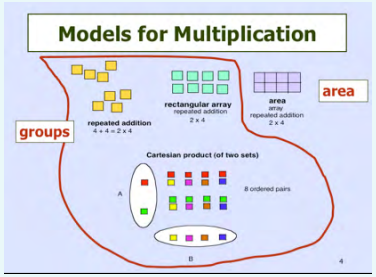
Knowing Mathematics for Teaching

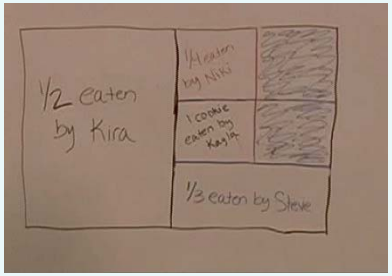
Deborah Loewenberg Ball

Hyman Bass

University of Michigan

Center for Proficiency in Teaching Mathematics





Cornell University

February 24, 2005

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Overview of Session

1. A brief field trip into a third grade mathematics classroom
2. Mathematical knowledge for teaching: A theory in progress, and recent developments
3. How mathematicians can contribute to mathematics education (without converting)

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1. A Short Visit to a Classroom

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Context for Field Trip

- Third graders (ages 8 - 10)
- Mathematics: even and odd numbers, definitions, showing why something is true
- Multi-lingual and multi-cultural class

Easy: The teacher should have …

More difficult: What mathematics do you notice here? What is going on mathematically? What might a teacher need to be sensitive to?

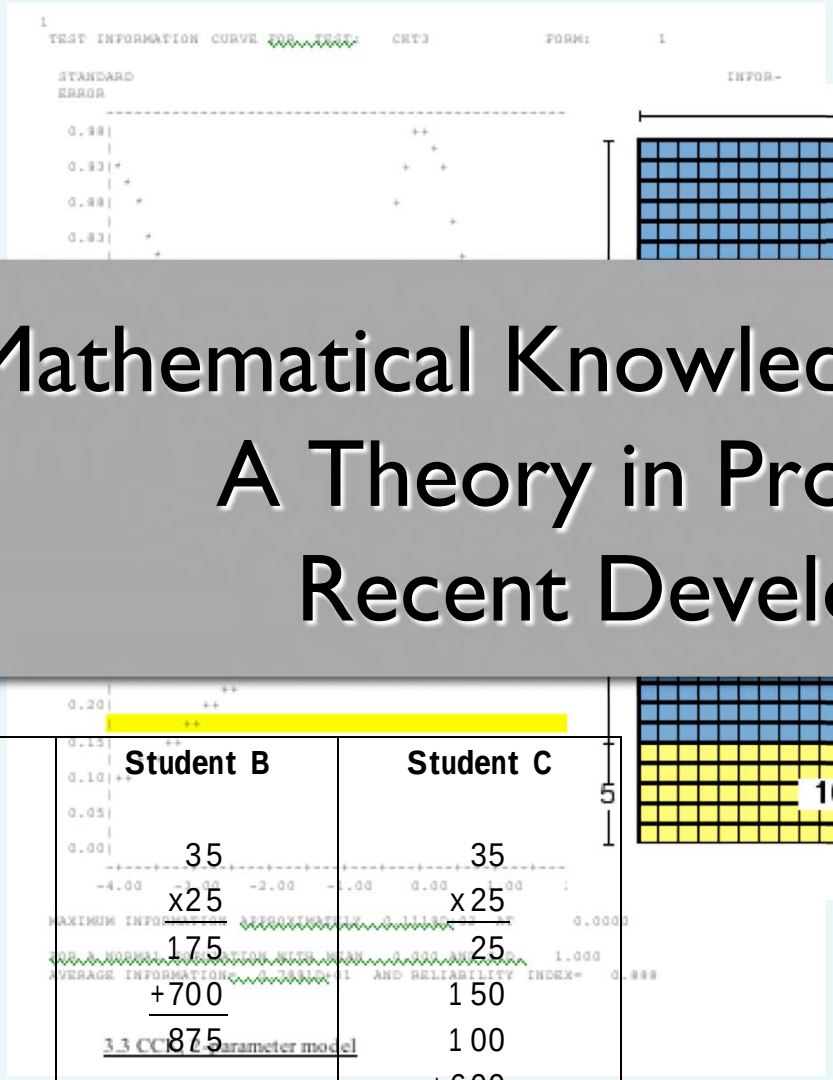
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What Perspectives Can Research Mathematicians Bring?

