

Mit einem Auge auf den mathematischen Horizont:
Was der Lehrer braucht für die Zukunft seiner Schüler

Deborah Löwenberg Ball und Hyman Bass

University of Michigan U.S.A.

43. Jahrestagung für Didaktik der Mathematik

Oldenburg, Deutschland• 2 - 6 März 2009



SCHOOL OF EDUCATION UNIVERSITY OF MICHIGAN

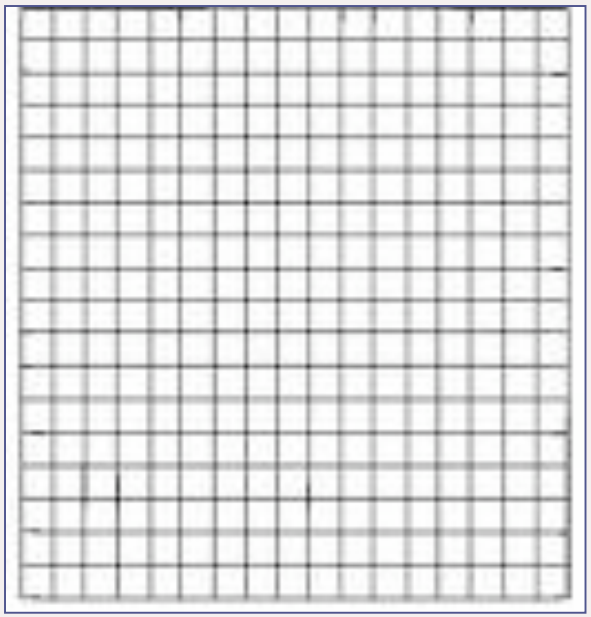
© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

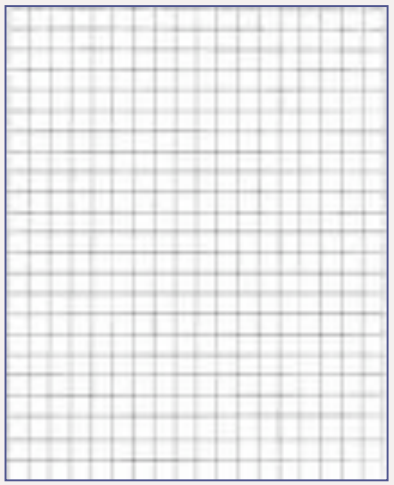
Drei Beispiele des Mathematischen Lernens

© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Messung einer Fläche im ersten Schuljahr







© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Von Vermutungen zu Beweisen im dritten Schuljahr

Der Vermutung von Betsy:

‘Addieren von zwei ungeraden Zahlen ergibt immer eine gerade Zahl.’

© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission



© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Horizont

ho•ri•zon |həˈrīzən|

noun

1 [usu. in sing.] the line at which the earth's surface and the sky appear to meet : *the sun rose above the horizon.*

• (also **apparent** or **visible horizon**) the circular boundary of the part of the earth's surface visible from a particular point, ignoring irregularities and obstructions.

• (also **celestial horizon**) Astronomy a great circle of the celestial sphere, the plane of which passes through the center of the earth and is parallel to that of the apparent horizon of a place.

2 (often **horizons**) the limit of a person's mental perception, experience, or interest : *she wanted to leave home and broaden her horizons.*

© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

What does ‘three fourths’ mean?

