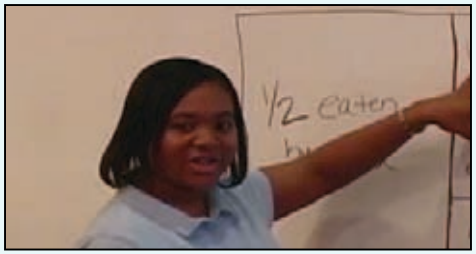
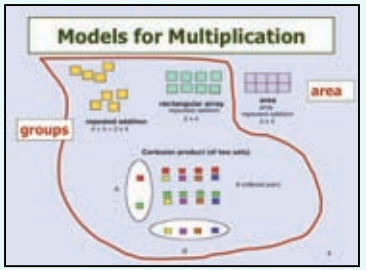
Learning the Mathematical Work of Teaching



Deborah Loewenberg Ball

University of Michigan

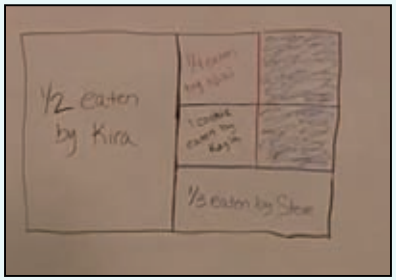
Center for Proficiency in Teaching Mathematics

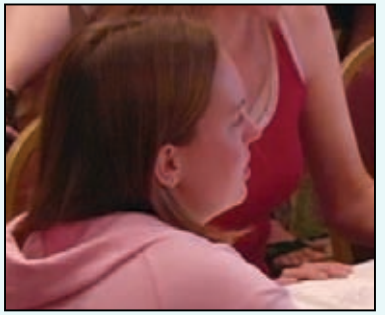


Annual Meeting of the

National Council of Supervisors of Mathematics

April 4, 2005





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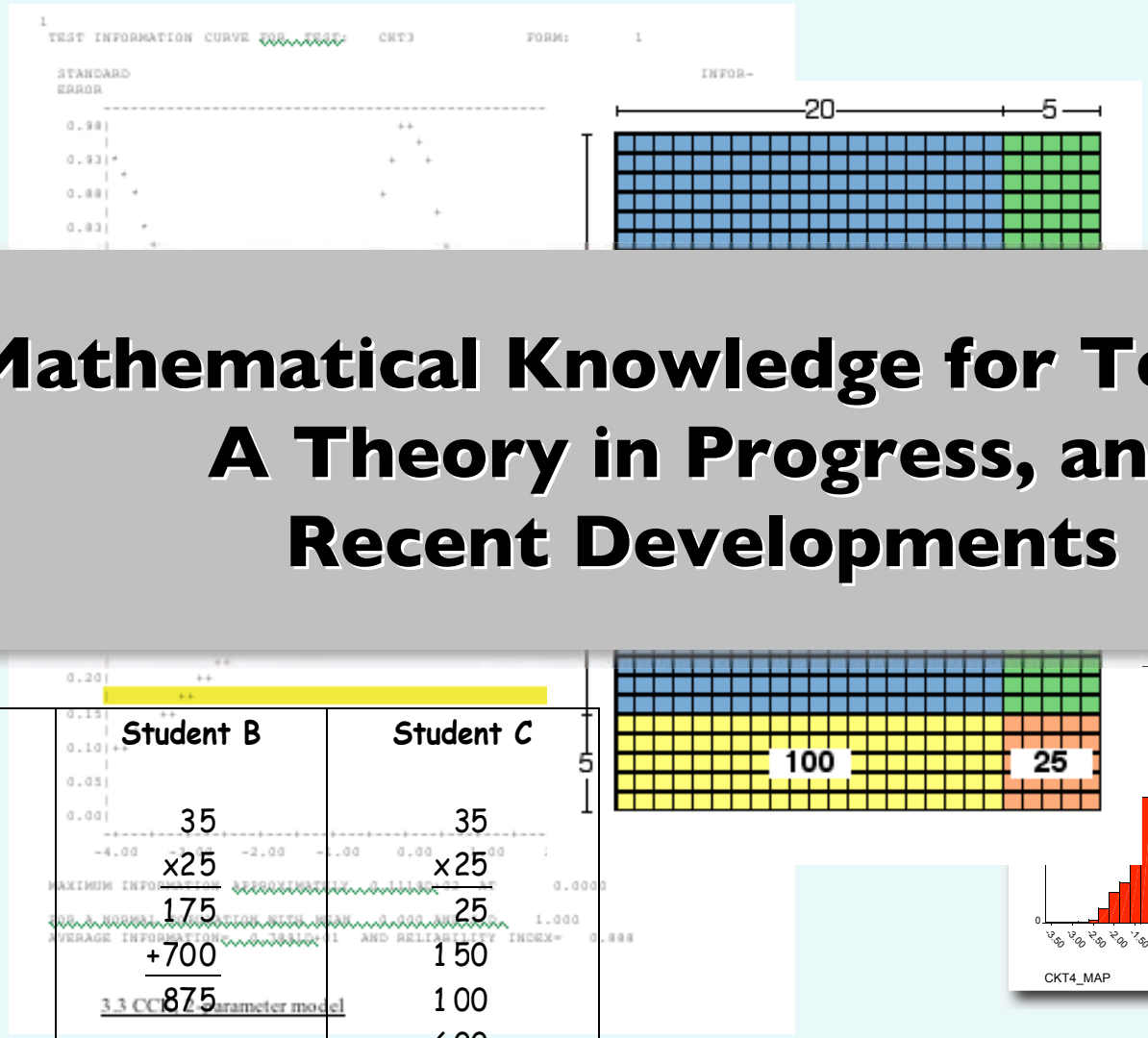
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Overview of Session

