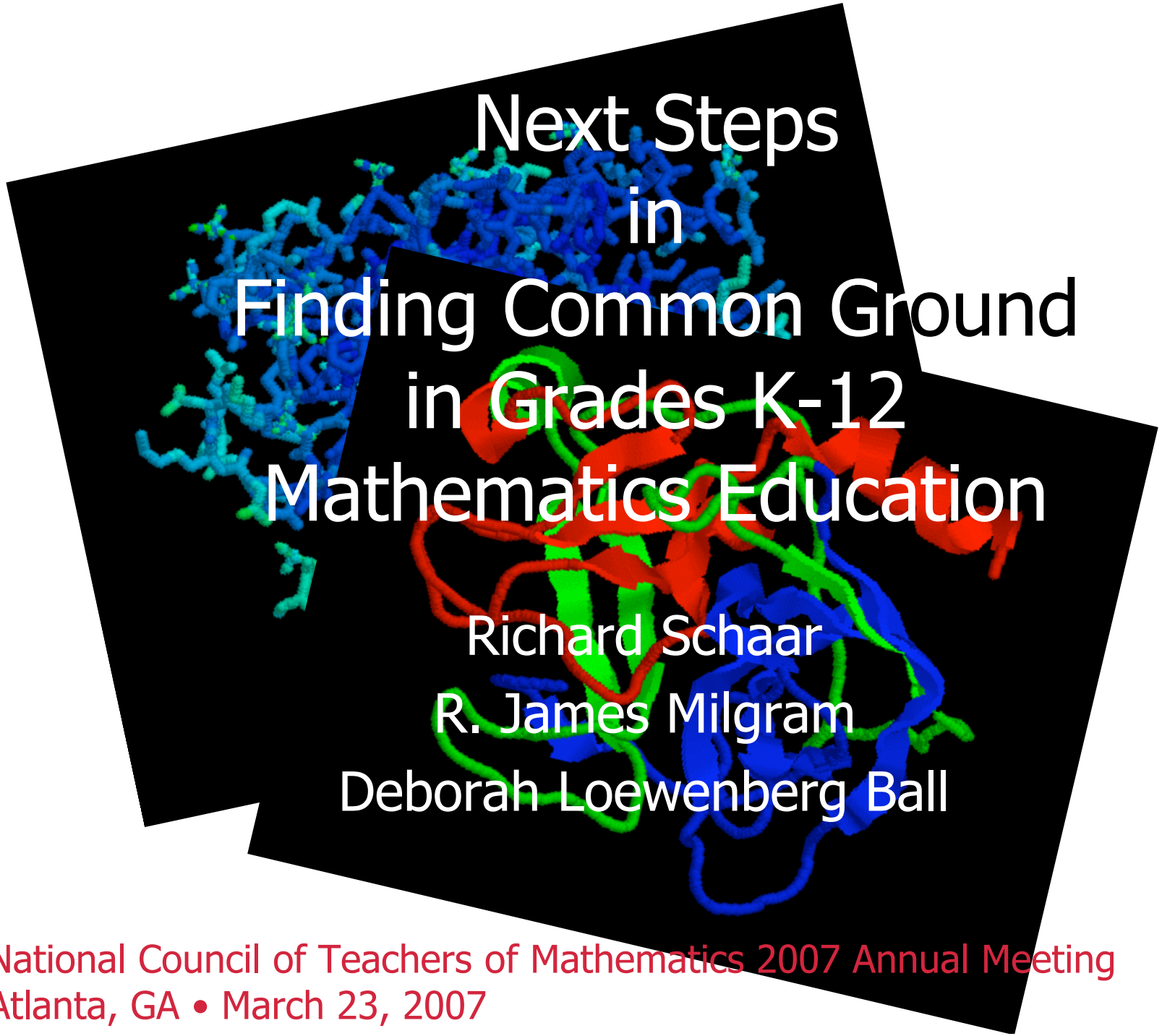


Next Steps in Finding Common Ground in Grades K-12 Mathematics Education

Richard Schaar
R. James Milgram
Deborah Loewenberg Ball

National Council of Teachers of Mathematics 2007 Annual Meeting
Atlanta, GA • March 23, 2007

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Introductions

- Deborah Loewenberg Ball
  - Dean of the School of Education
  - University of Michigan
  - Ann Arbor, Michigan
- R. James Milgram
  - Professor Emeritus of Mathematics
  - Stanford University
  - Palo Alto, California

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Finding Common Ground: Agenda

- Background/History
- Process
- The Document
- Accomplishment/results
  - Dean Ball
  - Professor Milgram
- Next Steps

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Where this point of view is coming from…

- Former President of Educational & Productivity business
- Chairman, Workforce and Human Resource Policy Committee, American Electronics Association (AeA)
- Business Roundtable Taskforce on Mathematics Education





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"Finding Common Ground"

- Working at the State level to promote best practices in K-12 math and science education.
  - Found it difficult to accomplish goals due to each state choosing sides
  - Impediment to math and science education improvement

- Developed the idea that if a few experienced, knowledgeable scholars on both sides and the middle of the conflict met in an informal setting, there might be some significant common ground

- After a traveling around the US, chose five people from around the United States to participate in the process

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Original FCG Team

- Deborah Loewenberg Ball
- Joan Ferrini-Mundy
- Jeremy Kilpatrick
- R. James Milgram
- Wilfried Schmid

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December 2004 to June 2005

- Chose controversial K-8 topics that both needed common ground and could be analyzed on general principles.
- Finalized initial paper which is on the MAA web site and has been published in a number of places including by NCTM and AMS.
- Discussed next steps in dissemination of process and paper and how to add to the total of common ground concepts.

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Structure of the FCG team

- Our discussions had a facilitator.
- Detailed notes were taken by one of the participants.
- Projected notes as needed so everyone could review.
- Observers were present.

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Process, Part 1

- Began by exploring "flash point" topics to understand positions.
  - Limited to topics that involved professional expertise and knowledge
  - Not topics that could be empirically tested
- Probed our own positions as well as others.
- Determined where we agreed initially and discovered other areas of agreement.
- Listen closely, frequently asking for clarification or specific examples.

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Process, Part 2

- Tested group understanding and agreement by proposing statements.
- Drafted document jointly, projecting it so everyone could understand and participate.
- Could take issue with particular words and reshape as appropriate.
- Forced the team to look closely at its language to seek common ground.