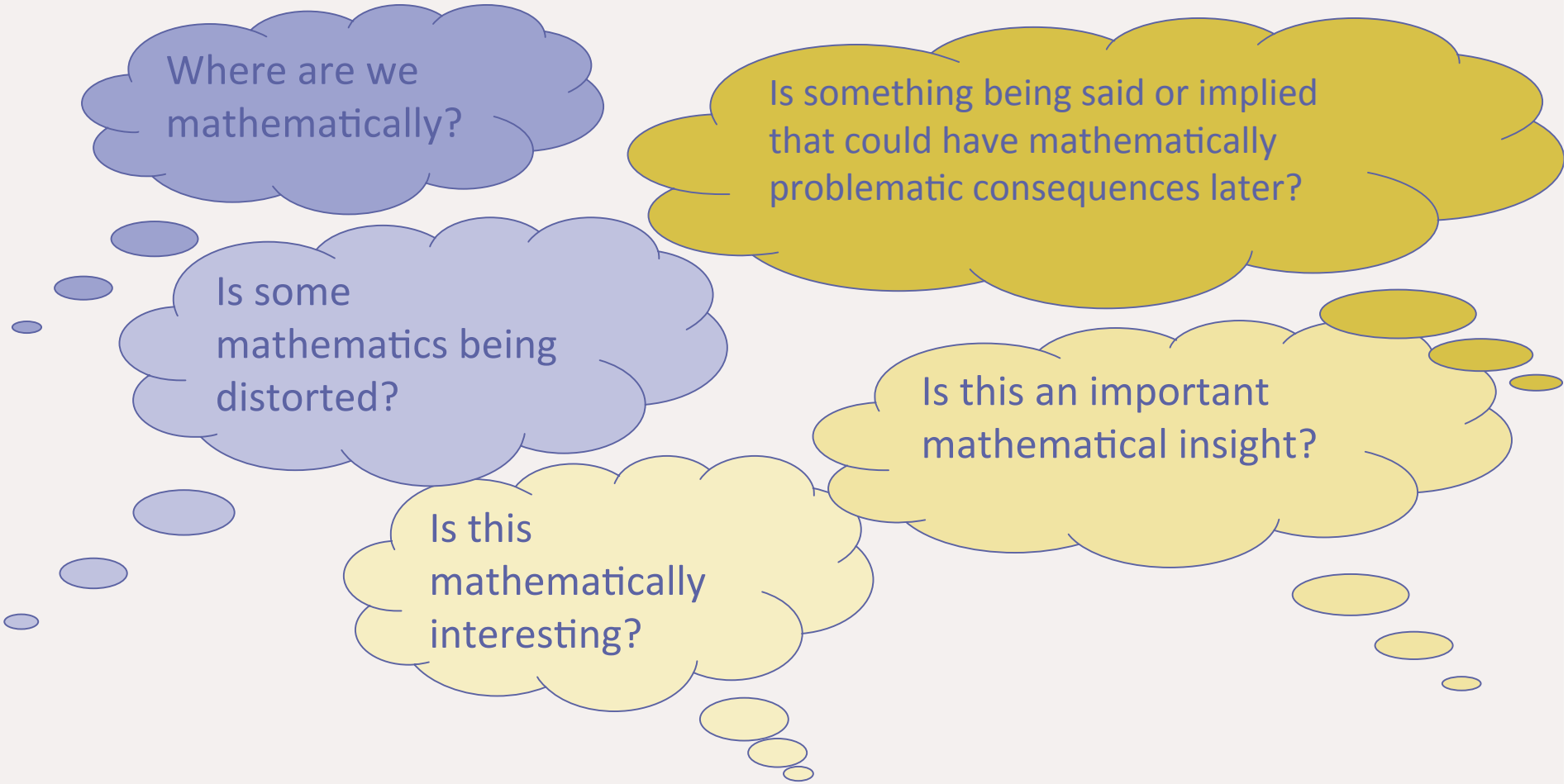
$$\frac{3}{4}$$

© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission



© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Unterricht mit einem Auge auf den mathematischen Horizont



© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Zwei Bedingunden des Unterrichts

1. Intellektuelle Ehrlichkeit
2. Respekt vor und Verpflichtung zum Lernen von jedem Schüler

© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Unterricht der verantwortlich ist und bei den Schülern gut ankommt

- Hears the mathematics in students' representations and talk
- Notices misconceptions in students' representations and talk
- Balances mathematical rigor with generosity toward emerging ideas
- Manages mathematical opportunities wisely

© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Mathematische Verantwortung

© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Was ist Mathematik für das Unterrichten?