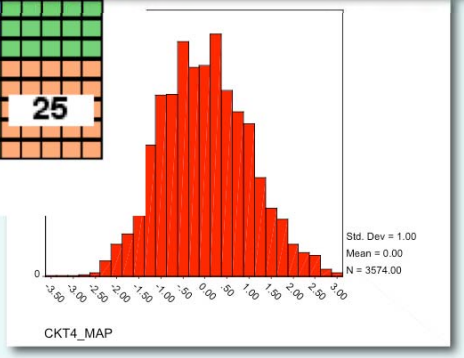
- Three different definitions of "even number" in use, unreconciled
- Shea has an idea in need of a name; is instead appropriating "even and odd" to label his idea
- Shea makes a claim about 6; Lin generalizes
- Students expecting to listen, interpret, and respond to others' claims
- Imperative for justification
- What is a mathematically appropriate and usable definition of "even" and "odd" numbers for third graders?
- A small step from mod-2 to mod-4 arithmetic.

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2. Mathematical Knowledge for Teaching: A Theory in Progress, and Recent Developments







Student A	Student B	Student C
35 x 25 1 25 +75 875	35 x25 175 +700 875	35 x 25 25 1 50 1 00 +600 875

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Common Framing of the Problem → Failure to Solve the Problem

- Teachers do not know mathematics well enough (evidence and causes vary)
- Concentration of under-qualified teachers in urban and high poverty schools

- Increase number and rigor of mathematics requirements for teacher certification
- Recruit mathematically-trained people into teaching

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Clarifying the Problem

The quality of mathematics teaching and learning

Teachers' knowledge of mathematics and their ability to use it in teaching

What mathematical resources do teachers need in order to teach mathematics to all students effectively?

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A Practice-Based Approach to Answering the Question

1. Study instruction, and identify the mathematical work of teaching
2. Analyze what mathematical knowledge is needed to do that work effectively, and how it must be understood in order to be useful for the work
3. Develop, test, and refine approaches to helping teachers develop and use mathematical knowledge for teaching, study results

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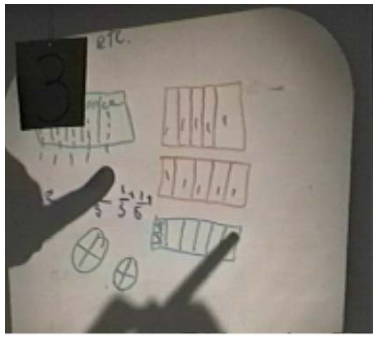
Linking Teacher Knowledge and Student Achievement: New Results

- Questionnaire consisting of 30 items
- Model: Student Terra Nova gains predicted by:
  - Student descriptors (family SES, absence rate)
  - Teacher characteristics (math methods/content, content knowledge)
- Teacher content knowledge significant
  - Small effect (< 1/10 standard deviation): 2 - 3 weeks of instruction
  - But student SES is also about the same size effect on achievement
    - (Hill,Rowan, and Ball, in press)

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What Do we Mean By "Mathematical Knowledge for Teaching"?





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Mathematical Knowledge for Teaching

What do we mean when we use this term, "mathematical knowledge for teaching"?

- Mathematical knowledge, skill, habits of mind that are entailed by the work of teaching

What do we mean by the "work of teaching"?

- The tasks in which teachers engage, and the responsibilities they have, to teach mathematics, both inside and outside of the classroom

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What *is* "Mathematical Knowledge for Teaching"? An Example from Multiplication of Decimals

Multiply:

$$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline \end{array}$$

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Analyzing Incorrect Answers for $\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline \end{array}$

Why do you multiply before you add?

(a)

$$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline 255 \\ 80 \\ \hline 10.55 \end{array}$$

(b)

$$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline 62.5 \end{array}$$

Why do you have as many decimal places in the answer as the total number of decimal places in the problem?

