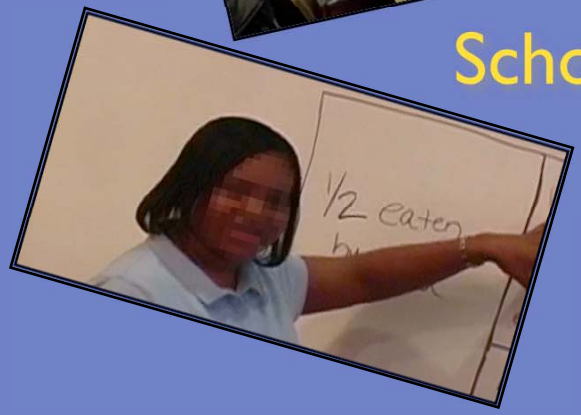
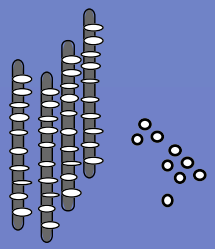
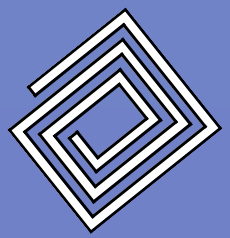
Who Knows Math Well Enough to Teach Third Grade — and How Can We Decide?











Presentation to the
Wolverine Caucus, Lansing
Deborah Loewenberg Ball
School of Education — University of Michigan
March 15, 2006

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Overview

1. The "problem" of teacher knowledge
2. Knowing mathematics for teaching
3. What do we know about the role of mathematical knowledge in teaching?
4. Teacher education for better mathematics teaching

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Education in the U.S. in 2006



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Clarifying the Problem

The quality of mathematics teaching and learning

Teachers' knowledge of mathematics and their ability to use it in teaching

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But: U.S. Teachers Often Lack Sufficient Mathematical Knowledge

- What's the evidence?
  - Anecdotes
  - Research
- What does it tell us and what is not well understood?
  - Lack of a system
  - This is a societal problem, widespread

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Some Approaches to Solving the Problem

- Require more mathematics for certification
  - More mathematics courses
  - A major (or minor) in mathematics
  - Mathematics test
- Recruit mathematically trained people into teaching
  - Engineers, accountants, mathematicians, …
- Fund mathematically focused professional development

But -- these approaches assume we know <u>what</u> mathematical knowledge and skill is needed for high-quality teaching.

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Clarifying the Problem

The quality of mathematics teaching and learning

Teachers' knowledge of mathematics and their ability to use it in teaching

~~How MUCH mathematics do teachers need to know?~~

But: What mathematical knowledge and skill does high quality teaching require?

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Mathematical Knowledge for Teaching

What do we mean when we use this term, "mathematical knowledge for teaching"?

- Mathematical knowledge, skill, habits of mind that are entailed by the work of teaching

What do we mean by the "work of teaching"?

- The tasks in which teachers engage, and the responsibilities they have, to teach mathematics, both inside and outside of the classroom

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Knowing Multiplication

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline \end{array}$$

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Knowing Multiplication for Teaching: Analyzing Student Errors

(a)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 405 \\ 108 \\ \hline 1485 \end{array}$$

(b)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 225 \\ 100 \\ \hline 325 \end{array}$$

(c)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 1250 \\ 25 \\ \hline 1275 \end{array}$$

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Knowing How to Order Decimals

Put these numbers in order:

.7 .2 3.4 .05

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Knowing Decimals for Teaching: Designing Quiz Questions

A. .5 7 .01 11.4

B. .60 2.53 3.12 .45

C. .6 4.25 .565 2.5

D. These lists are all equally good for assessing whether students understand how to order decimal numbers.

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Knowing Radical Expressions

Simplify: $\sqrt{150}$

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Knowing Radical Expressions for Teaching: Choosing Numerical Examples

