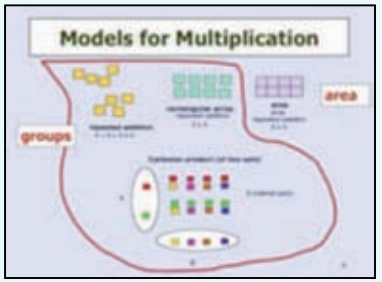
Who Knows Mathematics Well Enough to Teach It?



Deborah Loewenberg Ball



University of Michigan

Center for Proficiency in Teaching Mathematics

Annual Meeting of

the Association of State Supervisors of Mathematics

Anaheim, CA

April 4, 2005

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Overview of Session

1. Setting the policy context
2. A brief field trip into a third grade mathematics classroom
3. Mathematical knowledge for teaching: A theory in progress
4. Developing and using measures of mathematical knowledge for teaching

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1. Setting the Context



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Challenges to Teacher Education

1. To claim professional jurisdiction for the preparation and development of teachers (Grossman, 2005)
2. To articulate characteristics of "high quality" teaching, and the knowledge and skill demands for teachers
3. To seek evidence on effects

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2. A Short Visit to a Classroom

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Context for Field Trip

- Third graders (ages 8 - 10)
- Mathematics: even and odd numbers, definitions, showing why something is true
- Multi-lingual and multi-cultural class

Easy: The teacher should have . . .

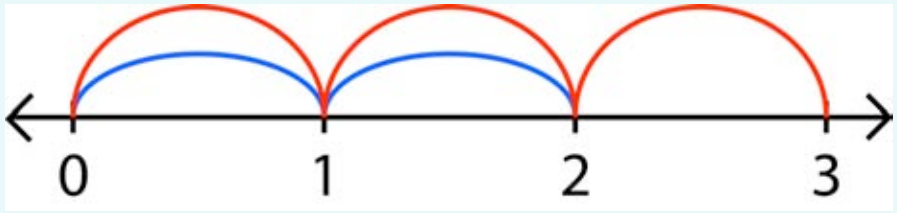
More difficult: What mathematics do you notice here? What is going on mathematically? What might a teacher need to be sensitive to?

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What Mathematics is at Play Here?

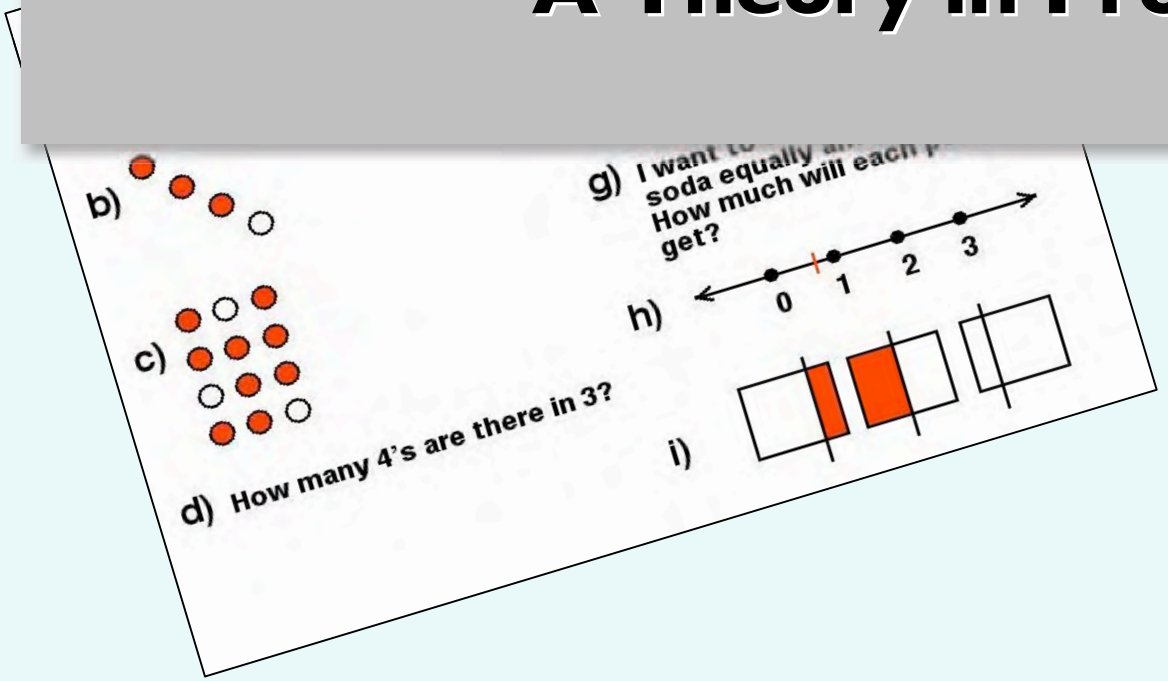
- Three different definitions of "even number" in use, unreconciled
- Shea has an idea in need of a name; is instead appropriating "even and odd" to label his idea
- Shea makes a claim about 6; Lin generalizes
- Students expecting to listen, interpret, and respond to others' claims
- Imperative for justification
- What is a mathematically appropriate and usable definition of "even" and "odd" numbers for third graders?
- A small step from mod-2 to mod-4 arithmetic.