Mathematical Knowledge for Teaching—MKT

SCHOOL OF EDUCATION UNIVERSITY OF MICHIGAN

© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Die Grundbegriffe unseres Ansatzes, der auf die Praxis aufbaut

1. Das Feststellen von dem mathematischen Unterbau des Unterrichts
2. Analyze what mathematical knowledge is involved in the work of teaching (MKT)
3. Develop measures of MKT, validate teacher scores against practice and against student achievement gains
4. Develop and evaluate approaches to helping teachers learn mathematical knowledge for teaching (MKT)

© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Knowing multiplication

Multiplizieren Sie einmal diese Zahlen:

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline \end{array}$$

© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Knowing multiplication for teaching

Welche mathematischen Begriffe wurden dabei verwendet?

(a)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 405 \\ 108 \\ \hline 1485 \end{array}$$

(b)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 225 \\ 100 \\ \hline 325 \end{array}$$

(c)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 1250 \\ 25 \\ \hline 1275 \end{array}$$

© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Knowing division with fractions

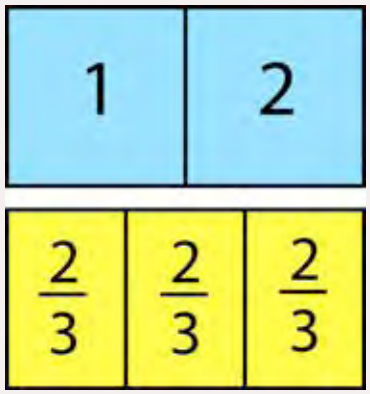
$$2 \div \frac{2}{3}$$

© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

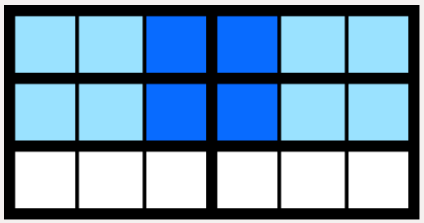
Knowing division of fractions for teaching

Which of these <u>can</u> represent $2 \div \frac{2}{3}$?

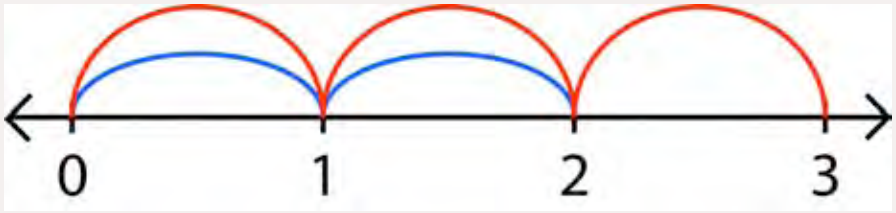
(a)



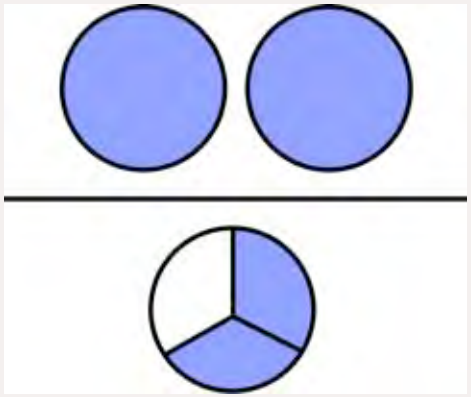
(b)



