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Using Records of Practice to Tackle the Unsolved Problem of Teachers' Mathematical Knowledge

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Overview

1. Background: The "problem" of teachers' mathematical knowledge
2. Mathematical knowledge for teaching: A different approach
3. Investigating what teachers need to do, and know, to teach mathematics
4. Re-framing the problem

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1. Background: The "Problem"

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The "Problem" of Teachers' Mathematical Knowledge

- Many criticisms of teachers for not knowing mathematics
- Evidence: teachers' scores, research on teacher knowledge, anecdotes, mathematicians' shock
- Remedy: increase and make more rigorous the mathematics requirements for teachers
- Although logical, common remedy does not work well -- why not?

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Barriers to Solving the Problem of Teachers' Mathematical Knowledge

1. Many unqualified teachers teaching mathematics
2. Underdeveloped theory of mathematical knowledge for teaching and multiple warrants for claims about what teachers need to know
3. No large-scale common means to measure what teachers (or others) know
4. Lack of tools for studying knowledge growth or for modeling instruction
5. Weak interventions in professional education and resources

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2. Mathematical Knowledge for Teaching: A Different Approach

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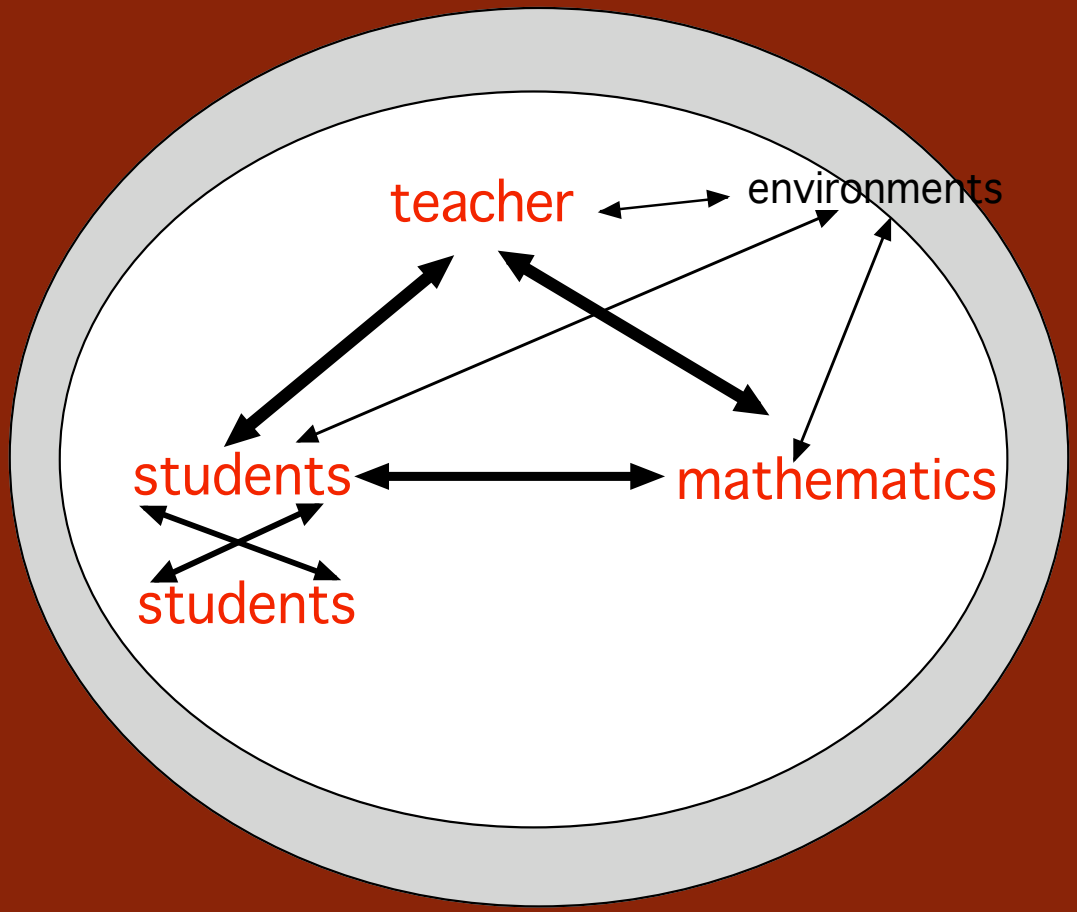
What is the actual problem that we are trying to address?