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$$2 \div \frac{2}{3}$$

3. Mathematical Knowledge for Teaching: A Theory in Progress



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Common Framing of the Problem
→ Failure to Solve the Problem

- Teachers do not know mathematics well enough (evidence and causes vary)
- Concentration of under-qualified teachers in urban and high poverty schools

- Increase number and rigor of mathematics requirements for teacher certification
- Recruit mathematically-trained people into teaching

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Clarifying the Problem

The quality of mathematics teaching and learning

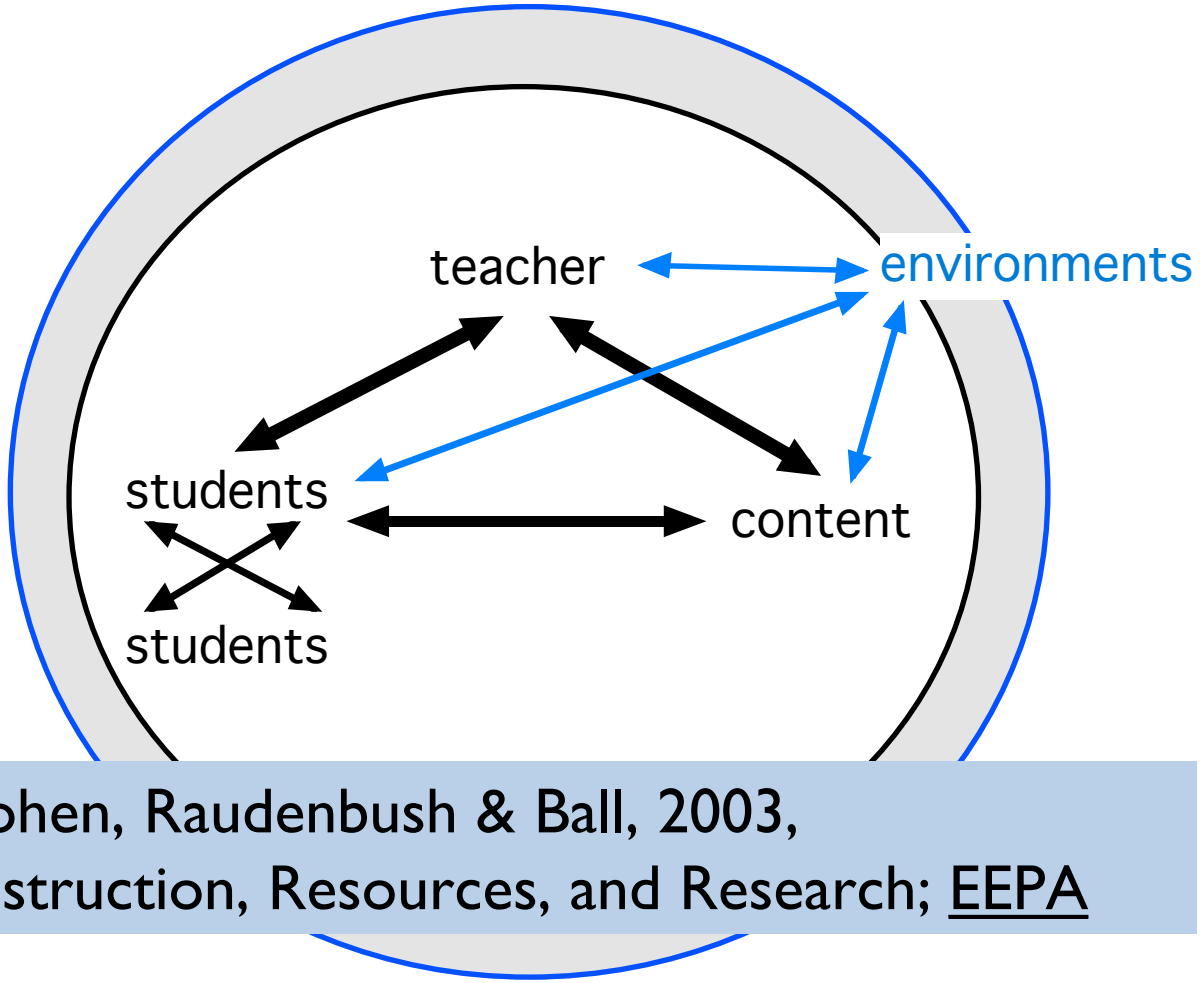
Teachers' knowledge of mathematics

and their ability to use it in teaching

What mathematical resources do teachers need in order to teach mathematics to all students effectively?

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Using Knowledge IN Instruction



Cohen, Raudenbush & Ball, 2003,
Instruction, Resources, and Research; EEPA

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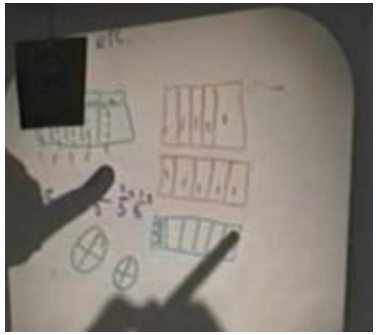
A Practice-Based Approach to Answering the Question

1. **Study instruction**, and identify the mathematical work of teaching
2. Analyze **what mathematical knowledge** is needed to do that work effectively, and **how** it must be understood in order to be useful for the work
3. Develop, test, and refine **approaches to helping teachers** develop and use mathematical knowledge for teaching, study results

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What Do we Mean By "Mathematical Knowledge for Teaching"?





