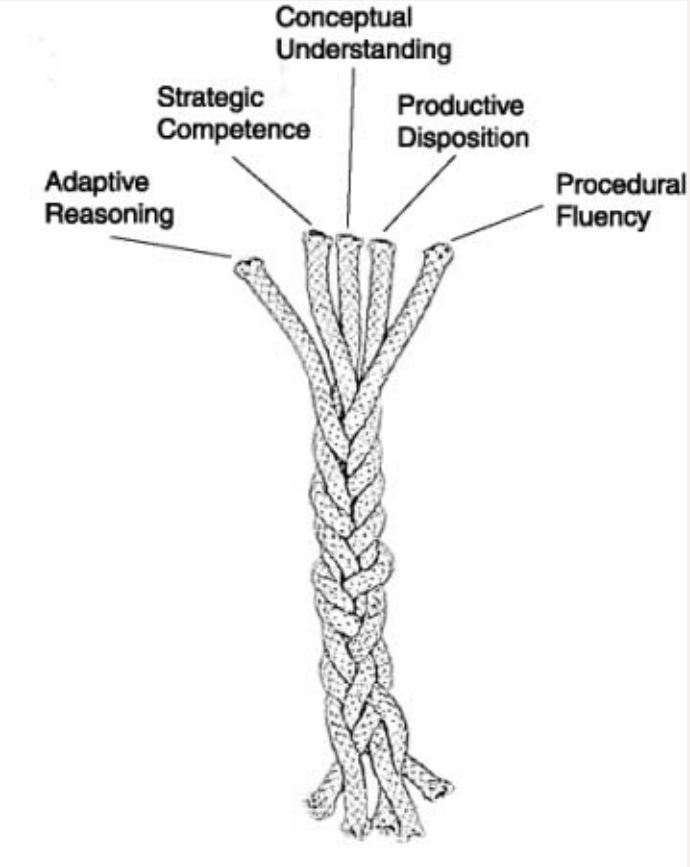
1. Enormous gaps in learning opportunities and disparities in achievement (within U.S. and in international comparisons)
2. Rapidly changing school population
3. Higher, more complex academic goals
4. High expectations for all students






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# Strands of mathematical proficiency




- ***Conceptual understanding*** - comprehension of mathematical concepts, operations, and relations
- ***Procedural fluency*** - skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- ***Strategic competence*** - ability to formulate, represent, and solve mathematical problems
- ***Adaptive reasoning*** - capacity for logical thought, reflection, explanation, and justification
- ***Productive disposition*** - habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

Kilpatrick, J., J. Swafford, and B. Findell. (2001). *Adding It Up: How Children Learn Mathematics*. Washington, DC: National Academy Press.



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# Essential characteristics and skills of the Iowa core

- Teaching for understanding
- Problem-based instructional tasks
- Distributed practice that is meaningful and purposeful
- Mathematical modeling (secondary school emphasis)
- Deep conceptual and procedural knowledge
- Rigor and relevance
- Effective use of technology
- Assessment for learning
- Connected and coherent content

1. Problem solving
2. Communication
3. Reasoning and proof
4. Ability to recognize, apply, and make connections
5. Ability to construct and apply multiple connected representations



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# Key practices of teaching to enable mathematics achievement

1. Expecting and enabling complex mathematical work of each student
2. Coordinating mathematical, school, and students' language
3. Using contexts with sensitivity to students' funds of knowledge and to the integrity of the mathematics
4. Creating a respectful mathematically-focused learning environment



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**Expecting and enabling complex mathematical work of each student,**



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# Enacting high expectations in mathematics instruction

1. Broaden what it means to be successful in math class
2. Make mathematical practices explicit
3. Support students' mathematical work both publicly and privately
   - Listen carefully to students' talk
   - Notice and improve ambiguous talk
   - Ask mathematical questions
4. Teaching students to be "people who study mathematics"



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## What is the work of teaching?

1. ENACTING HIGH EXPECTATIONS IN CLASS
2. SCAFFOLDING STUDENTS' EXPECTATIONS AND STANDARDS
3. USING MATHEMATICAL TASKS



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# 1. Enacting high expectations in class

What is the work of teaching?