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Fundamental Premises

- Basic skills with numbers continue to be vitally important for a variety of everyday uses.
- Mathematics requires careful reasoning about precisely defined objects and concepts.
- Students must be able to formulate and solve problems.

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Areas of Agreement #1

- Automatic recall of basic facts: Automaticity.
- Calculators: Useful role.
- Learning algorithms: Use the basic algorithms of whole number arithmetic.
- Fractions: Understanding the number meaning.

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Areas of Agreement – II

- Teaching mathematics in "real world" contexts: Not always useful.
- Instructional methods: Mixture of direct instruction, structured investigation, and open exploration.
- Teacher knowledge: Depends on a solid understanding of the material.

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Reaction to Paper

- Policymakers were finally beginning to recognize the competitive crisis because of the ground swell of comments.
  - Tapping America's Potential from Business Roundtable
  - Rising above the Gathering Storm from NRC
- Have taken the FCG document and are building policy around it because they must act quickly.
- Many began the process of thinking of ways to collaborate to help students learn mathematics.

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March 2006 Meeting

- Must expand the number of people involved in the process, must expand the topics, and must delve into previous topics in more depth.
- Brought 50 – 60 experts in K-12 mathematics education to IUPUI to accomplish this goal.
- Everyone wanted to work together, and there was progress in basic communications.
- Output of this work is also on the MAA web site.

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Accomplishments/Results

- A collaborative dialogue has been started between all "sides," and agreements are being forged.
- The NCTM Focal Points is an example of the output of such an effort.
- The National Math Panel had its genesis in discussion at the Department of Education around FCG.

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Accomplishments/Result

- More mathematicians, mathematics educators, and others are getting involved to help teachers teach and students learn.
- When mathematicians and mathematics educators work together, good things happen.
  - Solid, coherent programs
  - Richardson ISD, Texas

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Richardson ISD - Results

- 33% of the students passed 2006 TAKS after failing last year.
- The teachers' mathematics knowledge significantly increased after a year of collaboration and professional development.
- Teacher mathematics knowledge and growth in this knowledge were both positively correlated to the TAKS performance of their students.

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Richardson ISD – Results

- TI technology increased student engagement and increased their algebra readiness.
- The double blocks provided more time for hands-on learning and problem solving.
- Teachers reported that the math training improved their understanding so they could better explain connections to students and understand the mathematics process.

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Next Steps

- Our group work has halted while our individual work is ongoing.
- Will continue to emphasize meaningful collaborations to increase teacher knowledge and student achievement.
- These collaborations will give teachers more opportunity to learn the mathematics critical to teaching while mathematicians will learn more about what is needed in classrooms for student learning.

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Richard Schaar
rschaar@ti.com

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It's a New World Out There



Our students need to be prepared for that world.

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Mathematics is key.

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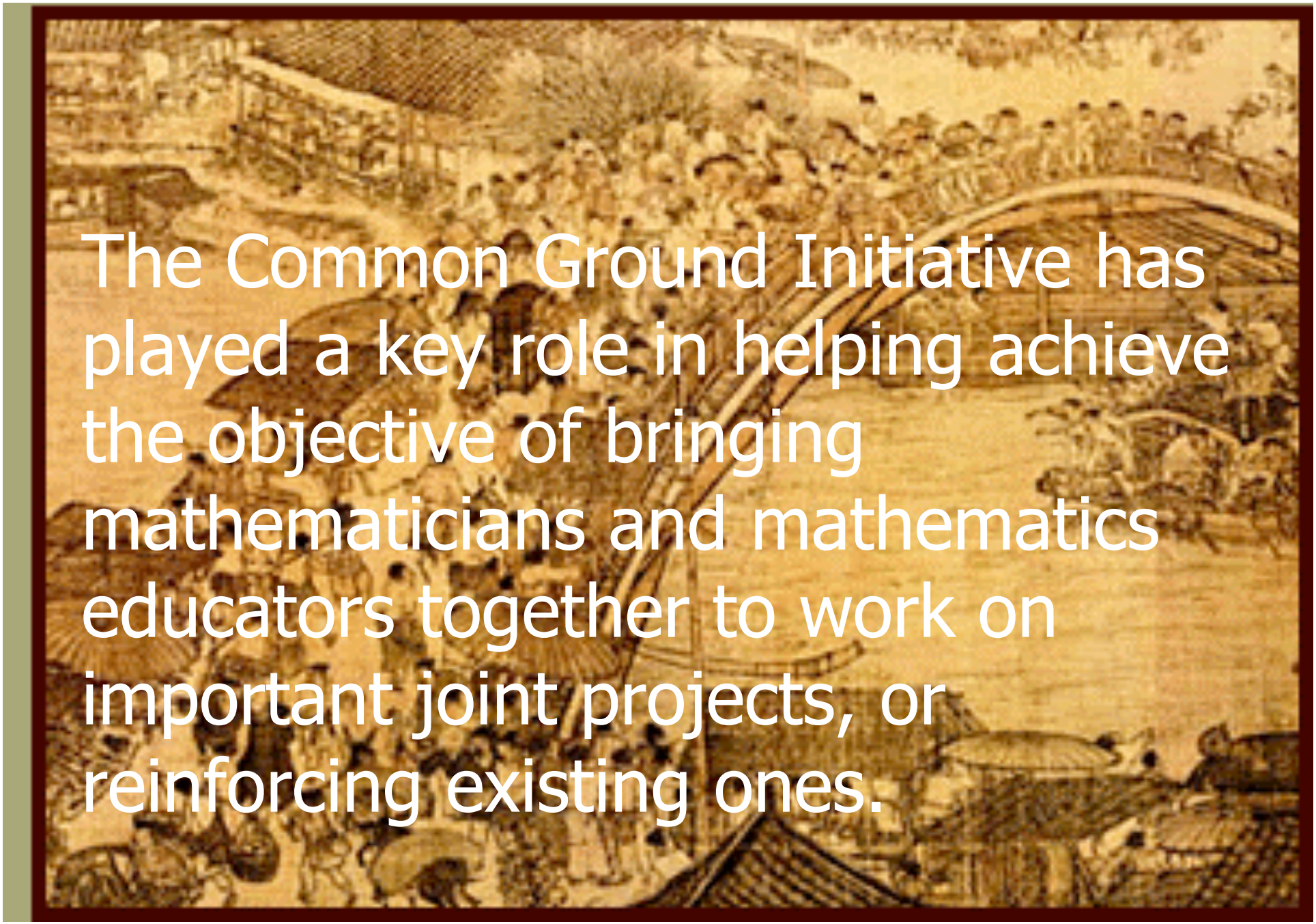
But without agreement between mathematicians and math educators there is no way our young people can learn what they need to know.