(a) $\sqrt{54}$

(b) $\sqrt{156}$

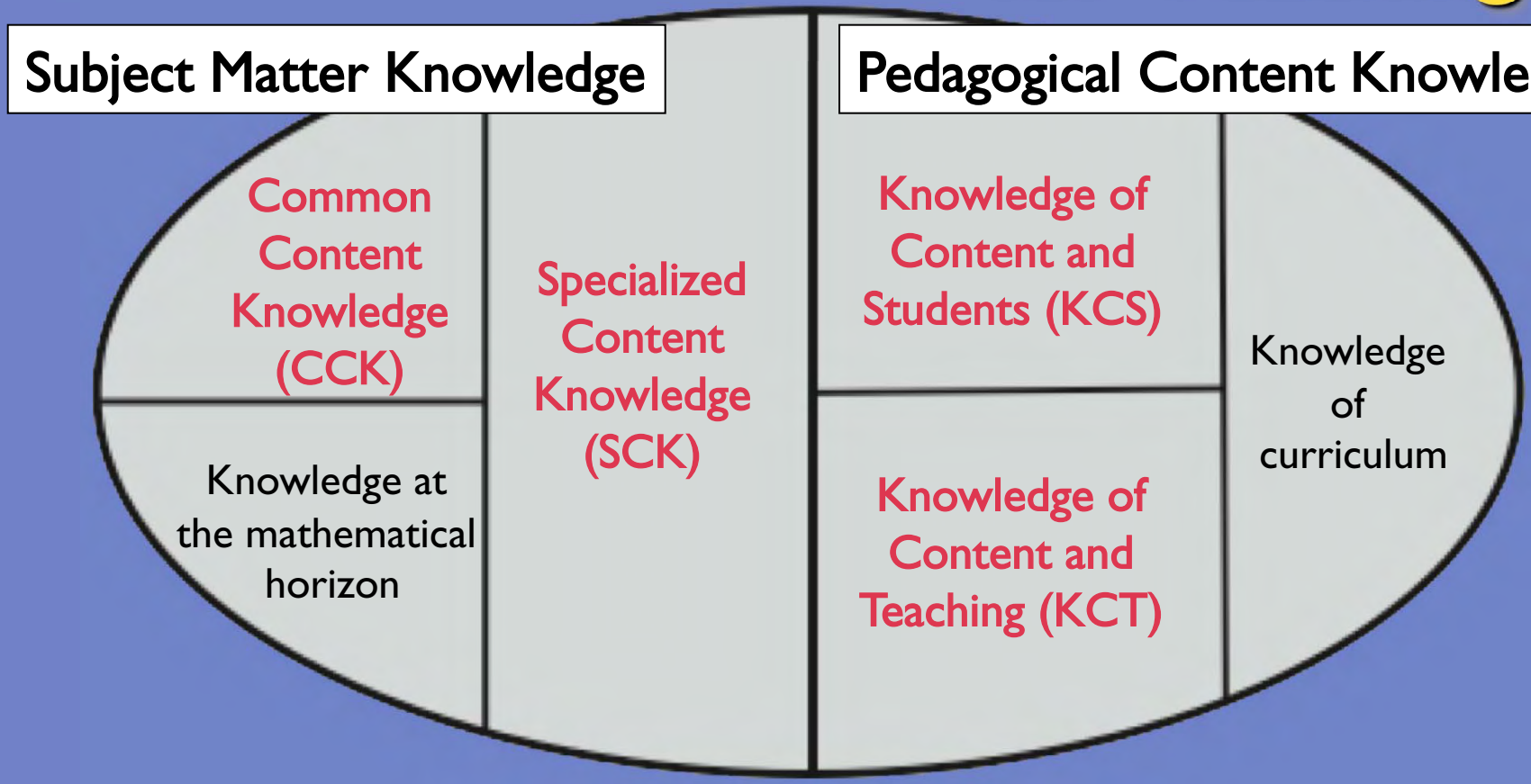
(c) $\sqrt{128}$

(d) These examples all work equally well.