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Mathematical Knowledge for Teaching

What do we mean when we use this term, "**mathematical knowledge for teaching**"?

- Mathematical knowledge, skill, habits of mind that are entailed by the work of teaching

What do we mean by the "**work of teaching**"?

- The tasks in which teachers engage, and the responsibilities they have, to teach mathematics, both inside and outside of the classroom

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What *is* "Mathematical Knowledge for Teaching"? An Example from Multiplication of Decimals

Multiply:

$$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline \end{array}$$

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Analyzing Incorrect Answers for

$$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline \end{array}$$

Why do you multiply before you add?

(a)

$$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline 255 \\ 80 \\ \hline 10.55 \end{array}$$

(b)

$$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline 62.5 \end{array}$$

Why do you have as many decimal places in the answer as the total number of decimal places in the problem?

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Analyzing Correct Answers for

$$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline \end{array}$$

(a)

$$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline .25 \\ 1.5 \\ 1 \\ 6 \\ \hline 8.75 \end{array}$$

(b)

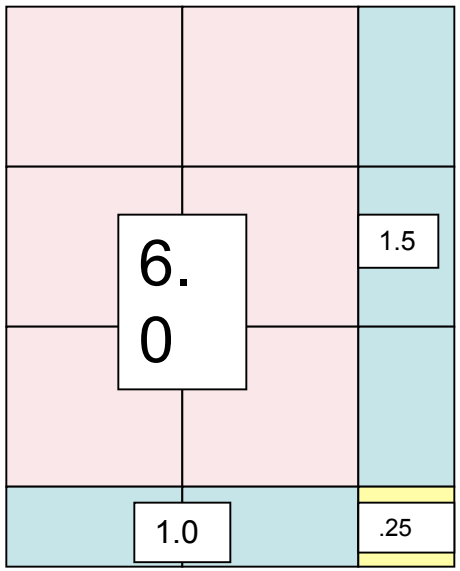
$$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline 1.25 \\ 7.5 \\ \hline 8.75 \end{array}$$

Is there a method?
Does it work for
all decimal numbers?
How do you know?

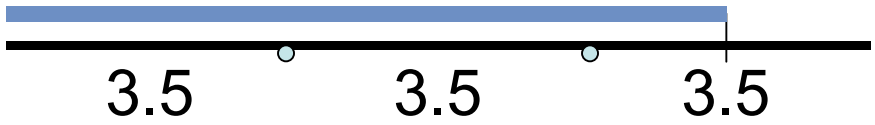
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Using Representations for $\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline \end{array}$

(a)



(b)



Explain:
Where is the 3.5?
Where is the 2.5?
Where is 8.75?

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Design Quiz Questions

A. .5 7 .01 11.4

B. .60 2.53 3.12 .45

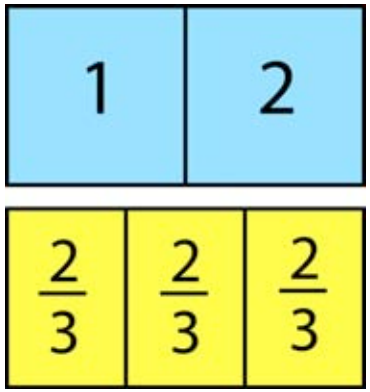
C. .6 4.25 .565 2.5

D. These lists are all equally good for assessing whether students understand how to order decimal numbers.

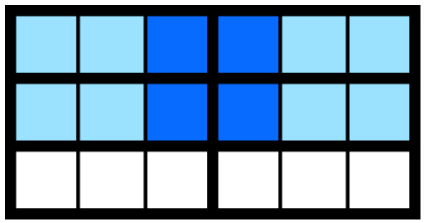
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Choose, Develop, and Use Representations

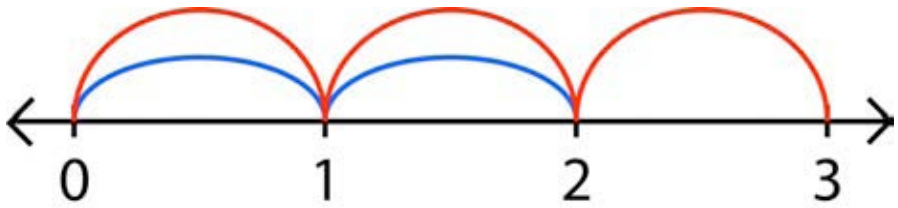
(a)



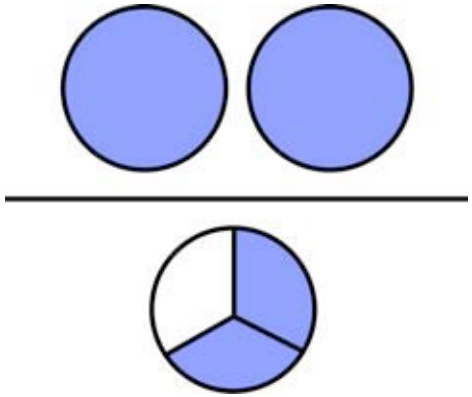
(b)



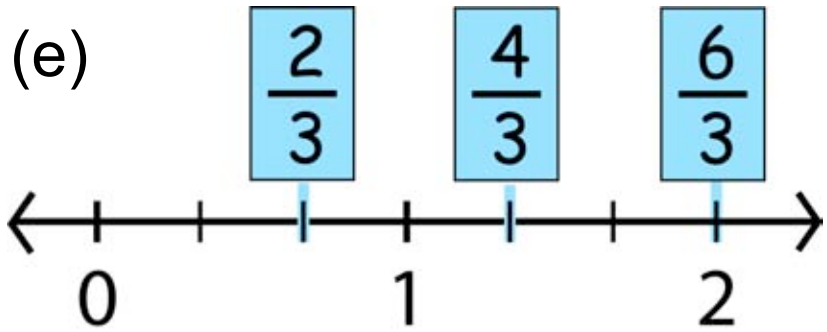
(c)