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Over the past 3 years these two communities have begun to cooperate to bring the strengths of international curricula to the United States.

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In parallel With Common Ground

- Park City, Utah in the summer Common meetings AMS and NCTM
  - Teachers come for professional development and meet with mathematicians
  - NCTM group meets with AMS group hammering out agreements on core mathematics
- Deborah Ball runs a laboratory on elementary mathematics teaching

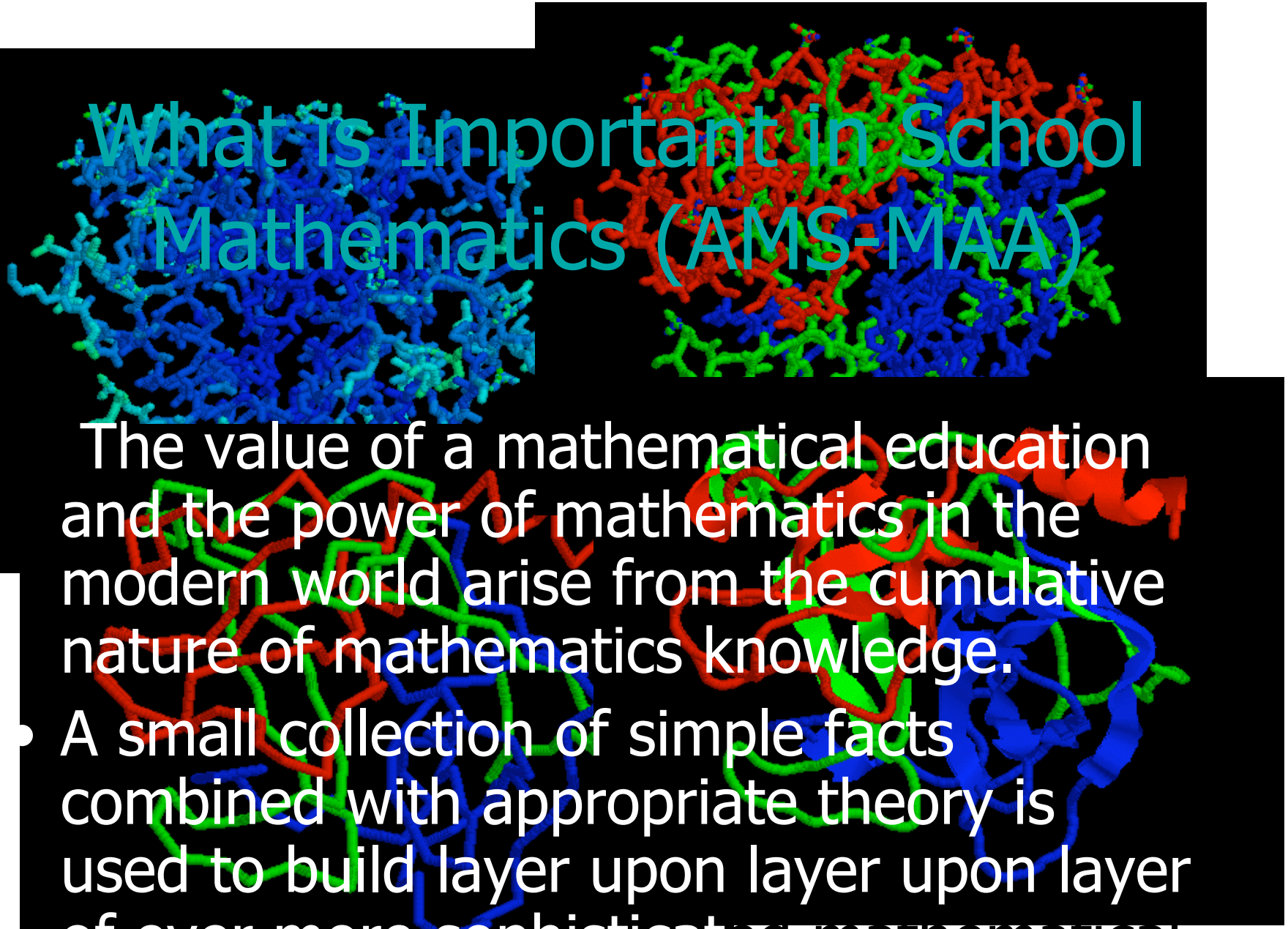
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AMS-MAA Project to create list of key topics for K-12 curriculum

- First started 3 years back by Roger Howe, Herb Clemens, and Alan Tucker.
- Developed by group of involved mathematicians and critiqued by an NCTM committee headed by Johnny Lott, with many joint meetings.