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Mathematical Knowledge for Teaching



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Common Content Knowledge

The mathematical knowledge and skill expected of any well-educated adult

Teachers need to be able to:

- Recognize wrong answers
- Spot inaccurate definitions in textbooks
- Use notation correctly
- Do the work assigned to students

Example Items

What is $1\frac{1}{4} \div \frac{1}{2}$?

What number is halfway between 1.1 and 1.11?

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Specialized Content Knowledge

The mathematical knowledge and skill needed by teachers in their work and beyond that expected of any well-educated adult

Teachers need to be able to:

- Analyze errors and evaluate alternative ideas
- Give mathematical explanations and use mathematical representations
- Be explicit about mathematical language and practices

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Specialized Content Knowledge

Which of these students is using a method that could be used to multiply any two whole numbers?

Student A	Student B	Student C
35 x 25 125 +75 875	35 x25 175 +700 875	35 x 25 25 150 100 +600 875

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Knowledge of Content and Students
Knowledge of Content and Teaching (PCK)

Combines knowledge of mathematics with knowledge of students or knowledge of teaching

Teachers need to be able to:

- Anticipate student errors and common misconceptions
- Interpret student thinking
- Predict what students are likely to do with specific tasks and what they will find interesting or challenging

Teachers need to be able to:

- Sequence content for instruction
- Recognize instructional pros and cons of different representations
- Size up mathematical issues in responding to students' novel approaches

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Knowledge of Content and Students

Ms. Violeta was looking carefully at her students' papers, and she saw the following responses to the problem:

$$8 + 4 = ___ + 5$$

i) 12 ii) 17 iii) Can't do it iv) 1

Which of the following is the most likely explanation of the difficulty the students are having? (Mark ONE answer.)

a) They do not know their basic addition facts.
b) They cannot do multi-step problems.
c) They do not know that addition is commutative.
d) They do not understand the meaning of the equals sign.

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Knowledge of Content and Teaching

To introduce the idea of grouping by tens and ones with young learners, which of the following materials or tools would be most appropriate? (Choose ONE.)

a. A number line
b. Plastic counting chips
c. Pennies and dimes
d. Straws and rubber bands
e. Any of these would be equally appropriate for introducing the idea of grouping by tens and ones.

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Contrasting Knowledge Common, Specialized, and PCK

Common	Specialized	Students	Teaching
Recognize incorrect answers	Analyze errors	Know common errors	Know what to do next
307 - 168 261	307 - 168 169	307 - 168 261	307 - 168 261

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Linking Teacher Knowledge and Student Achievement

- Questionnaire consisting of 30 items (scale reliability .88)
- Model: Student Terra Nova gains predicted by:
  - Student descriptors (family SES, absence rate)
  - Teacher characteristics (math methods/content, content knowledge)
- Teacher content knowledge significant
  - Small effect (< 1/10 standard deviation): 2 - 3 weeks of instruction
  - But student SES is also about the same size effect on achievement

    (Hill,Rowan, and Ball, AERJ, 2005)