The quality of mathematics teaching and learning

Teachers' knowledge of mathematics and their ability to use it in their teaching

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Knowing and Using Mathematics in Teaching



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What *is* "Mathematical Knowledge for Teaching"?

Calculate:

$$\begin{array}{r} 35 \\ \times\ 25 \\ \hline \end{array}$$

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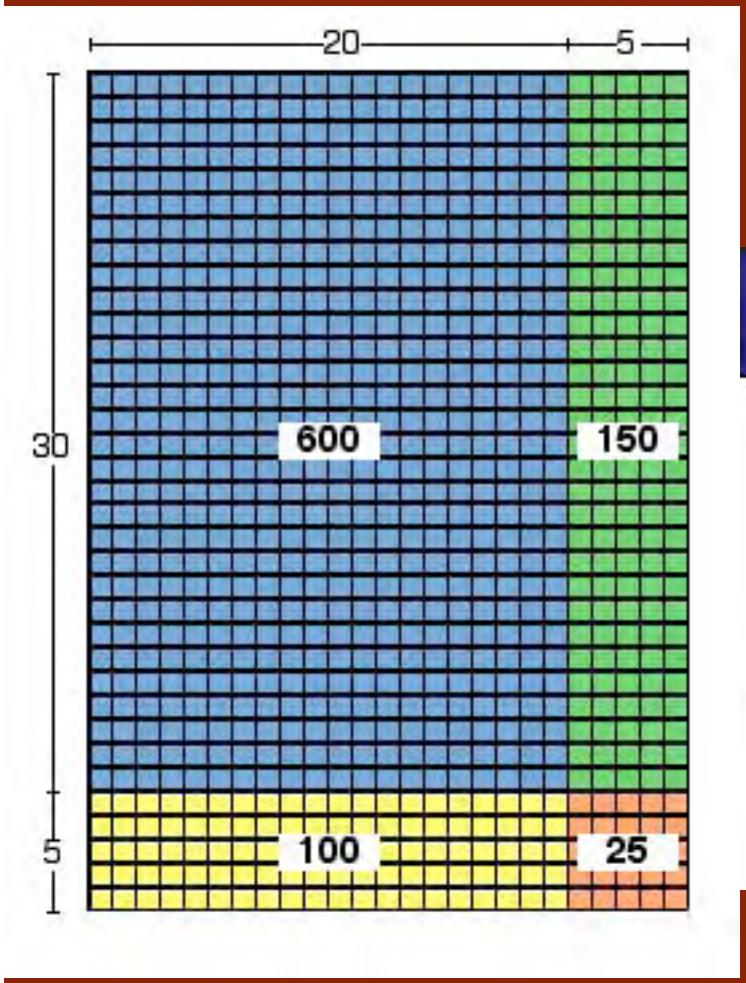
Evaluating and Appraising Different Methods

Student A	Student B	Student C
35 x25 125 +75 875	35 x25 175 +700 875	35 x25 25 150 100 +600 875

Which of these students is using a method that could be used to multiply any two whole numbers?

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Using Representations to Model and Reason about Procedures



A

35
x 25
125
+75
875

B

35
x 25
175
+700
875

C

25
x 35
25
100
150
+ 600
875

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Analyzing Error

A

$$\begin{array}{r} 35 \\ \times\ 25 \\ \hline 175 \\ 70 \\ \hline 245 \end{array}$$