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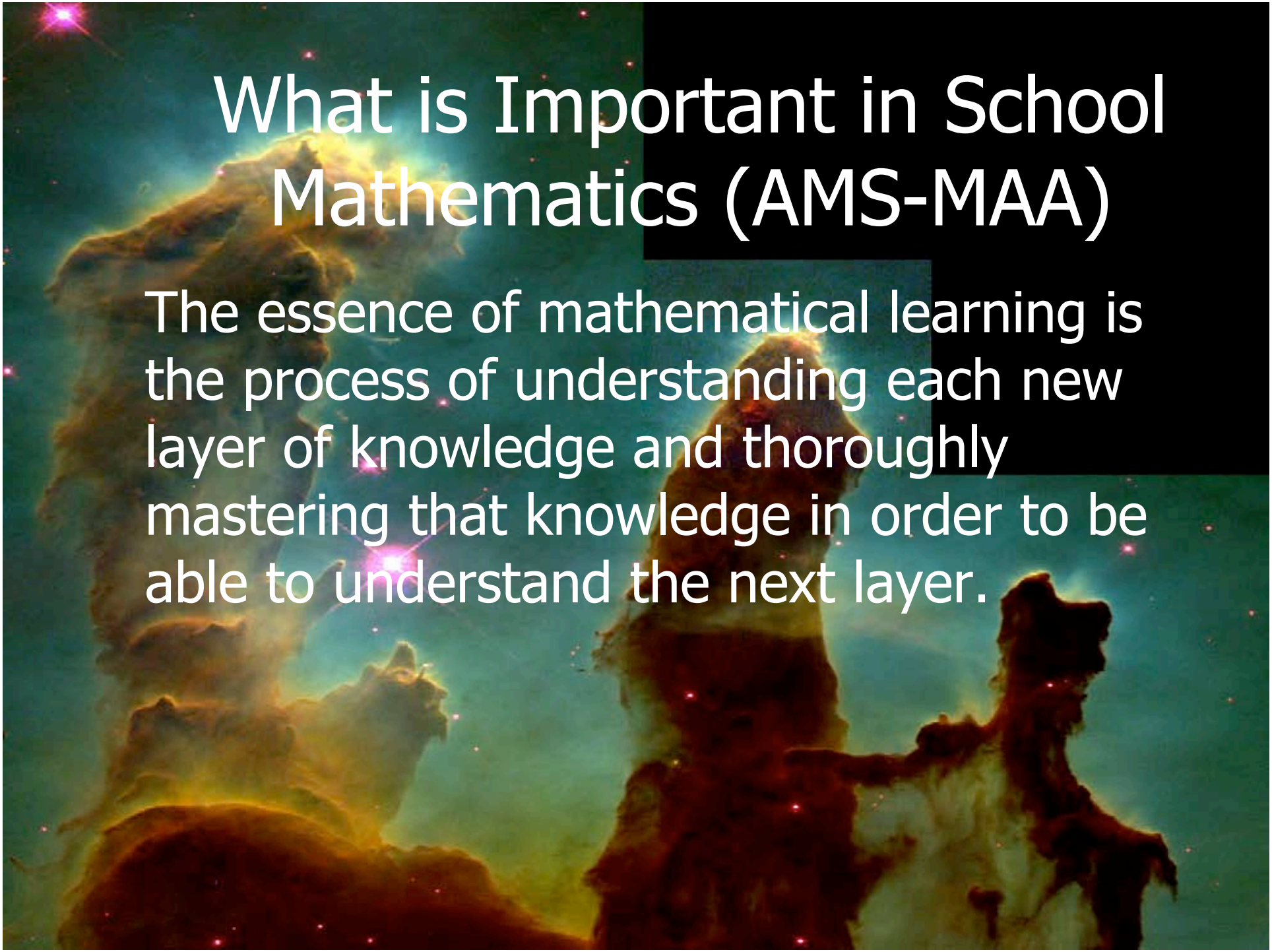
What is Important in School Mathematics (AMS-MAA)



The value of a mathematical education and the power of mathematics in the modern world arise from the cumulative nature of mathematics knowledge.

- A small collection of simple facts combined with appropriate theory is used to build layer upon layer upon layer of ever more sophisticated mathematical knowledge.

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(AMS-MAA) Principles for School Mathematics (1)

- Whole number arithmetic and the place value system are the foundation for school mathematics with most other mathematical strands evolving from this foundation. This foundation should be the subject of most instruction in early grades.

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(AMS-MAA) Principles for School Mathematics (2)

- In every grade, the mathematics curriculum needs to be carefully focused on a small number of topics. Most mathematics instruction should be devoted to developing deepening mastery of core topics through computation, problem-solving and logical reasoning.

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(AMS-MAA) Principles for School Mathematics (3)

- Instruction should be mathematically rigorous in a grade-appropriate fashion. All terms should be defined with language that is mathematically accurate. Key theorems and formulas should be proved, whenever possible.

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(AMS-MAA) Principles for School Mathematics (4)

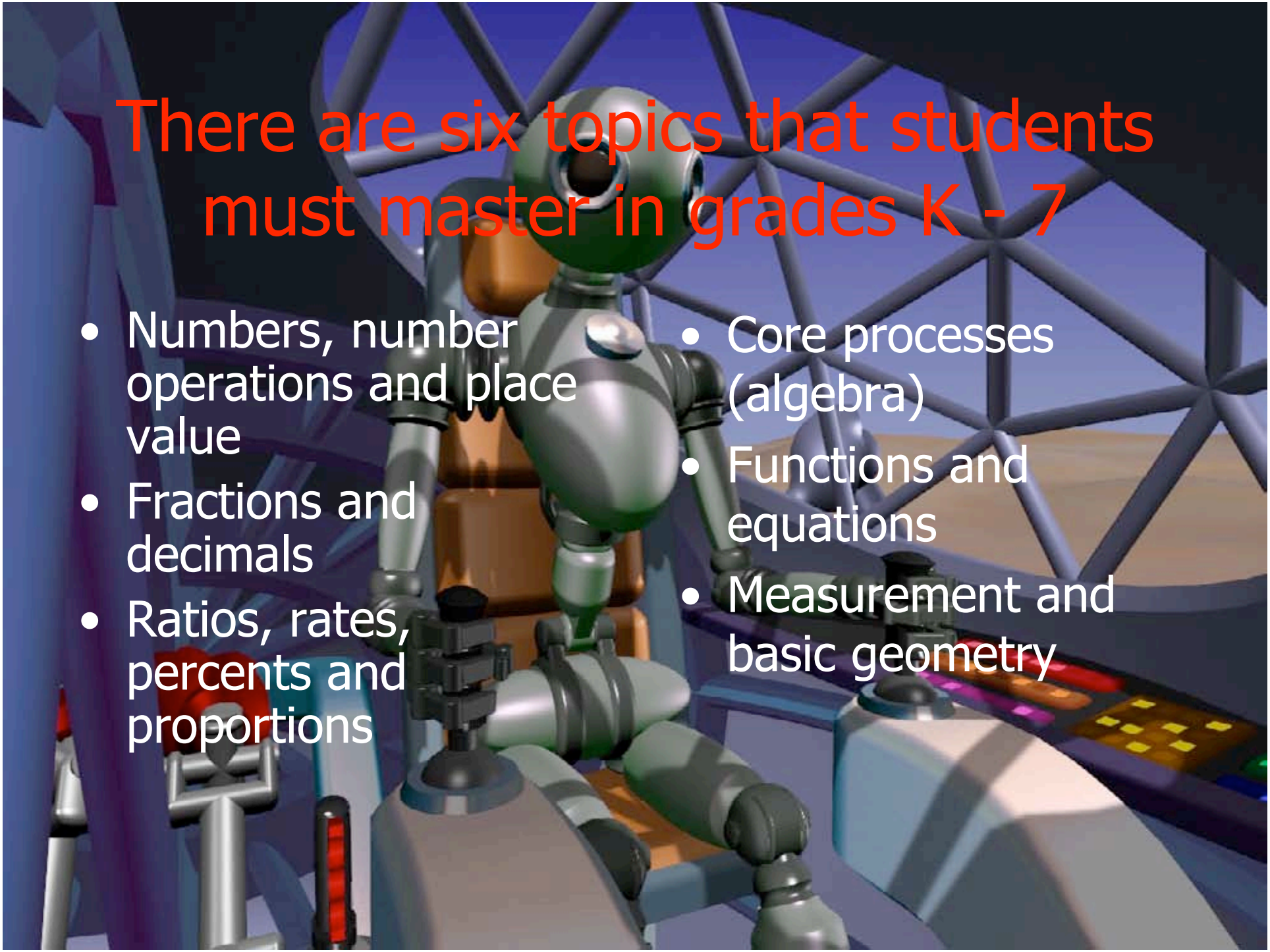
- Disciplined, mathematical reasoning is one of the most important goals of a school education.
- Although it is difficult to assess on statewide tests, it must permeate all mathematical instruction.