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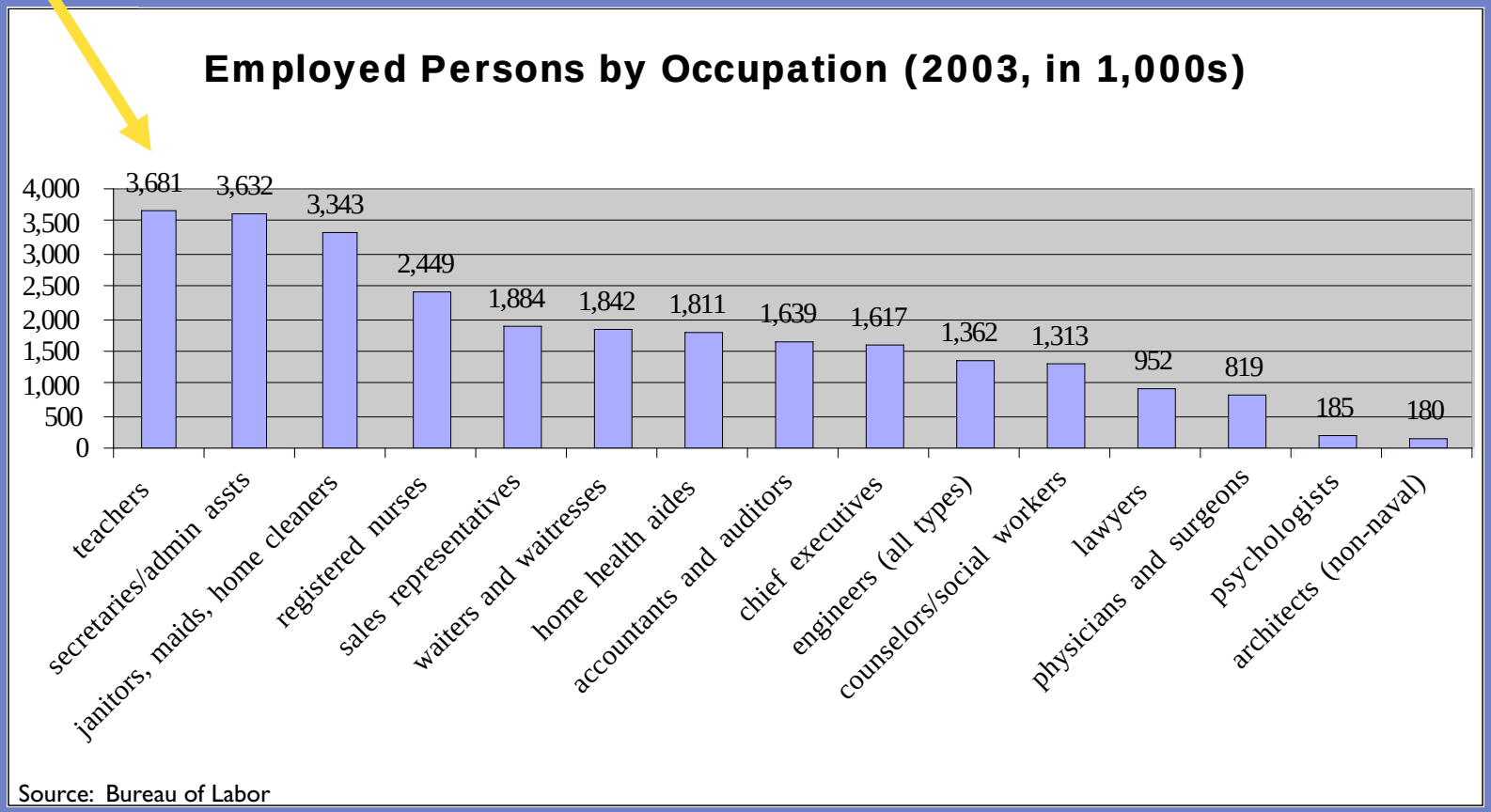
What Do We Know About What Teachers Need to Learn?

- Mathematical knowledge for teaching is the knowledge needed for the work of teaching.
- A kind of content knowledge distinct from that required by other mathematically-intensive professions
- Importance of building this theory IN practice
- Possible to write valid and reliable survey measures of MKT
- Credibility of this approach with teachers
- This kind of testing is professional

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Teachers: The Largest Group in the U.S. Workforce