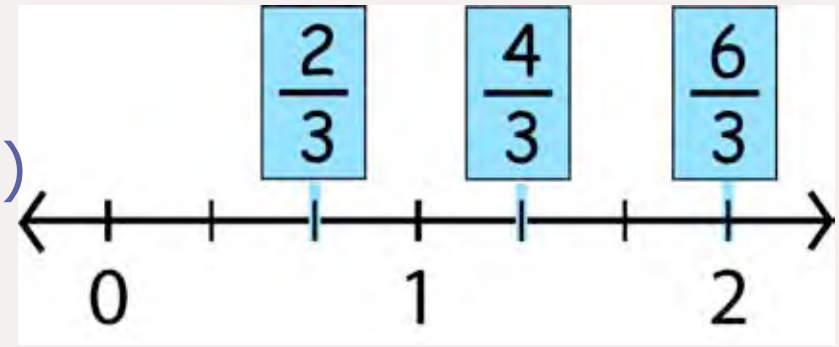
(c)



(d)



(e)



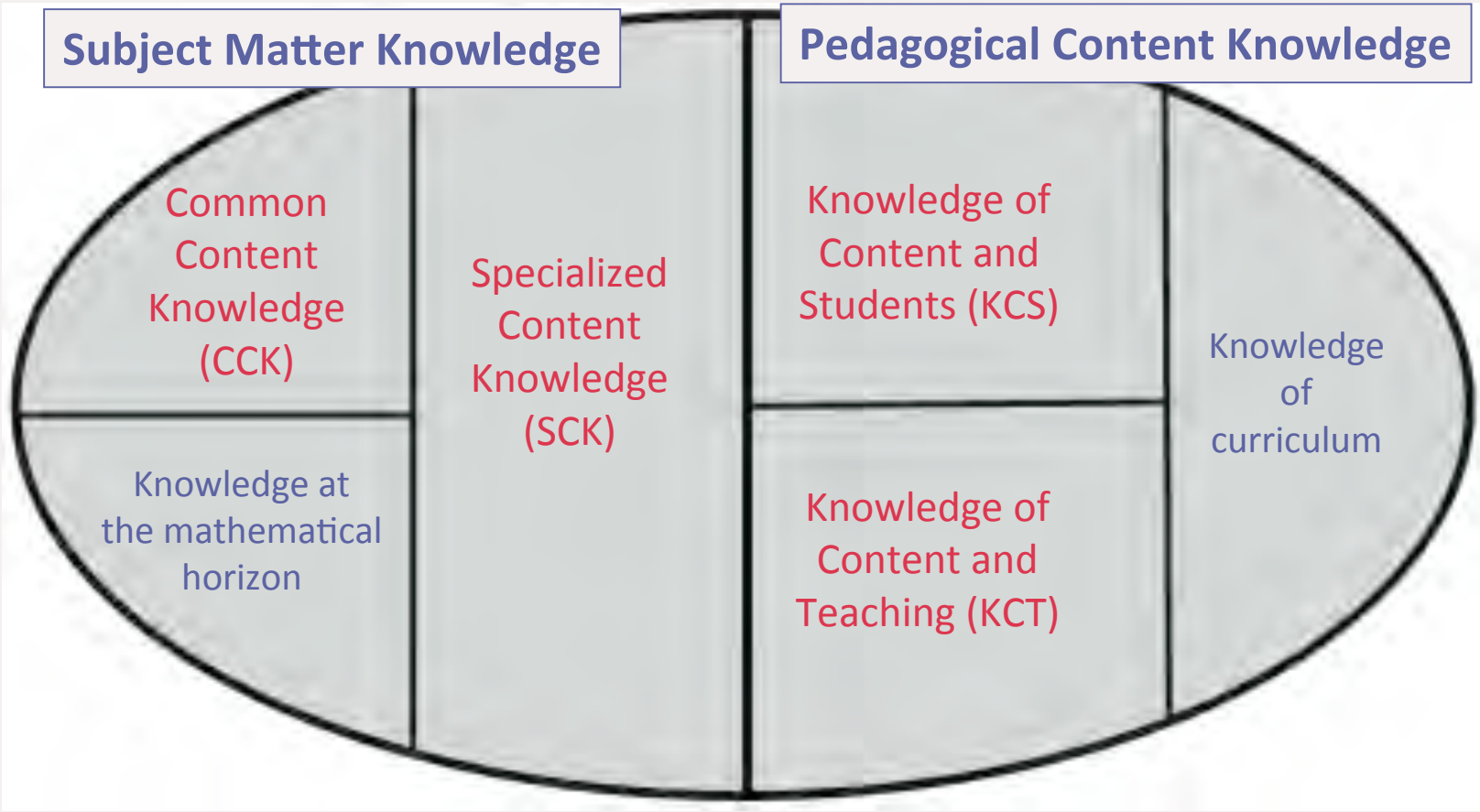
© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Beispiele aus dem Unterricht