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(AMS-MAA) Principles for School Mathematics (5)

- Most students should be taught the mathematical knowledge and reasoning skills needed to succeed in college.
- Students planning for a Bachelor's degree in a quantitative discipline should take a more demanding mathematics track in high school which prepares them to start calculus when they enter college.

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Instrumental in Development of Focal Points

- The AMS-MAA material helped guide the selection
- The effective cooperation between the two key communities facilitated a common effort on both groups part to create the Focal Points.

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Focal Points: Numbers

K	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5
Represent, order compare whole numbers	Addition and subtraction	Base10 number system, place value	Multiplication division	Quick recall multiplication division facts, fluency with multiplication	Understand and become fluent with whole number division
Describe shapes and spaces	Whole number relationships and grouping	Quick recall of basic addition subtraction facts	Fractions, equivalent fractions	Decimals, connection between fractions and decimals	Understand and become fluent with addition, subtraction of fractions
Order objects by measurable attributes	Composing and decomposing geometric shapes	Measuring lengths	Properties of 2 dimensional shapes	Understand area, determine area of 2 dimensional shapes	Volume and surface area for three dimensional shapes

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Focal Points: Fractions

K	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5
Represent, order compare whole numbers	Addition and subtraction	Base10 number system, place value	Multiplication division	Quick recall multiplication division facts, fluency with multiplication	Understand and become fluent with whole number division
Describe shapes and spaces	Whole number relationships and grouping	Quick recall of basic addition subtraction facts	Fractions, equivalent fractions	Decimals, connection between fractions and decimals	Understand and become fluent with addition, subtraction of fractions
Order objects by measurable attributes	Composing and decomposing geometric shapes	Measuring lengths	Properties of 2 dimensional shapes	Understand area, determine area of 2 dimensional shapes	Volume and surface area for three dimensional shapes

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Focal Points: Fractions, Ratios

Grade 6	Grade 7	Grade 8
Understanding and fluency multiplying and dividing fractions and decimals	Understand and apply proportionality and similarity	Analyze and represent linear equations. Solve linear equations and systems of linear equations
Connect ratio and rate to whole number multiplication and division	Understand and use formulas to determine surface area and volume	Use distance and angle to analyze figures in two and three dimensions
Write, interpret and use expressions and equations	Solve linear equations and operate with all rational numbers, including negative numbers	Analyze and summarize data sets

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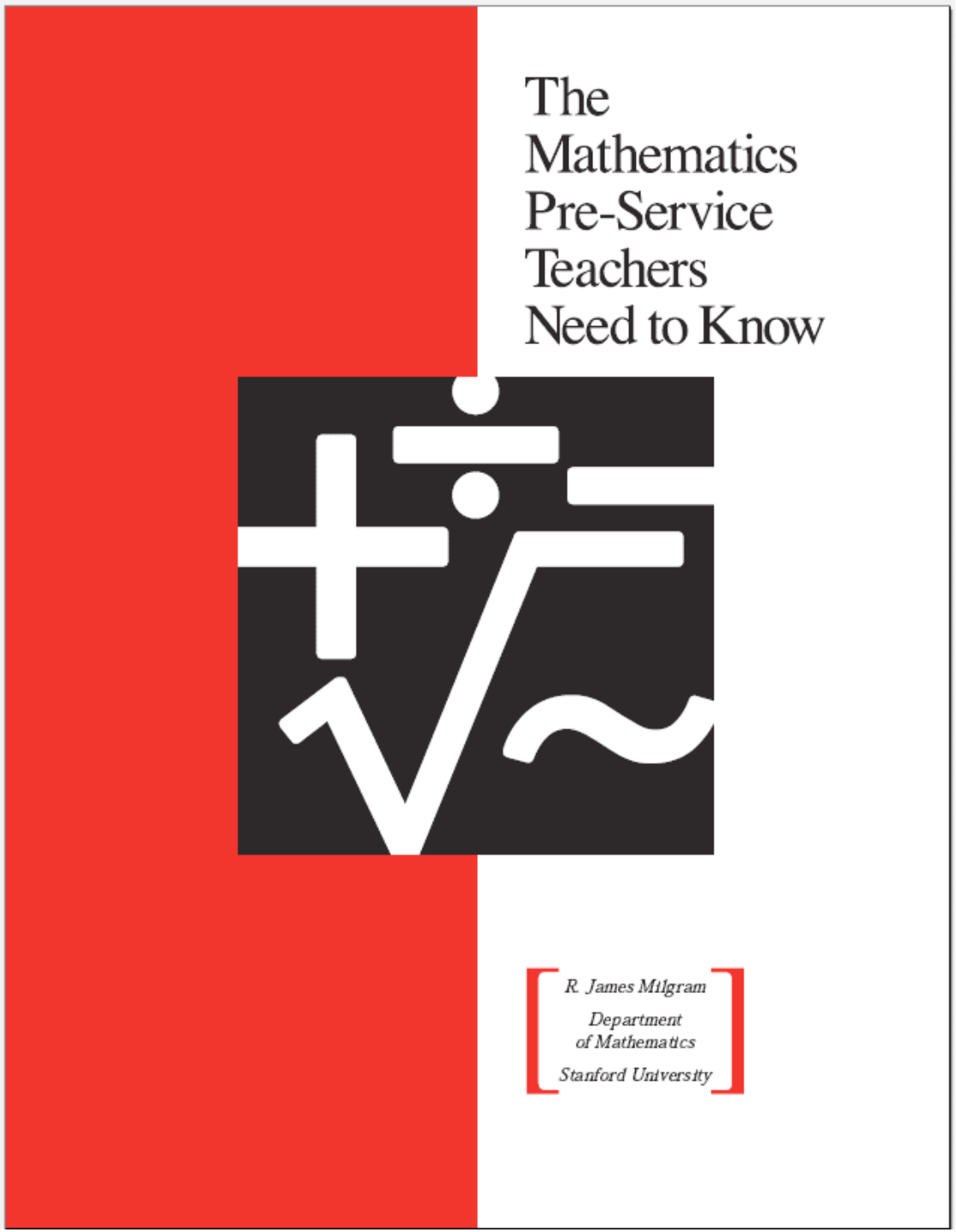
Focal Points: Algebra, Data

Grade 6	Grade 7	Grade 8
Understanding and fluency multiplying and dividing fractions and decimals	Understand and apply proportionality and similarity	Analyze and represent linear equations. Solve linear equations and systems of linear equations
Connect ratio and rate to whole number multiplication and division	Understand and use formulas to determine surface area and volume	Use distance and angle to analyze figures in two and three dimensions
Write, interpret and use expressions and equations	Solve linear equations and operate with all rational numbers, including negative numbers	Analyze and summarize data sets

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It was also developed through a joint collaboration

The book presents the key mathematics - tied to the six areas mentioned - that K-8 teachers must know together with most of the core pedagogical issues involved

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Download:

ftp://math.stanford.edu/pub/papers/milgram/FIE-book-high-res.pdf

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States are revising standards guided by Focal Points

- Among those furthest along in the process are Florida and Utah

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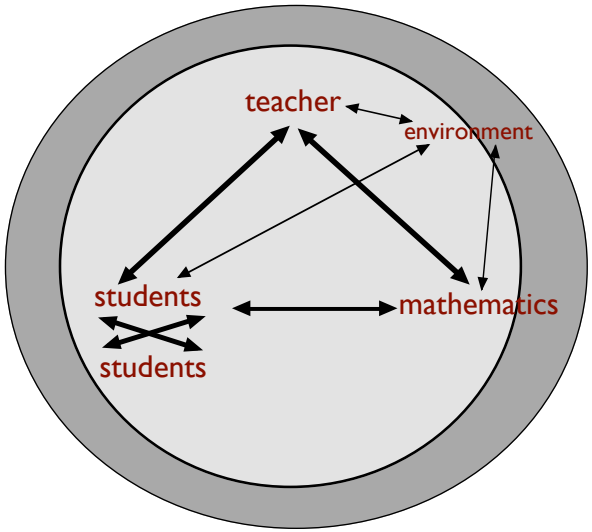
It is clear that we are achieving Common Ground on standards and the core content everyone needs to know.

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But . . . there is more to improving mathematics learning than agreeing on content standards

AGREED

Mathematics content



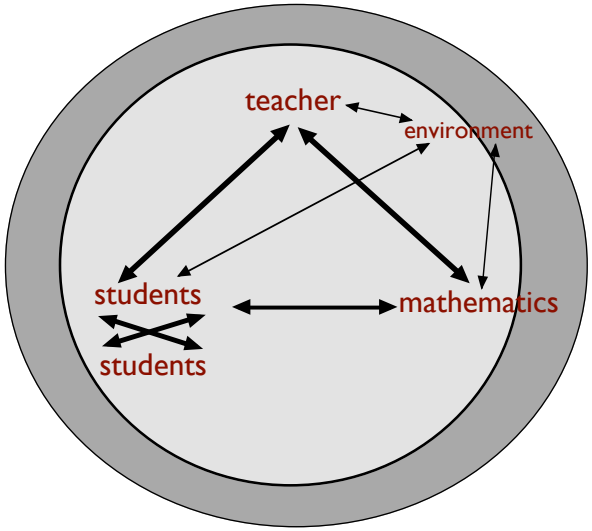
- Teaching strategies, moves, routines
- Teachers' capacity and opportunity to learn
- Teacher <u>developers'</u> capacity and opportunity to learn
- Assessment
- Closing achievement disparities and broadening who is successful at mathematics

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Examples

- Bill McCallum's new Institute on Mathematics Education, University of Arizona
- Elementary Mathematics Laboratory class at Park City Mathematics Institute
- The development of measures of mathematical knowledge for teaching (MKT)
- Mathematicians and mathematics educators (higher ed and teachers) team-teaching in teacher education and professional development