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Analyzing Correct Answers for $\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline \end{array}$

(a)
$$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline .25 \\ 1.5 \\ 1 \\ 6 \\ \hline 8.75 \end{array}$$

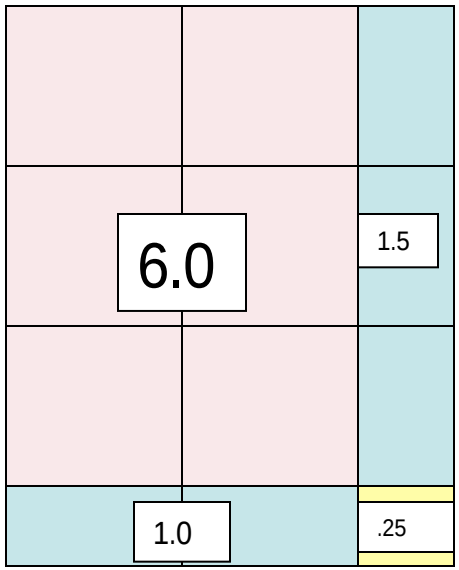
(b)
$$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline 1.25 \\ 7.5 \\ \hline 8.75 \end{array}$$

Is there a method?
Does it work for
all decimal numbers?
How do you know?

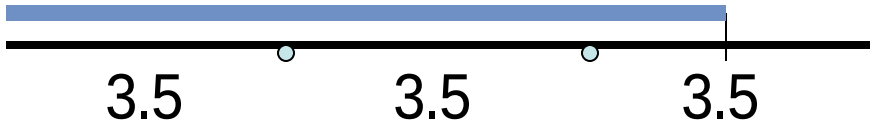
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Using Representations for $\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline \end{array}$

(a)



(b)



Explain:
Where is the 3.5?
Where is the 2.5?
Where is 8.75?

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Design Quiz Questions

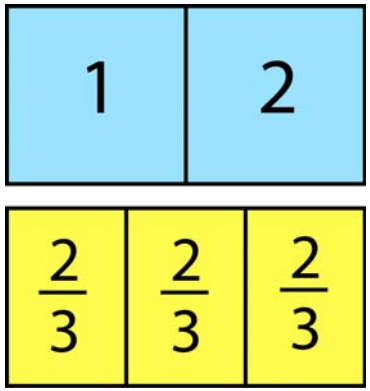
A.	.5	7	.01	11.4
B.	.60	2.53	3.12	.45
C.	.6	4.25	.565	2.5

D. These lists are all equally good for assessing whether students understand how to order decimal numbers.

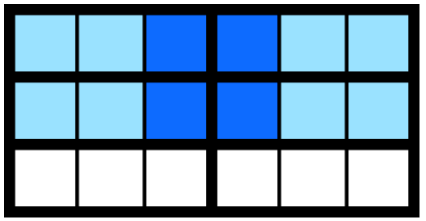
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Choose, Develop, and Use Representations

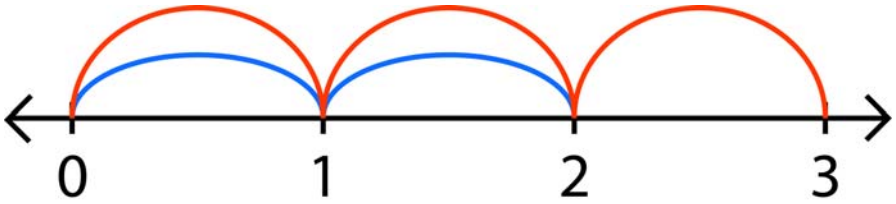
(a)



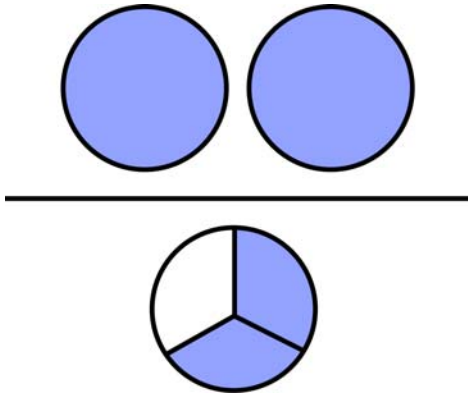
(b)



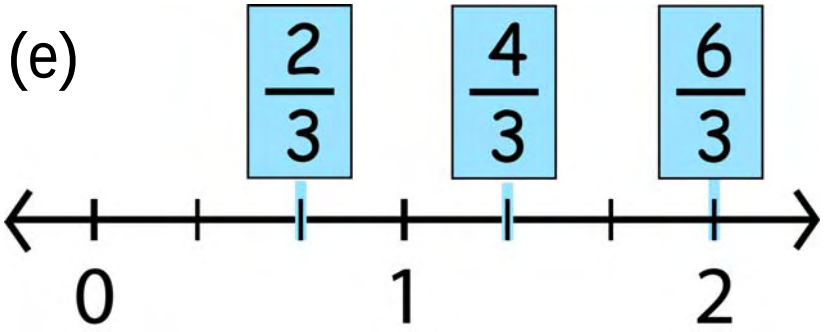
(c)



(d)



(e)



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Use and Teach Definitions

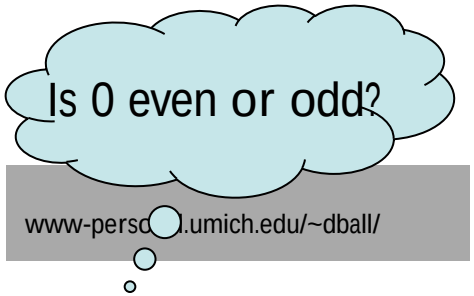


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Examine Textbook Definitions

How well does any of these help to answer the the question?

1. An even number is a number of the form 2k, where k is an integer.
2. An even number is a natural number that is divisible by 2.
3. An even number is any multiple of 2.
4. An even number is a number that has 0, 2, 4, 6, or 8 in the ones place.



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Define Terms

Write a definition of "even number" that is mathematically precise and also usable by third graders.

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3. How Mathematicians Can Contribute to Mathematics Education (Without Converting)

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Date: Mon, 12 Feb 1996
14:42:45 -0500
To: hb@math.columbia.edu
From: Deborah Loewenberg Ball <dball@msu.edu>
Subject: project idea

Dear Hyman,

I have an idea for a project we might do together. I have been studying videotapes of teaching and trying to uncover and understand the mathematics that is important in teaching elementary school mathematics. It would be very helpful to this work if I could interest you in this project and in watching some tapes, and generating mathematical commentaries on them. Would you be interested in doing this?

thanks, Deborah

Date: Thu, 15 Feb 1996
18:17:23 - 0500
To: dball@msu.edu
From: Hyman Bass <hb@math.columbia.edu>
Subject: Re: project idea

Dear Deborah,

Thank you for your message. This sounds interesting, but I have some questions:

1.What is the nature of the videotapes that I would watch? What methods would I use?

2.What would I be expected to produce?

3.How would you use what I do?

I am also not sure that I have much to contribute to this work. I would be willing to discuss this further to learn what you have in mind.

best,

hy

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Roles for Mathematicians in Education

1. **Consumer-Critic**: Reading, using, critiquing, reviewing, learning from mathematics education research, materials, policy documents
2. **Consultant**: Consulting on mathematics education research, development, and policy projects
3. **Teacher-Researcher**: Engaging in systematic inquiry into own practice, developing own materials
4. **Research Collaborator**: Collaborating on mathematics education research projects
5. **Materials Author:** Writing mathematics curriculum for students or teachers
6. **Teacher Educator/Professional Developer:** Working with teachers

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Observations About Mathematicians' Work in Education and with Educators

1. Mathematicians can contribute significant perspectives and insight to research and development in mathematics education.
2. Work on mathematics education problems is intellectually challenging and interesting to many mathematicians, in ways not unlike what fascinates them about mathematics itself.
3. Work that is concrete, centered on actual problems with specific examples or artifacts, helps to foster productive engagements.
4. Mathematicians' mathematical expertise is not instantly useful for work on or in mathematics education: mathematical knowledge must be unpacked, made more explicit, adapted.
5. Mathematical expertise, even retooled, is insufficient for solving problems of mathematics education. Mathematical expertise contributes in combination with other kinds of expertise.
6. Mathematicians' work on problems of mathematics education can be viewed as a form of specialized mathematical application. This require learning more about the contexts.

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Slides will be available at

http://www-personal.umich.edu/~dball/

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