B

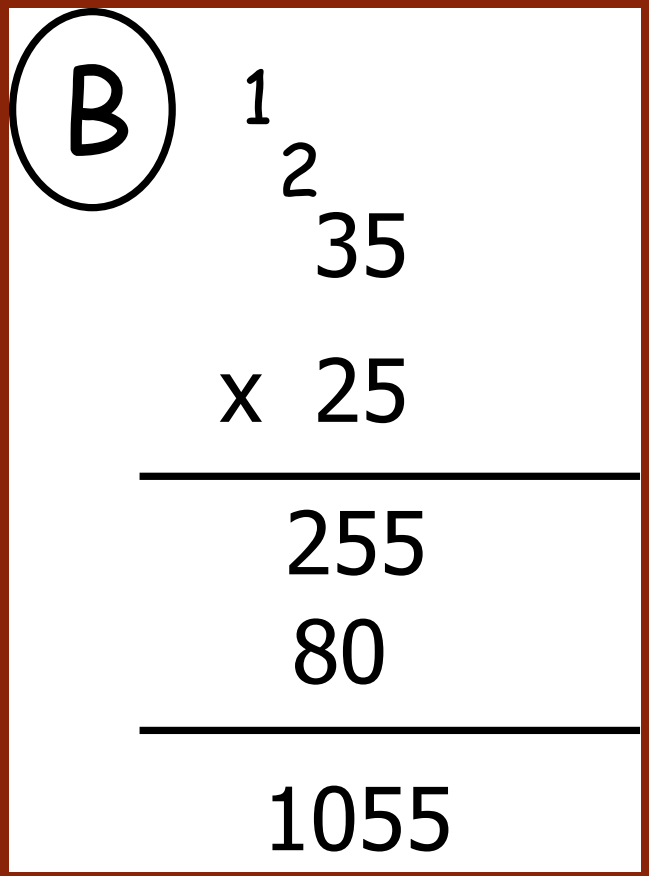
$$\begin{array}{r} {}^{1}{}_{2}35 \\ \times\ 25 \\ \hline 255 \\ 80 \\ \hline 1055 \end{array}$$

C

$$\begin{array}{r} 35 \\ \times\ 25 \\ \hline 625 \\ 85 \\ 105 \\ 60025 \end{array}$$

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Explaining



Why do you multiply before you carry?

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A Different Approach: Investigating What Mathematics is Used in Teaching, and How It Is Used

1. Use records of practice to study teaching, and identify the mathematical work that teaching entails
2. Analyze what mathematical knowledge — topics and skills — is needed to do that work
3. Analyze also the qualities of that knowledge (sensibilities) — how it must be understood and known to be serviceable for the work

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3. Using Records of Practice to Investigate What Teachers Have to Do, and Know, to Teach Mathematics

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Investigation #1: A Video Clip from a 3rd Grade Lesson on Fractions

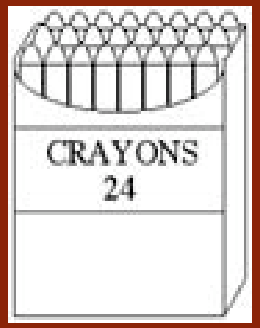
- Third graders (8 year-olds)
- Early May (ten days into fractions unit)
- Diverse classroom, many English language learners
- Orientation of the instruction:
  - To work on substantial mathematics and treat the mathematics with integrity
  - To take students' thinking seriously
  - To treat the construction of mathematical knowledge as the work of an intellectual collective, with justification crucial

From a mathematical perspective, what is important to notice about what the children and the teacher are doing?

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Mathematics Problems

Yesterday:



How many crayons in --

$\frac{1}{2}$ box $\frac{2}{3}$ box $1\frac{1}{2}$ boxes

$\frac{1}{4}$ box $\frac{3}{4}$ box

Today:

How many in three-quarters of a dozen?

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Discussing Teaching

- Easy: The teacher should have . . .

- More difficult: What is important mathematically about what is going on this clip? What do you see? What might be helpful for a teacher to see or be sensitive to?

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Investigating Mathematical Issues that Arise in Teaching (#1)

From a mathematical perspective, what is important to notice about what the children and the teacher are doing?