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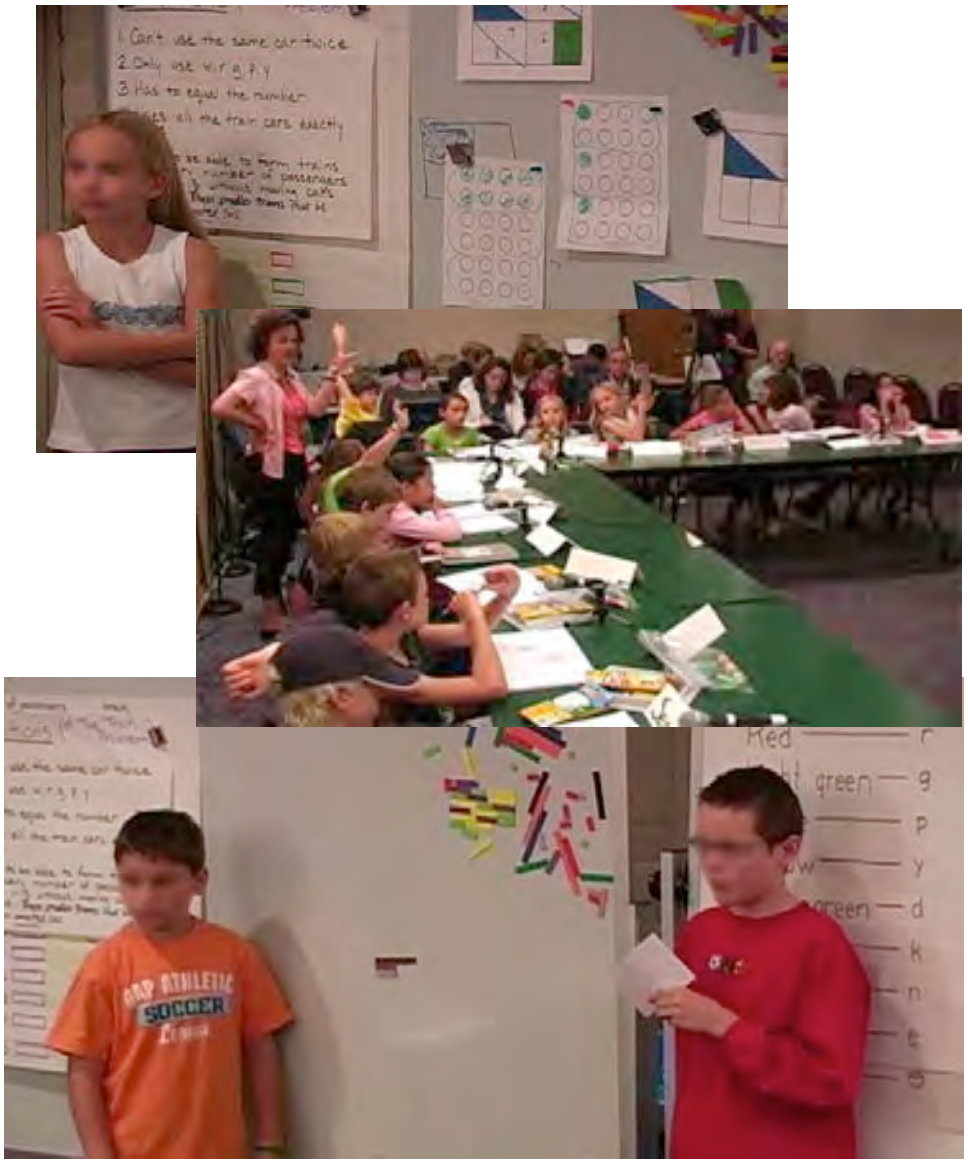


Examples

- Bill McCallum's new Institute on Mathematics Education, University of Arizona
- The development of measures of mathematical knowledge for teaching (MKT)
- Mathematicians and mathematics educators (higher ed and teachers) team-teaching in teacher education and professional development
- Collaborative teams building curriculum for teacher development and for students
- Elementary Mathematics Laboratory class at Park City Mathematics Institute

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Elementary Mathematics Laboratory



- Inner circle: an elementary summer school class — 21 Park City students, entering fifth grade, wide range of mathematics achievement and dispositions
- 2.25 hours of instruction per day
- Mathematical content: Fractions, permutations, number line as a mathematical context
- Mathematical skills: explaining, representing, proving
- A laboratory for developing and studying mathematics instruction, and the mathematical demands for teachers
- Outer circle: mathematicians, K-12 teachers, math educators, international visitors

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ELM Design and planning

- Team of two experienced teachers and two mathematicians, adapted "lesson study" model
- One teacher who taught the class
- Close review of each day's class — the teaching, the students' work and talk, the task itself, by design team plus observer-participants
- Next steps in design for next day

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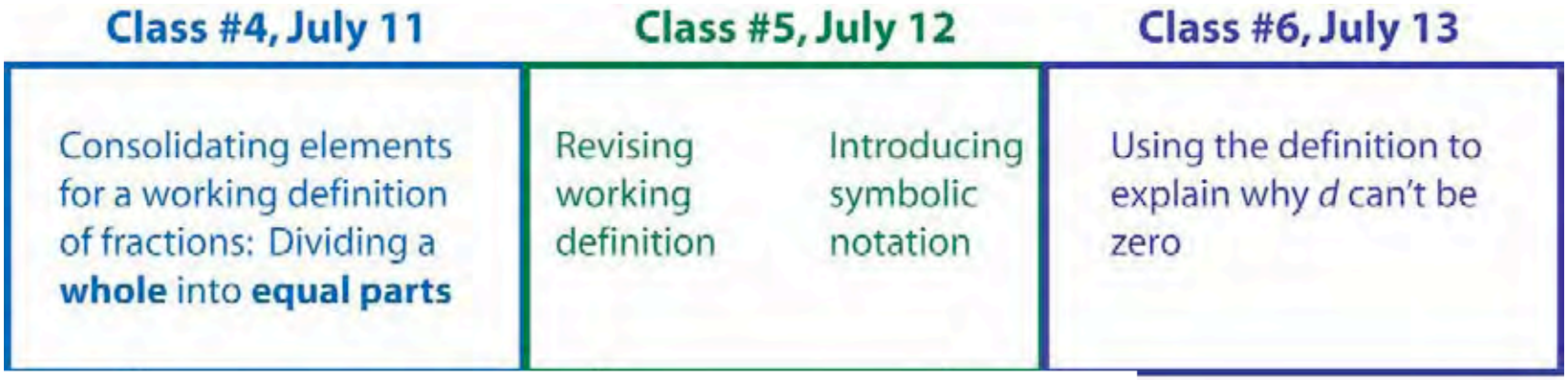
Learning about teaching: Exploring developing precise and usable definitions at elementary level

1. Can we write a definition of fraction that is correct and mathematically precise?
2. The usability of the definition: Does this seem usable by fourth graders? What problems might it present?
3. Using mathematics in teaching: What might it take to introduce this in a class, and make it usable with children? What does it gain for the students' learning?

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Development of fraction work in fourth grade class

In classes #2 and #3, the pupils solved fraction problems that surfaced central fractions concepts, such as the importance of the unit, or whole, and equal parts.