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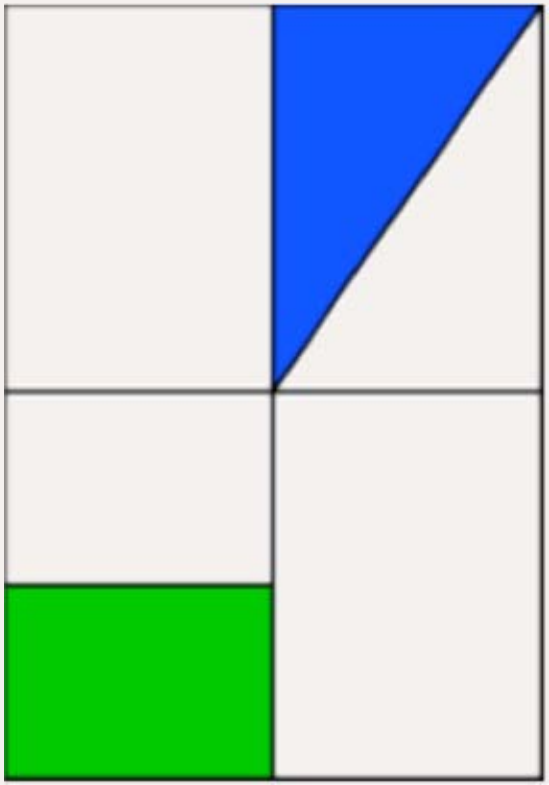
# Video clip

- Day 7, discussion of warm up problem
- Focused attention on equal parts
- Developing working ideas about fractions
  1. Identify the whole
  2. Make equal parts
  3. Count how many equal parts out of the whole



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# Setting up the problem



**What fraction of the big rectangle is shaded blue?**

**What fraction of the big rectangle is shaded green?**

**What fraction of the big rectangle is shaded altogether?**



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# Viewing focus

Where are opportunities taken or missed to:

1. Broaden what it means to be successful in math class
2. Use tasks and problems that encourage complex work
3. Make mathematical practices explicit
4. Support students' mathematical work both publicly and privately
5. Teach students to be "people who study mathematics"



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# The work of teaching in this clip

EXPECTING COMPLEX MATHEMATICAL WORK

1. Selecting tasks
2. Listening to and interpreting students' responses (Mamadou)
3. Relentless press on classroom mathematical practices
4. Supporting error as a fruitful site for mathematical work



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# 2. Scaffolding students' expectations and standards

What is the work of teaching?



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# Student contract

I understand that my ability to succeed in the Elementary Mathematics Laboratory class depends on my commitments and effort this summer. The following are the commitments asked of me to participate in the special class:

1. I will attend class every day.

2. I will keep neat detailed records of my mathematical thinking in my math notebook.

3. I will complete my homework carefully each night.

4. I will seek help or advice from my teacher or parents/caregivers if I have questions or concerns.

5. I will give my best effort to all the work I complete for this class.

I have read, understand and will follow through with the statements in this contract to the best of my ability.

Print student's full name: _____

Student signature:_____
Date:_____



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# Teacher contract with

## Student's name

I understand that your ability to succeed in the Elementary Mathematics Laboratory class depends on my commitments and effort this summer. The following are the commitments I make to you as your teacher in this special class:

1. I will attend our class every day.
2. I will come to class prepared to teach interesting and appropriate material to you.
3. I will expect you to do your very best work all the time and I will help you learn to do so.
4. I will make sure that you understand the mathematics and problems we work on.
5. I will answer any questions or concerns that you may have about this class or our work.
6. I will keep careful records of my mathematical thinking and ideas.

I have read, understand and will follow through with the statements in this contract to the best of my ability.