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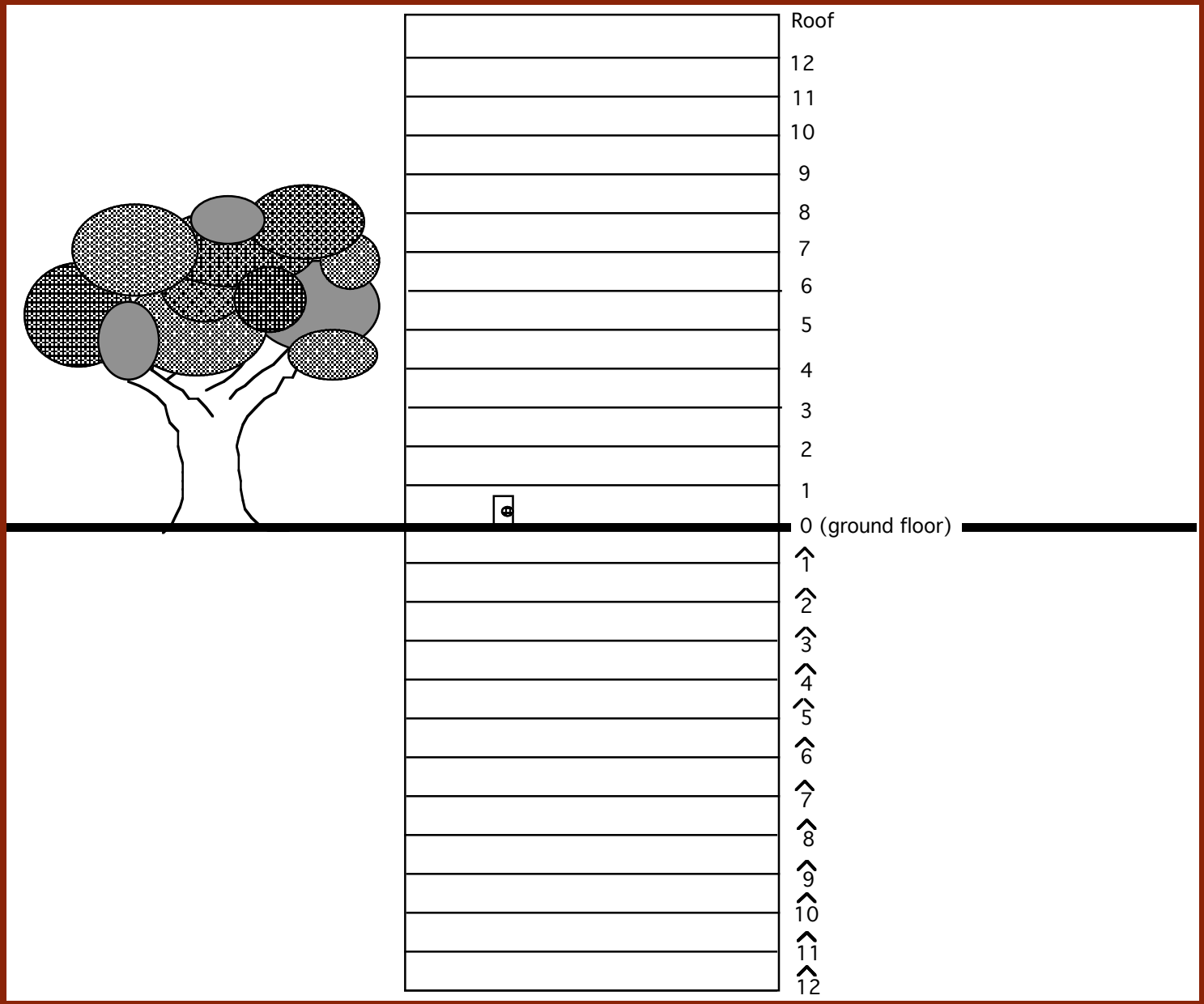
Insert video clip: 3/4 of 12

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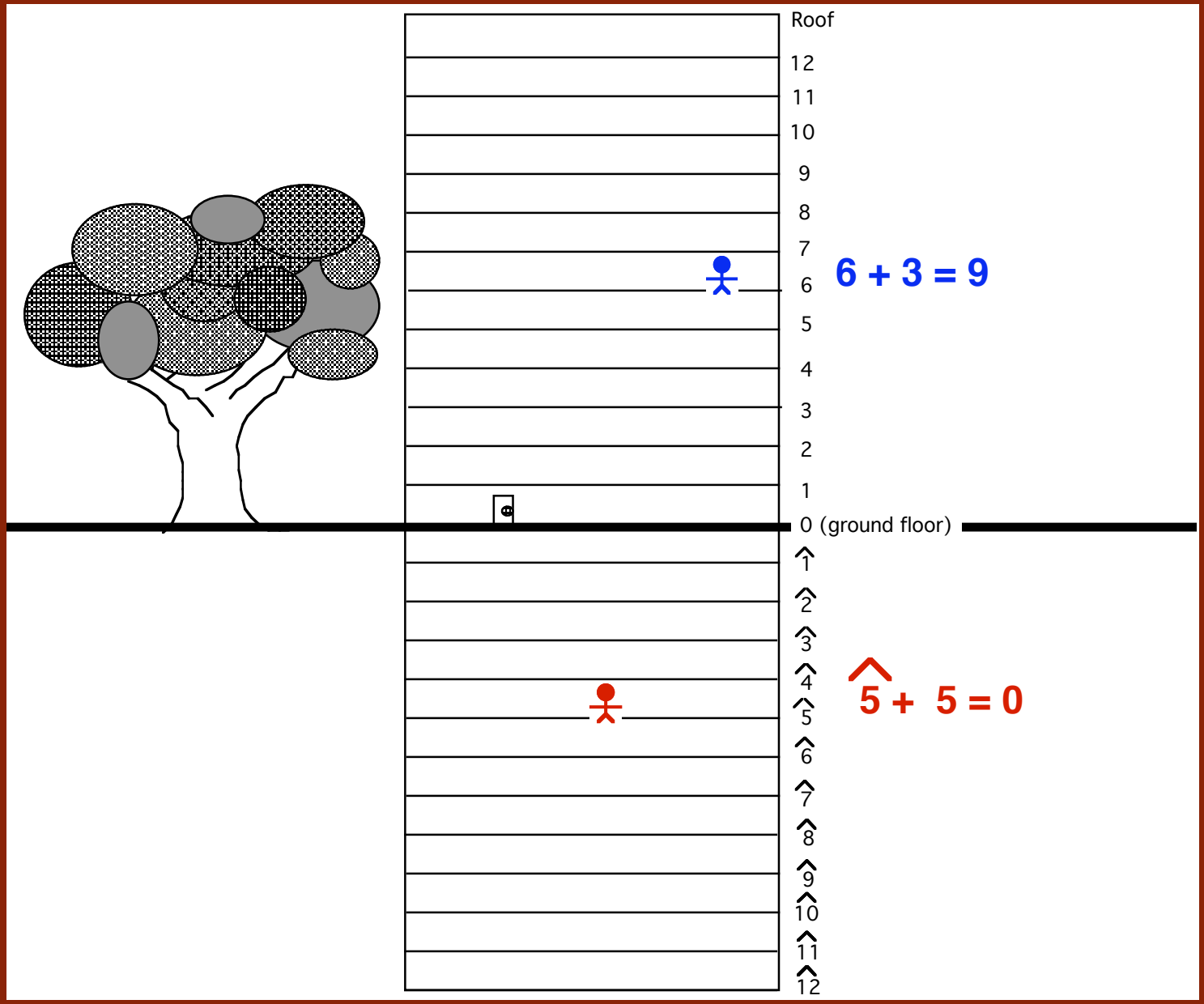
Investigation #2: A Video Clip from a 3rd Grade Lesson on Integers

- Same classroom, 7 months earlier (October)
- Near the end of two weeks of developing integers and addition and subtraction of integers
- Representations: elevator in a building with floors above and below 0 (ground), money (debt, assets)

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Investigating Mathematical Issues that Arise in Teaching (#2)

- Analyzing students' mathematical thinking
- Choosing and using representations (and making them viable in the classroom)

What issues do you see with either of these in this segment?

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Insert video of 6 + (-6)

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Analyzing Students' Mathematical Thinking
Choosing and Using Representations

- Why does Lin think that 6 + (-6) = 9?

- What are different possible representations for addition of negative numbers? What does each offer? What are the problems of each? Which are usable by 8-year olds?

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4. Re-Framing the Problem: Knowing and Using Mathematics in Teaching

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The Work of Teaching Mathematics

- Giving and evaluating explanations
- Modeling operations, concepts
- Creating environment in which mathematical reasoning is central
- Hearing and interpreting students' mathematical ideas
- Analyzing pupils' errors
- Evaluating claims and solutions (often quickly)
- Appraising and modifying textbooks
- Inspecting equivalences
- Judging representations
- Using mathematical notation and representations
- Choosing and developing usable definitions

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Qualities of Mathematics Knowledge Useful for the Work of Teaching

- Respectful of the integrity of the discipline
- Able to be extended and opened up for learners — "unpacked"
- Justified, reasoned
- Connected within and across domains, building on earlier ideas and anticipating more advanced topics
- Organized psychologically as well as logically

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Re-Framing the Problem of Teachers' Mathematical Knowledge

1. Studying records of practice helps to uncover the mathematics needed for the work of teaching mathematics.
2. Mathematics teaching involves specialized mathematical problem solving.
3. Knowledge needed for teaching is different from what is needed for other occupations or professions where mathematics is used (e.g., physics, mathematics, carpentry, tailoring, business).
4. Teachers' mathematical knowledge must be usable for the mathematical work that teachers do.
5. Learning mathematics for teaching involves learning mathematical content in and beyond the pupils' curriculum, and learning to apply that content to solve mathematical problems that arise in teaching.

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