1. Selecting/designing tasks
2. Identifying and working toward the mathematical goal of the lesson
3. Listening to and interpreting students' responses
4. Analyzing student work
5. Teaching students what counts as "mathematics" and mathematical practice
6. Making students' errors into good mathematical work
7. Attending to the ambiguity of specific words
8. Deciding what to clarify, what to make more precise, what to leave in student's own language

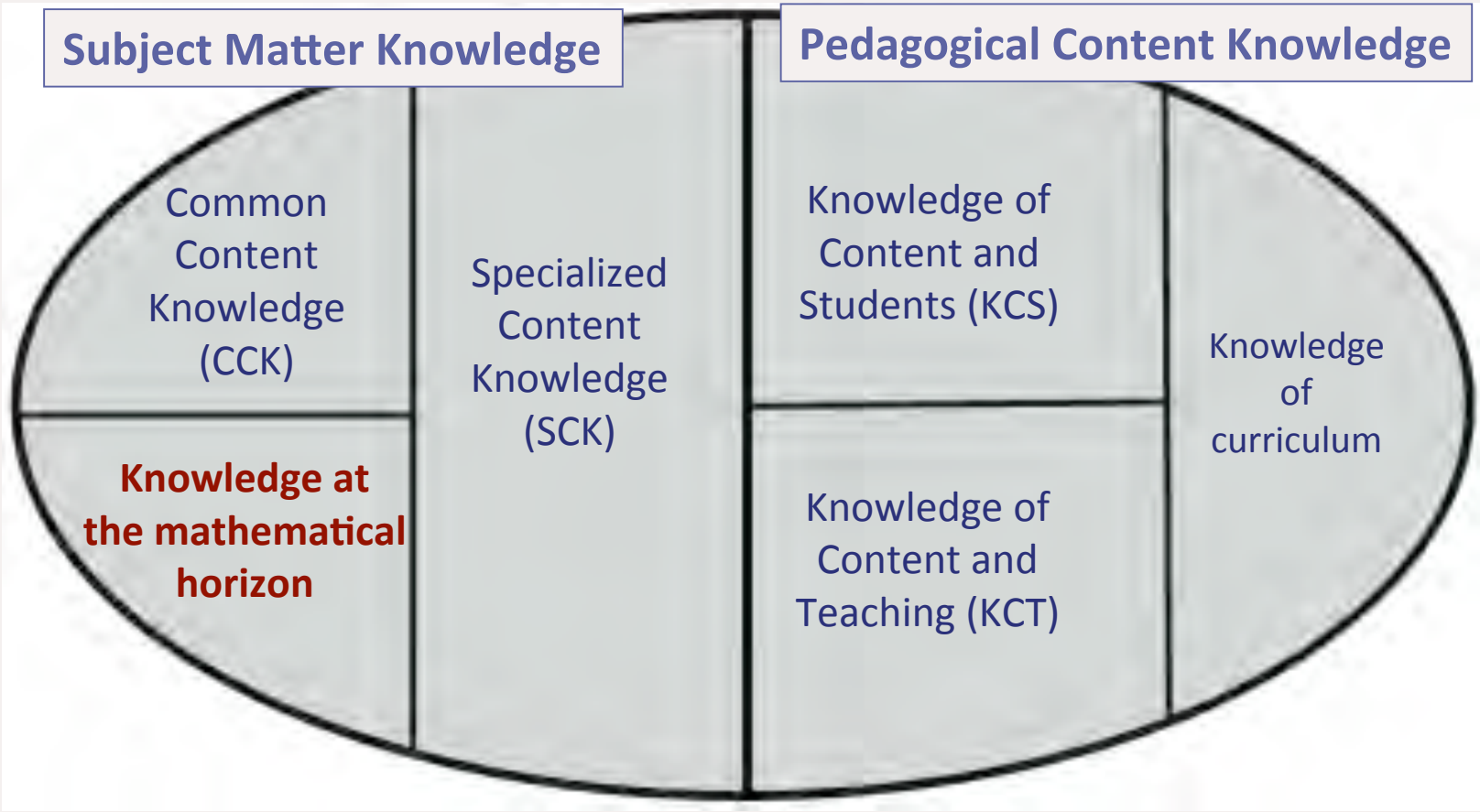
© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Mathematik für das Unterrichten (MKT)



© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Mathematik für das Unterrichten (MKT)



© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Wo (oder was) ist der "mathematische Horizont" und wie sollte es die Arbeit der Lehrer beeinflussen?

© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Video

- Drittes Schuljahr, U.S.A., 8 – 9 Jahre
- Untersuchung der geraden und ungeraden Zahlen
- Diskussion einer überraschenden Idee von einem Schüler
- Mathematische Tätigkeiten (vermuten, beweisen, definieren)
- Norms of respect

© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Bitte achten Sie auf diese Aspekte

1. Welche sind die wichtigen mathematischen Themen?
2. Wo ist der "mathematische Horizont"?
3. Welche Themen könnten die Arbeit der Lehrerin beeinflussen?

© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Die Idee von Shea

Die Zahl 6 kann auch ein gerade und ein ungerade Zahl sein.

© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission



© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Welche Mathematik sehen Sie in diesem Video?

that the teacher—

- might need to be aware of
- might need to deal with
- might use to help navigate the situation?

Auf was sollte die Lehrerin achten (bei der Mathematik)?

© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Welche Mathematik sehen Sie in diesem Video?

- Reasoning, and critical listening; imperative for mathematical justification
- Die Zahl 6 kann auch eine gerade Zahl und eine ungerade Anzahl sein.
- drei Definitionen für gerade und ungerade Zahlen: 'fair share' — 2•n und (2•n) + 1; 'pair'— n•2 und (n•2) +1; alternierend
- What would be mathematically precise definitions of even and odd usable by third graders?
- Lin states Shea's idea, then generalizes it in order to convince him it is unreasonable
- What are these "Shea numbers?"

© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Mathematische Verantwortung

- –Make judgments about mathematical importance
- –Hear what students are saying
- –Highlight and underscore key points
- –Anticipate and make connections
- –Notice mathematical opportunities
- –Catch mathematical distortions or possible precursors to later mathematical confusion or misrepresentation

© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Welche Mathematik sollte ein Lehrer kennen für die Zukunft seiner Schüler?

1. A sense of the mathematical environment surrounding the current "location"
2. Major disciplinary ideas and structures
3. Mathematische Tätigkeiten

1. Core mathematical values and aesthetics

1. Definitions, factorization, modular arithmetic

2. Number systems, Körper Axiome, equations
3. Establishing equivalence, choosing representations, proving
4. Precision, consistency, efficiency

© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Current thoughts about horizon knowledge as a domain of MKT

- Horizon knowledge is advanced knowledge that equips teachers with perspective for their work; it is not knowledge of the kind they need to understand in order to explain it to pupils
- Knowledge of the horizon does not create an imperative to act in any particular mathematical direction
- Some horizon knowledge is topics, some is practices, some is values
- Wir wissen nicht die Antworten auf mehrere wichtige Fragen
  - How far out or in what direction the useful horizon extends
  - How much detail is needed
  - How horizon knowledge can be acquired and developed
  - How horizon knowledge is related to student achievement

© 2009 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission