Overview of Session

1. Introductions
2. Overview and framing of the day: Checking in with you?
3. Sharing your work on mathematical explanation with teachers
4. Investigation #1: Assessing preservice teachers' learning of mathematical explanation: An exam task
5. Investigation #2: What does it take, mathematically, to assess students' learning? A third-grade quiz
6. Investigation #3: What does it take, mathematically, to listen to and help students develop their ideas?
7. Tasks for preservice teachers: practice in analyzing errors
8. Wrap up, comment cards

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Sharing Your Work

1. What have you tried to do — or, what are you trying to do — in trying to emphasize mathematical explanation in your work with teachers?
2. What successes have you had in emphasizing and developing their skills with mathematical explanation?
3. What challenges are you facing in this work?

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#1a: Assessing Preservice Teachers' Learning of Mathematical Explanation

- Do problem #1 yourself. Your partner does one task, you do the other.
- Meet with your partner and explain your solution orally. Comment on your partner's solution.
- What did you notice? What is involved in this task? What opportunities for "seeing" mathematical explanation does it offer?

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#1b: Assessing Preservice Teachers' Learning of Mathematical Explanation

Viewing videotape clips of preservice teachers' explanations

- What stands out to you about their mathematical explanations?
- How would you evaluate each student's performance?
- What are the possibilities and limitations of this task for assessing preservice teachers' learning of mathematical explanation?

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#2a: Assessing Third Graders' Mathematics Learning

- Answer each of the questions on the quiz yourself.
- Use handout to analyze questions
  - What is the purpose of the question and the core mathematical ideas addressed?
  - What are characteristics of "good" answers at the third grade level?

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#2b: Assessing Third Graders' Mathematics Learning

- Read each of the three student responses to a particular question
- For each question discuss the following:
  - What does each student's response tell you about his or her mathematical understandings?
  - What follow-up questions would you like to ask each student?

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#3: Analyzing and Developing Students' Mathematical Thinking

- Study two clips of tape
- Consider what Tina is saying: What can you hear and see?
- What is the work for the teacher in attending to and developing her ideas successfully?

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Mathematics Problem: Fractions and Division

PROBLEM 2:

(A) I have one dozen small raisin cookies. If I want to share them equally with my family at supper tonight, how many cookies would each person get? How do you know?

(B) How would this work out in YOUR family—how many cookies would each person get if YOU had a dozen cookies to share with them? How do you know?

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#4: Tasks For Preservice Teachers: Practice in Error Analysis

$\overset{1}{}$ 18 29 + 37 74	62 - 28 46	$\overset{8}{\cancel{9}}\overset{1}{0}5$ - 147 748	463 - 35 = 113

For each problem, discuss the following:

- What is the error?
- What might the error tell you about a student's understanding?
- What questions would you want to ask a student to probe his or her thinking?

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Three Common Student Errors

1

$$\begin{array}{r} 35 \\ \times\ 25 \\ \hline 175 \\ 70 \\ \hline 245 \end{array}$$

2

$$\begin{array}{r} {}^{1}\,{}_{2}\,35 \\ \times\ 25 \\ \hline 255 \\ 80 \\ \hline 1055 \end{array}$$

3

$$\begin{array}{r} 35 \\ \times\ 25 \\ \hline 625 \\ 85 \\ 105 \\ 60025 \end{array}$$

- What is the error?
- What produces this error, and what does that show about the student's understanding?

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A student solved the problem 92 - 37 in the following way:

I know that 92 – 40 = 52.
Since 37 is 3 less than 40,
I need to take 3 away from 52.
So the answer is 49.

- Analyze the student's method, and explain why it gives the incorrect answer. Why might a student think this method would work?
- How could you modify this method to make it work? (That is, how can you start with 92-40=52, and arrive at the correct answer?)
- What are some other erroneous subtraction methods that students might come up with?

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What Leads to All of These Answers?

$$
\begin{array}{r}
23 \\
3\overline{)609} \\
6 \\
\hline
9 \\
9 \\
\hline
0
\end{array}
$$

$$50 \times 40 = 200$$

$$.4 \times .2 = .8$$

$$
\begin{array}{r}
{}^{2}{}_{1}4.4 \\
\times\ 5.3 \\
\hline
132 \\
2200 \\
\hline
233.2
\end{array}
$$

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Analyzing Incorrect Answers for $\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline \end{array}$

(a)

$$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline 255 \\ 80 \\ \hline 10.55 \end{array}$$

(b)

$$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline 62.5 \end{array}$$

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Analyzing Correct Answers for $\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline \end{array}$

(a)

$$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline .25 \\ 1.5 \\ 1 \\ 6 \\ \hline 8.75 \end{array}$$

(b)

$$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline 1.25 \\ 7.5 \\ \hline 8.75 \end{array}$$

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Tell Us How This Went . . .

1. What was interesting or useful for you?
2. What worked well?
3. What was less interesting or useful?
4. What did not work well?
5. Other suggestions?

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Slides will be available at

http://www-personal.umich.edu/~dball/

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