Print teacher's full name: _____

Teacher's signature:_____ Date:_____



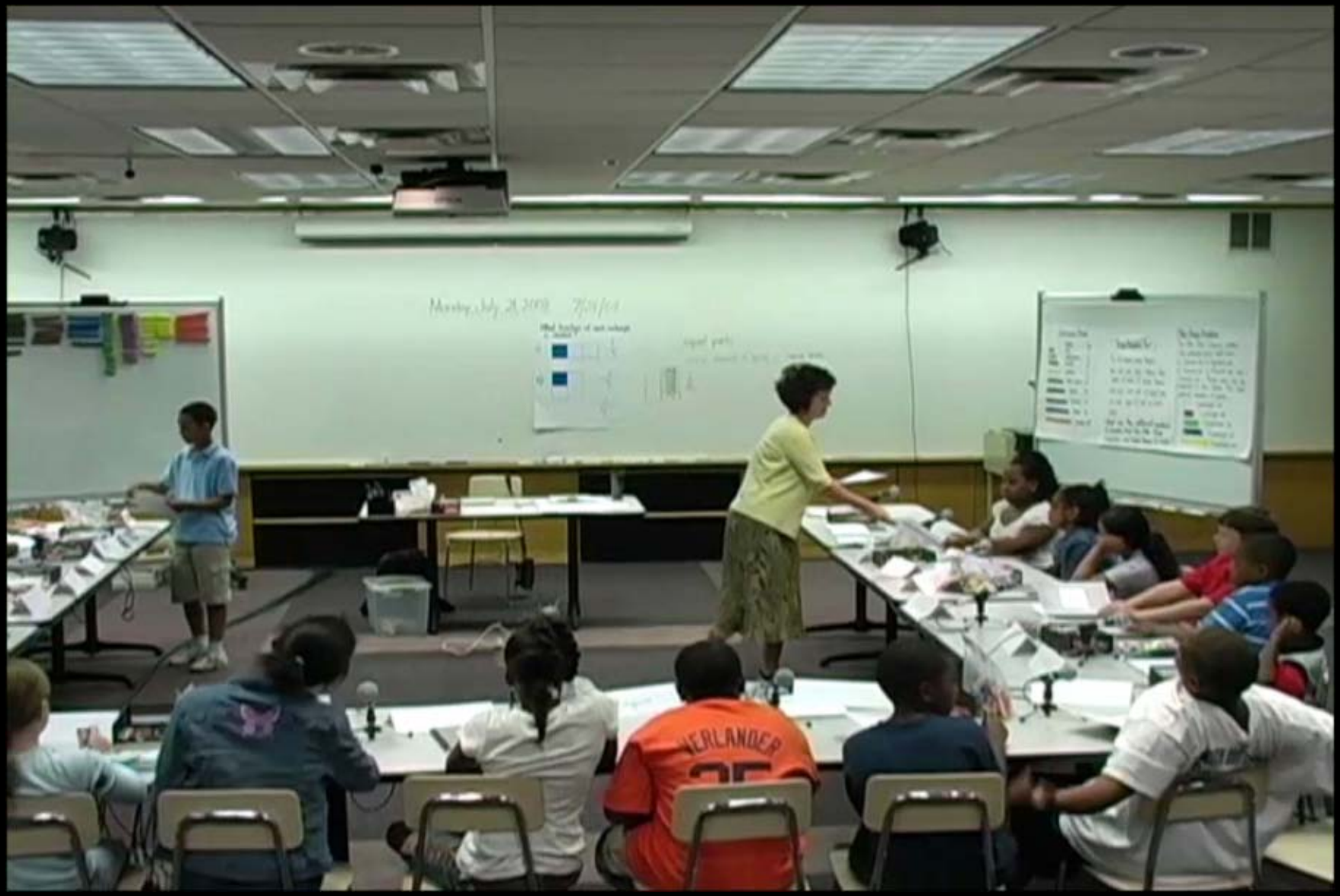
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# Viewing focus

- What is the teacher doing to expect **complex mathematical work** from students?
- How are high expectations being **communicated** to students?
- Where are **missed opportunities** to communicate such expectations?