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Developing possible definition of fraction

If some whole is divided into d equal parts, then we write $\frac{1}{d}$ to name one of those parts.

If a part is $\frac{1}{d}$ of a whole, then it takes d copies of $\frac{1}{d}$ to make the whole. $\frac{n}{d}$ is then defined as n • ($\frac{1}{d}$), or n copies of $\frac{1}{d}$.

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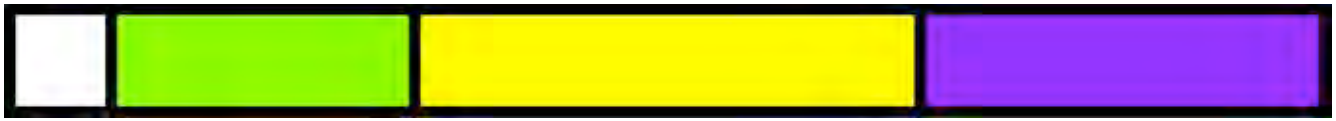
Experiment in reasoning and proving: Proving that a problem has no solution



Mr. Howe wants to order a special five-car train that uses one of each of the different-sized cars. He wants to be able to break apart his 5-car train to form smaller trains that hold exactly 1 to 15 people. In addition, he wants to be able to form these smaller trains using cars that are next to each other in the larger train.

Can the PCMI Train Company fill Mr. Howe's order? Explain how you know.

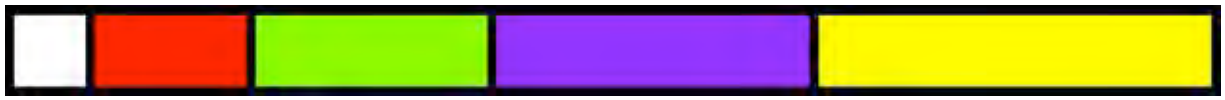
Ex: A train that holds 13 people:



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Learning about teaching: Helping students articulate the conditions of a problem

1. Can't use the same car twice
2. Only use w, r, g, p, y
3. Has to equal 0, 1, 2, 3, . . ., 14, or 15
4. Must use each rod exactly once
5. Has to be able to form trains for every number of passengers from 1-15 without moving cars around. These smaller trains must be built from connected cars.



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Learning about teaching: Exploiting the social nature of the problem as imperative for proof

Mr. Howe became discouraged with our lack of progress, so he hired a consultant to get some help with his order. The Consultant told him:

"To make a train like the one you ordered, the PCMI Train Company has to put the one-passenger car on one end and the two-passenger car on the other end."

Unfortunately, the Consultant left right away and she didn't explain her reason. Is she right, and if so, why? If she is not right, can you show why she is not?

Mr. Howe wants us to help him determine if the Consultant's claim is true and prepare a report to show him what we decide and explain it to him.

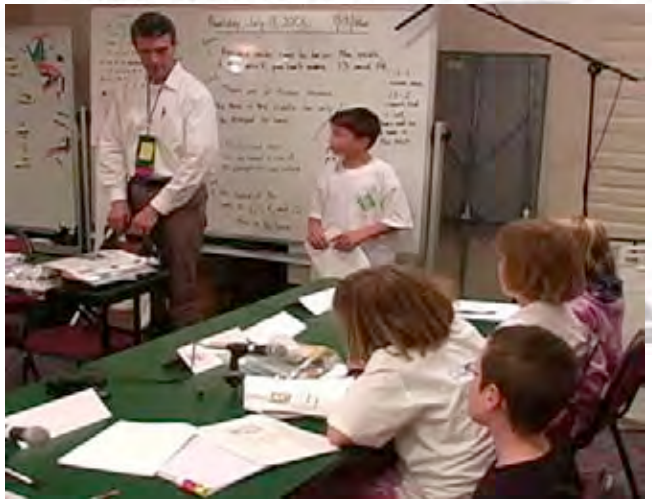
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PCMI Train Company

Dear Mr. Howe,

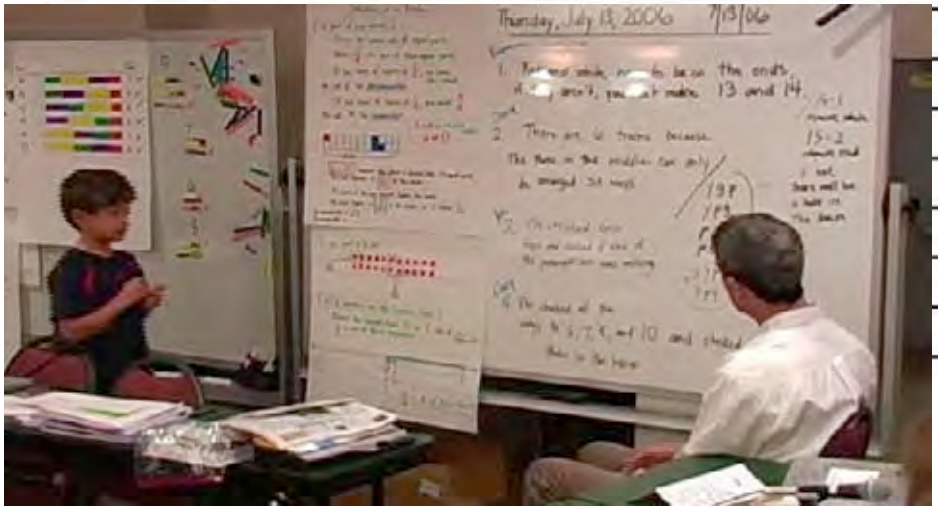
We Are Sorry Your Order Was Not Able To Be Completed. We Will Refund Your Money As Soon As Possible.

Please Turn The Page To See The Work We've Done To Prove That Your Order Cannot Be Filled.



The Companies crew:

Maddie ★ Cozy Hannah Luke
Art Sean Trevor Ben Sarah
Rebecca Rinnie David Paige
Yoni Brianna Ally
Jessica



Sign Here x Mr. Howe

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Conclusions about the potential for "common ground"

- Specific areas of agreement are possible when closer work on practice is done, instead of relying on slogans and ideology
- High quality work can be done in collaborations between mathematicians and mathematics educators
- Communication requires commitment and generosity in communication, as well as respect for different areas of expertise -- this is not easy

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