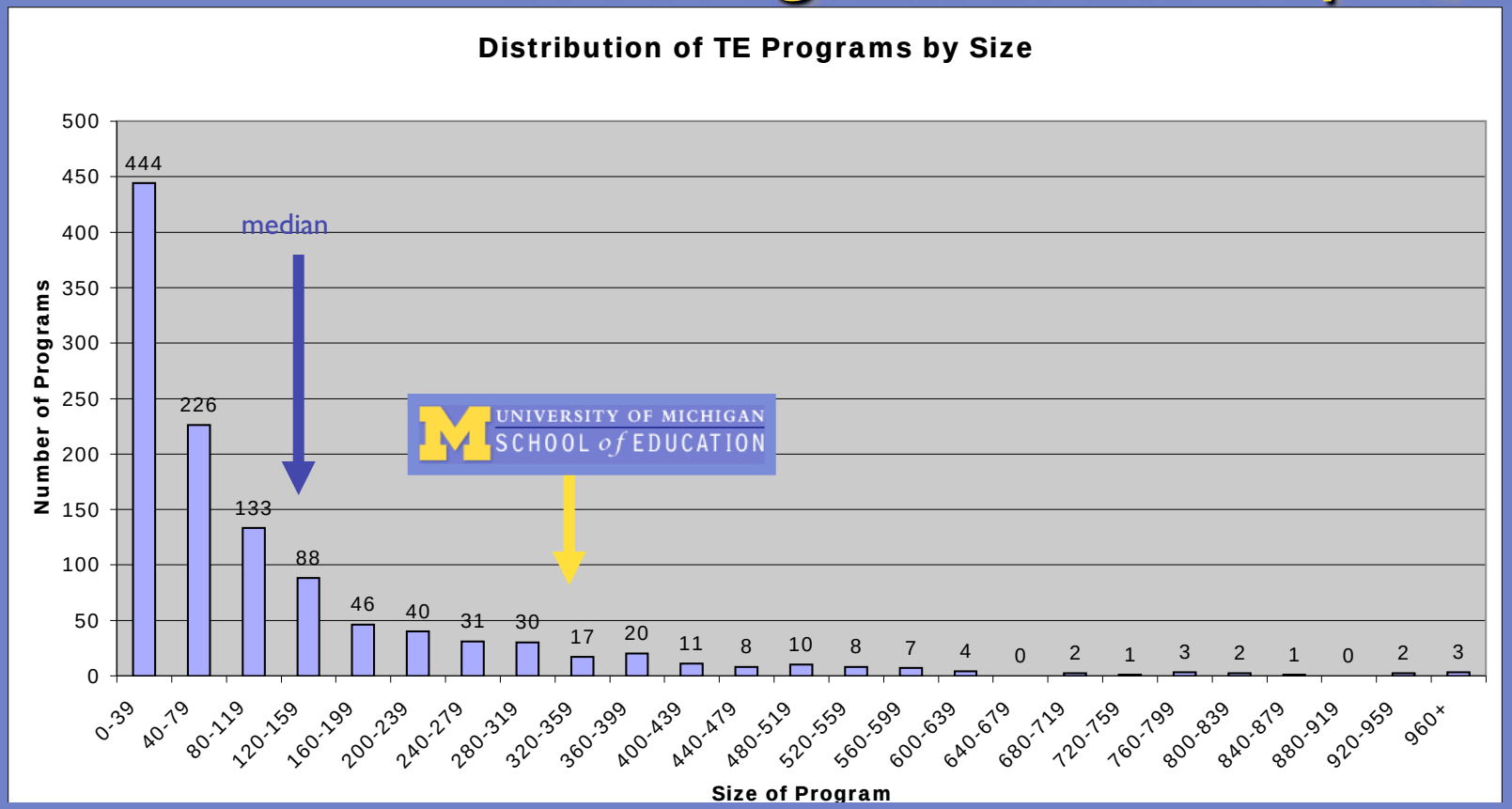


Source: Bureau of Labor

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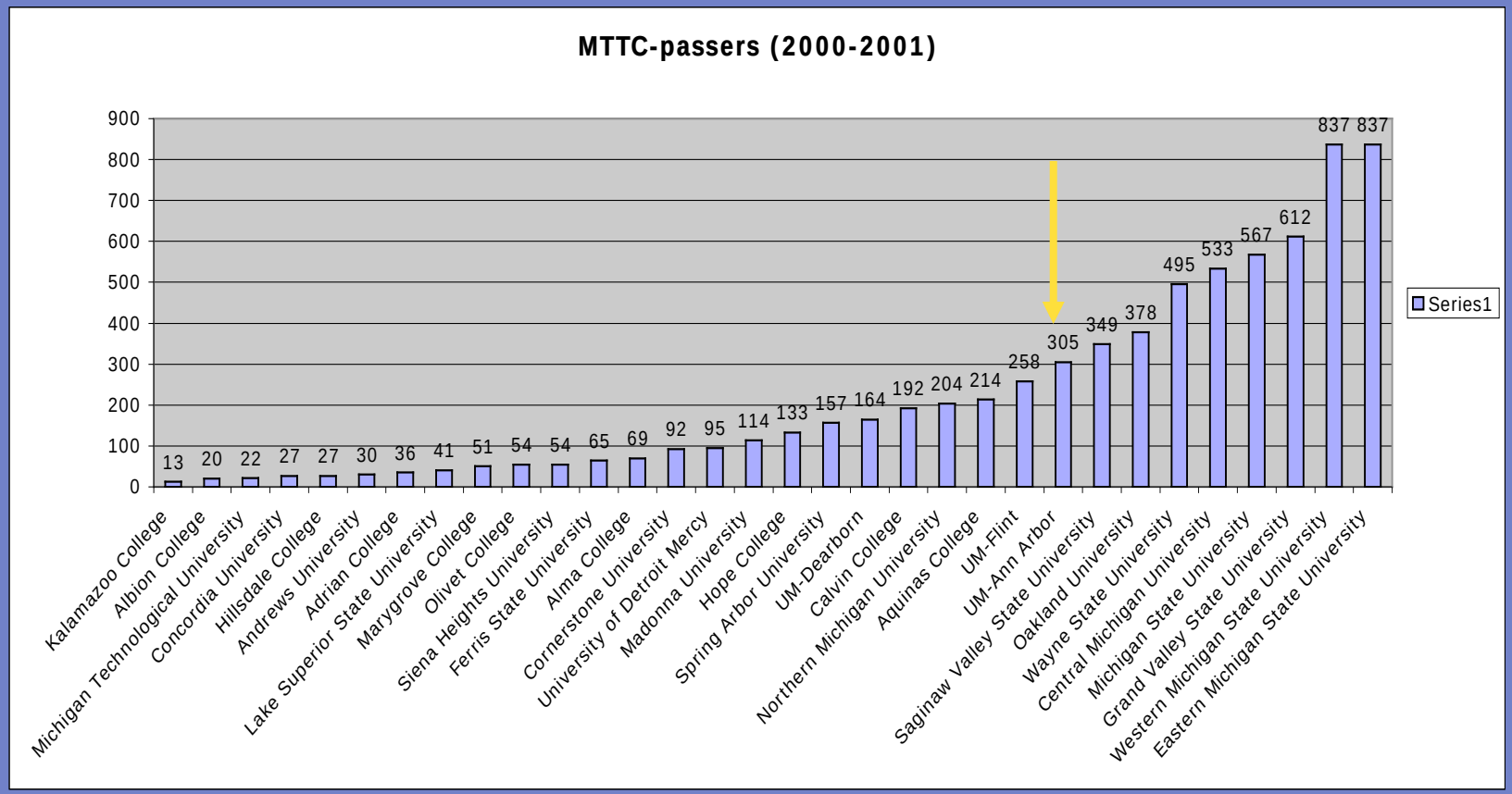
Tiny Size of Many Teacher Education Programs:
UM Program in the Top Quartile



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Teacher Education in Michigan



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What is Wrong with Teacher Education in the U.S.?

- Inappropriate subject matter preparation
- Inadequate preparation for diversity of U.S. classrooms
- Lack of focus on practice
- No professional system for training, licensure, certification, ongoing professional education

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Building a New Brand of Teacher Education at UM

Key elements

- Practice-based (focused on core practices, study of artifacts and records, use of a variety of school settings as labs)
- Knowledge of academic subjects for teaching
- Skills for working to address inequities
- Interpersonal and relational skills (diversity, families, colleagues)
- Preparation to manage and change school and policy environments
- New strategies for recruitment, admissions requirements
- Assessment of performance throughout the program
- Programs for early career professionals

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