(d)



(e)



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Use and Teach Definitions

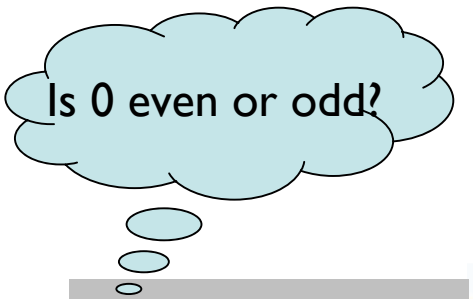


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Examine Textbook Definitions

How well does any of these help to answer the the question?

1. An even number is a number of the form 2k, where k is an integer.
2. An even number is a natural number that is divisible by 2.
3. An even number is any multiple of 2.
4. An even number is a number that has 0, 2, 4, 6, or 8 in the ones place.



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Define Terms

Write a definition of "even number" that is mathematically precise and also usable by third graders.



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Hypotheses About Knowledge of Mathematics for Teaching

PREVALENT HYPOTHESES

1. Teachers need knowledge of the curriculum, plus N levels more knowledge
2. Pedagogical content knowledge, curricular knowledge

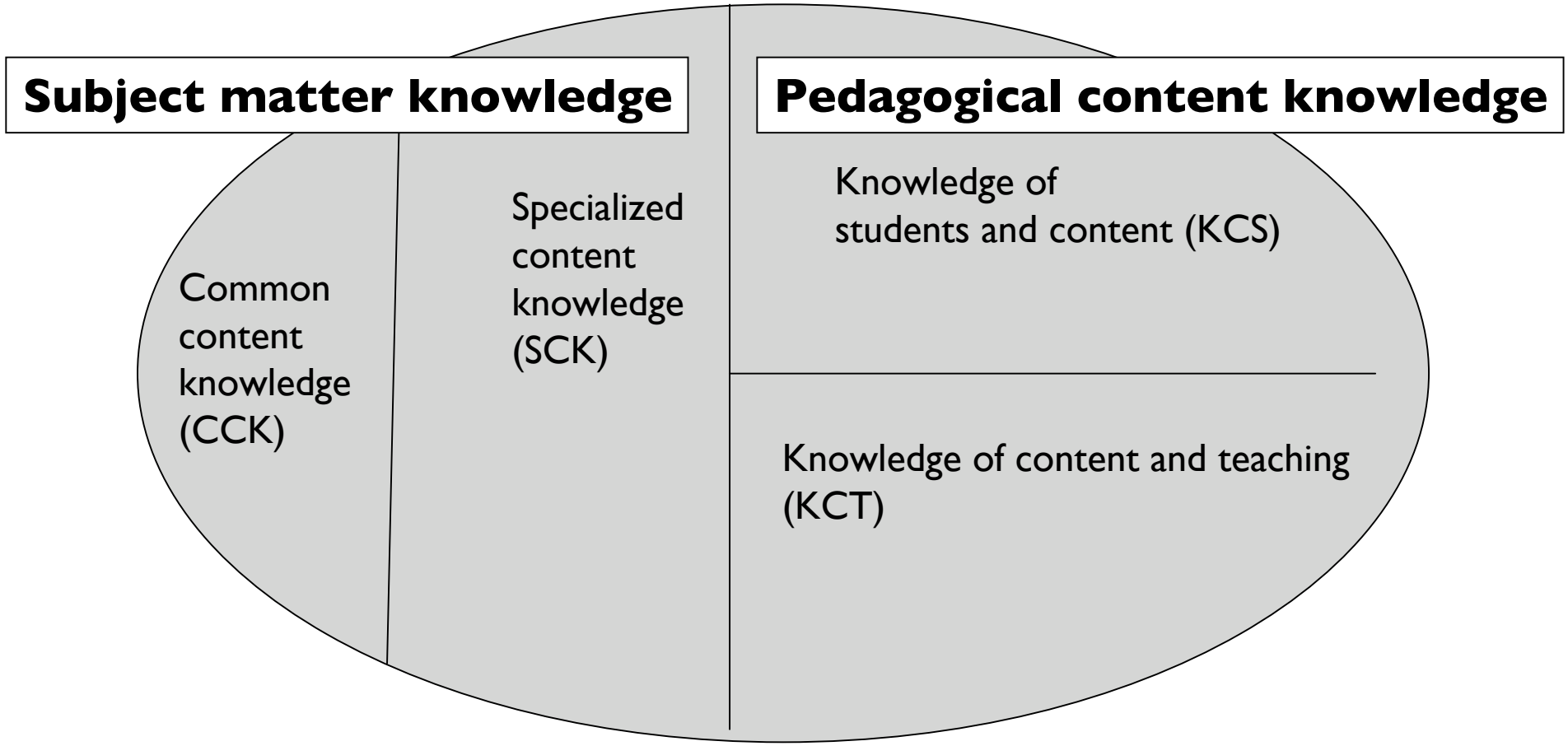
OUR CURRENT HYPOTHESES

- **Common** content knowledge
- **Specialized** content knowledge
- Knowledge of content and **students**
- Knowledge of content and **teaching and curriculum**

What does this have to do with "pedagogical content knowledge"?

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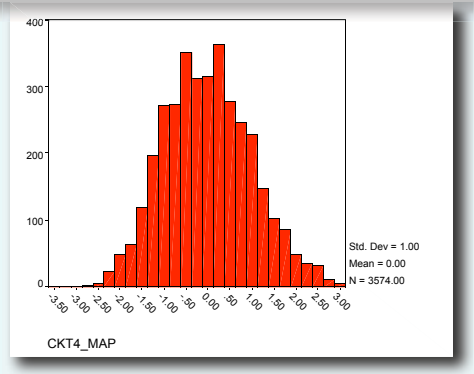
Shulman's Original Category Scheme (1985) Compared with Ours



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4. Developing and Using Measures of Mathematical Knowledge for Teaching

Student A	Student B	Student C
35 x25 125 +75 875	35 x25 175 +700 875	35 x25 25 150 100 +600 875



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Why Would We Want to "Measure" Teachers' Content Knowledge for Teaching?

- To understand role of teachers' content knowledge in students' performance
- To study and compare outcomes of professional development and teacher education
- To inform design of teachers' opportunities to learn content knowledge

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Measures of Teachers' Mathematical Knowledge for Teaching

OUR GOALS:

- Measure mathematical knowledge used in teaching
- Contextualize the items in tasks (or situations) that all teachers face
- Usable measures:
  - Large sample of teachers (N=5500) = multiple choice
  - Not grade-specific (grade 1 - 6 teachers)
  - Not represent any single view of teaching (e.g., "reform")
  - Items that vary in difficulty

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Developing and Using Valid Measures of Mathematical Knowledge of Teaching

- Interdisciplinary item development
- Piloting and analysis
  - Discriminating ***common content knowledge*** from ***specialized content knowledge***
- Content validity checks
- Cognitive tracing interviews with teachers, laypeople, mathematicians
  - Solving mathematical problems involving specialized content knowledge not always obvious or easy for mathematicians
- Examining videotapes of classroom teaching
- Using teachers' scores to predict student achievement

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Linking Teacher Knowledge and Student Achievement: New Results

- Questionnaire consisting of 30 items (scale reliability .88)
- Model: Student Terra Nova gains predicted by:
  - Student descriptors (family SES, absence rate)
  - Teacher characteristics (math methods/content, content knowledge)
- Teacher content knowledge significant
  - Small effect (< 1/10 standard deviation): 2 - 3 weeks of instruction
  - But student SES is also about the same size effect on achievement

    (Hill,Rowan, and Ball, in press)

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Conclusion: Who Knows Mathematics Well Enough to Teach It?

- Mathematical knowledge for teaching is the knowledge needed for the work of teaching. That work entails specialized content knowledge distinct from that required by other mathematically-intensive professions.
- Figuring out what teachers need to know requires multiple kinds of work, including research-based evidence.
- Work on these questions is central to the future of the profession, schools of education, and to the improvement of mathematics education.

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Slides will be available at

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