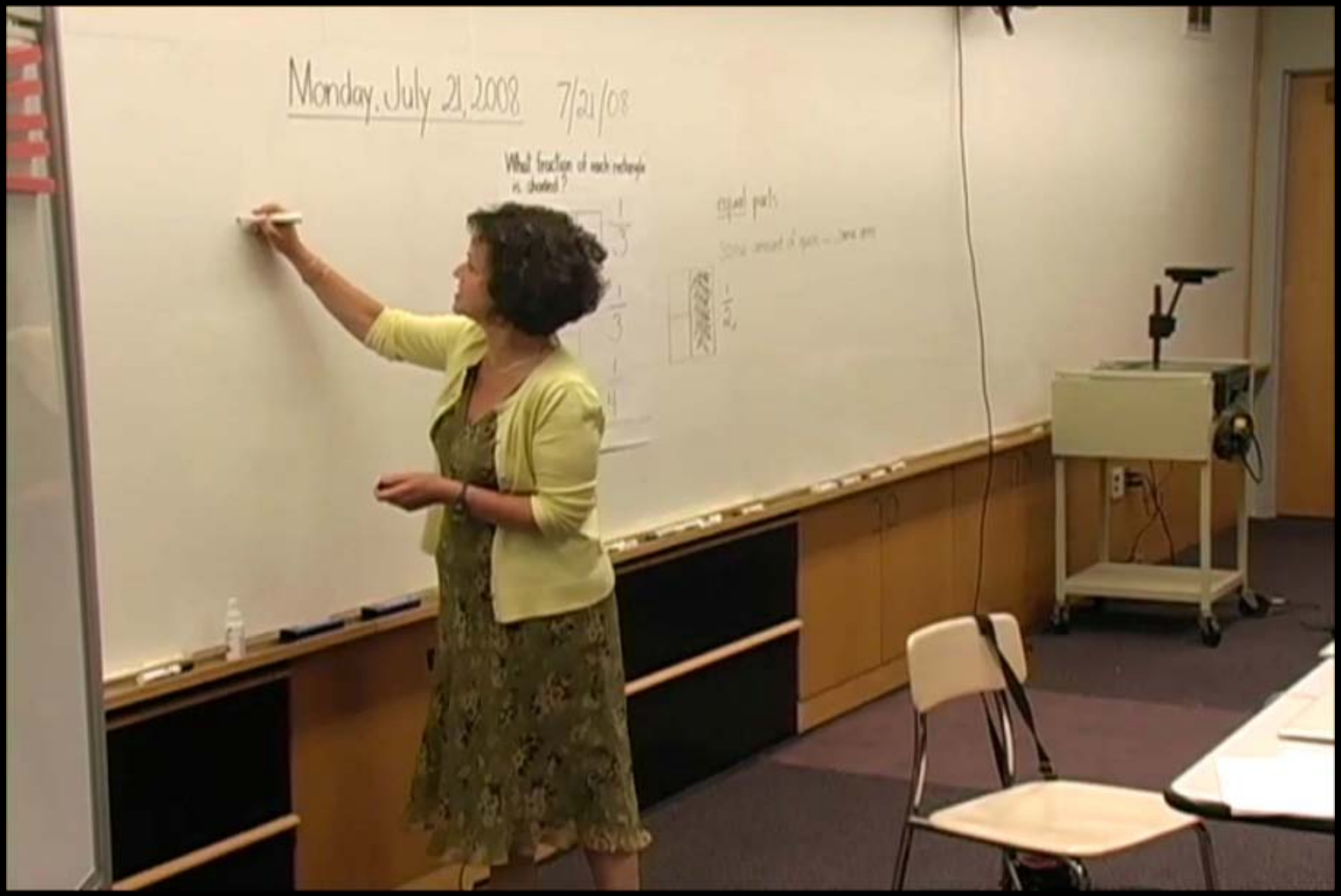


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# 3. Choosing mathematical tasks

What is the work of teaching?



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# Types of tasks in class and on homework: How are they similar and different?

## SIMILAR

- Involve the use of mathematical skills as well as concepts
- Routinely require explanation and reasoning
- May have more than one solution



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# Analyzing others' solutions

A fourth grade student did the calculations below. For each one, decide whether the answer is correct or incorrect and explain how you know. If the answer is incorrect, try to explain what the error is.

$$\begin{array}{r} 48 \\ +\ 27 \\ \hline 615 \end{array}$$

$$29 + 37 + 18 = 84$$

Is the answer correct or incorrect?

How do you know?

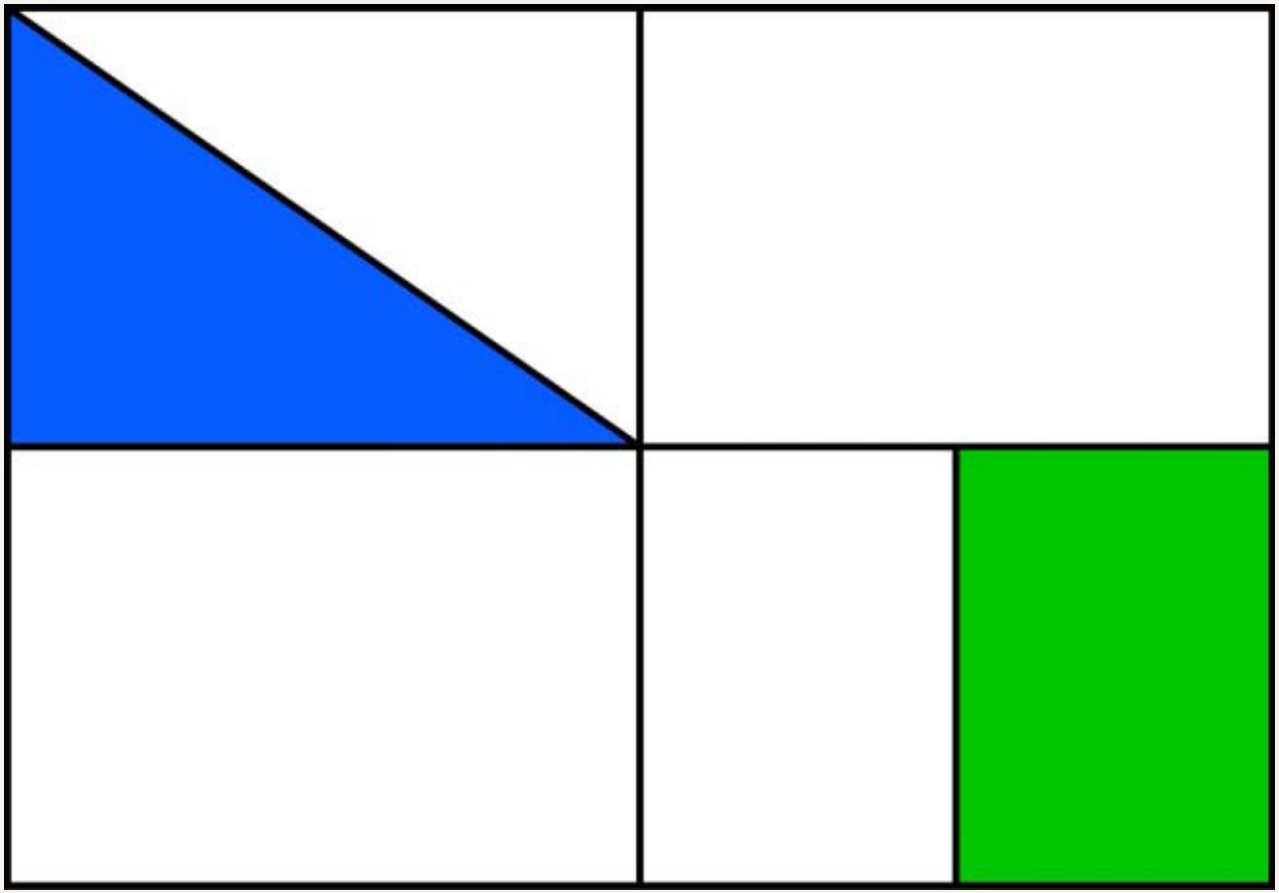
If the answer is incorrect, try to explain what the error is.

If the answer is incorrect, please do it correctly here:



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# Reasoning with representations





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# Reasoning with tools

Which Cuisenaire rod is three times as long as a red one?

Which rod is half as long as an orange one?

One rod is one-fourth as long as another rod. What colors might they be?

Which rod is one-third as long as the dark green one?

Which rod is five times as long as a red one?

The blue rod is how many times as big as the white Cuisenaire rod?



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# Differences between in-class and homework tasks

## HOMEWORK

- Can be done independently
- As unambiguous as possible
- Structures the workspace more explicitly

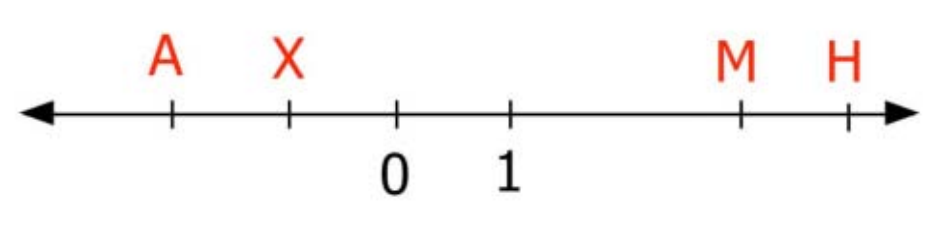
## IN-CLASS

- Profits from exchange with others
- Requires interpretation
- Requires more decisions about representation and recording
- Likely to yield multiple solution methods or solutions



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# Homework




Is A less than or more than H? How do you know?

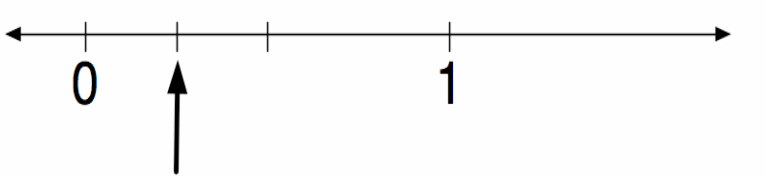
Is A less than or more than X? How do you know?

Label what numbers go at A, X, M, and H. Then, explain how you know what M is.



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# In-class




What number can be written at the mark where the blue arrow is pointing?

Circle your answer:

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{3}$ 1



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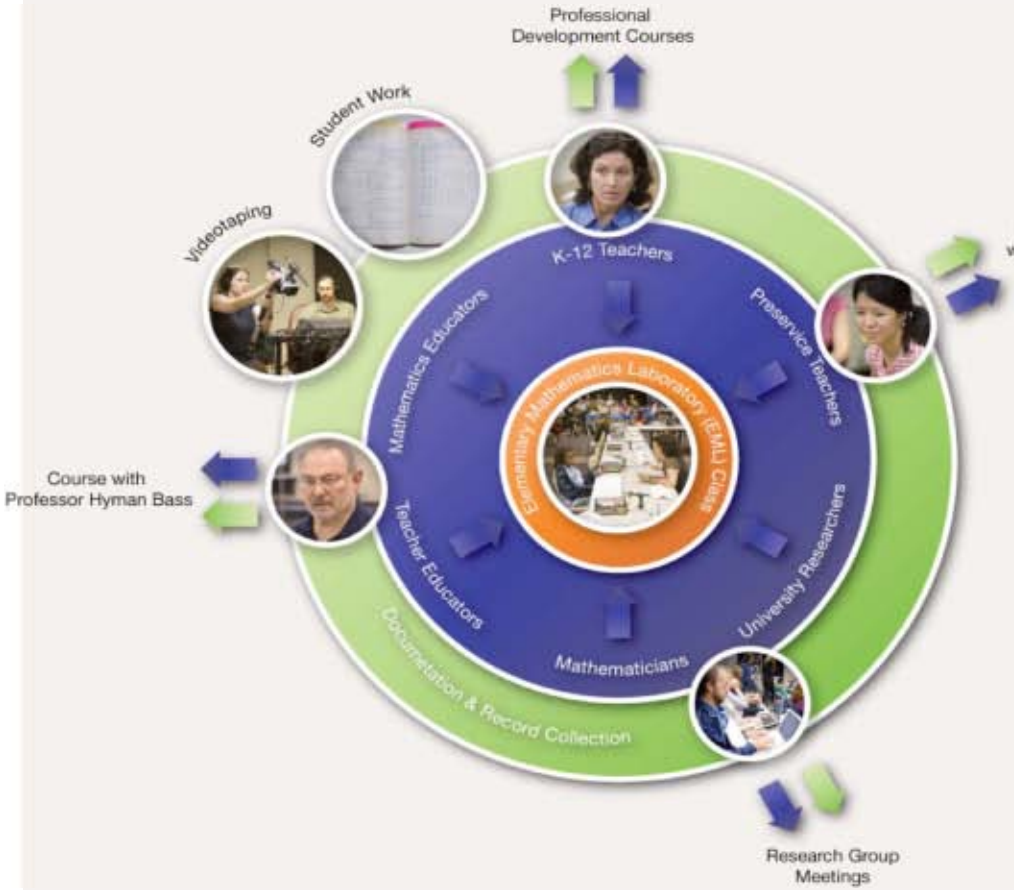
# Cautions about "high expectations"

- Helping students to do hard work
- Scaffolding complex work appropriately
- Making wise judgments about what to leave open and what needs to be made explicit
- Commenting on mathematics, and mathematical productions, not features of students
- Supporting error as a fruitful site for mathematical work, and teach students to use error productively



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# Elementary Mathematics Laboratory




- July 26 – August 6, 2010
- Ann Arbor, Michigan
- Professional development for teachers, teacher developers, cooperating teachers, and teacher leaders
- For details: Please contact eml2010@umich.edu



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# A closer look: The inner circle








## Inner circle: The children's class

- An elementary summer school class — 27 local students entering fifth grade, wide range of mathematics achievement and dispositions
- 2.25 hours of instruction per day
- Mathematical topics: fractions, permutations, place value, number line, equivalence
- Mathematical practices: explaining, representing, proving, defining
- Practices of learning math: recording, summarizing, attending to language & precision, studying



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# A closer look: The outer circle






**Outer circle: Research and TE**

- Diverse group of professionals interested in mathematics teaching and learning, instruction in general, and teacher education
- A laboratory for developing and studying mathematics instruction and the mathematical demands of the work
- An opportunity to make the work of teaching visible to researchers, student teachers, and teacher educators



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# Thank you!

- Google “Deborah Ball” for slides and resources related to this talk



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