1. Mathematical knowledge for teaching: A theory in progress, and recent developments
2. Developing mathematical knowledge for teaching: Exploring the design and teaching of explicit opportunities for teachers to learn



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1. Mathematical Knowledge for Teaching: A Theory in Progress, and Recent Developments

Student A	Student B	Student C
35 x25 125 +75 875	35 x25 175 +700 875	35 x25 25 150 100 +600 875

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Clarifying the Problem

The quality of mathematics teaching and learning

Teachers' knowledge of mathematics

and their ability to use it in teaching

What mathematical resources do teachers need
in order to teach mathematics to all students effectively?

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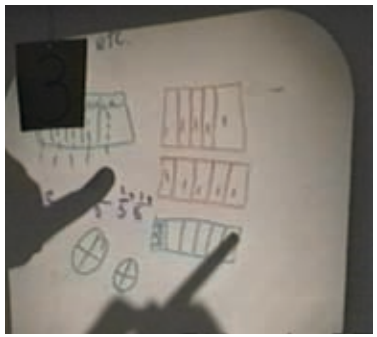
A Practice-Based Approach to Answering the Question

1. **Study instruction**, and identify the mathematical work of teaching
2. Analyze **what mathematical knowledge** is needed to do that work effectively, and **how** it must be understood in order to be useful for the work
3. Test and refine **theory of mathematical knowledge for teaching**
4. Develop, test, and refine **approaches to helping teachers** develop and use mathematical knowledge for teaching

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What Do we Mean By "Mathematical Knowledge for Teaching"?





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Mathematical Knowledge for Teaching

What do we mean when we use this term, "**mathematical knowledge for teaching**"?

- Mathematical knowledge, skill, habits of mind that are entailed by the work of teaching

What do we mean by the "**work of teaching**"?

- The tasks in which teachers engage, and the responsibilities they have, to teach mathematics, both inside and outside of the classroom

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What *is* "Mathematical Knowledge for Teaching"? An Example from Subtraction

Subtract:

$$\begin{array}{r} 307 \\ -\ 168 \\ \hline \end{array}$$

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Analyzing Incorrect Answers for

$$\begin{array}{r} 307 \\ -\ 168 \\ \hline \end{array}$$

(a)

$$\begin{array}{r} 307 \\ -\ 168 \\ \hline 261 \end{array}$$

(b)

$$\begin{array}{r} 307 \\ -\ 168 \\ \hline 169 \end{array}$$

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Analyzing Correct Answers for

$$\begin{array}{r} 307 \\ -\ 168 \\ \hline \end{array}$$

(a)

$$\begin{array}{r} 307 \\ -\ 168 \\ \hline -1 \\ -60 \\ 200 \\ \hline 139 \end{array}$$

(b)

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 307 \\ -\ 168 \\ \hline 139 \end{array}$$

(c)

$$\begin{array}{r} 307 \\ -\ 168 \\ \hline 2 \\ 30 \\ -107 \\ \hline 139 \end{array}$$

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Other Work of Teaching

- Choosing and using representations
- Choosing numerical examples
- Using and defining terms
- Using notation
- Explaining



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Developing and Using Valid Measures of Mathematical Knowledge of Teaching

- Interdisciplinary item development
- Piloting and analysis
  - Discriminating ***common content knowledge*** from ***specialized content knowledge***
- Content validity checks
- Cognitive tracing interviews with teachers, laypeople, mathematicians
  - Solving mathematical problems involving specialized content knowledge not always obvious or easy for mathematicians
- Examining videotapes of classroom teaching
- Using teachers' scores to predict student achievement

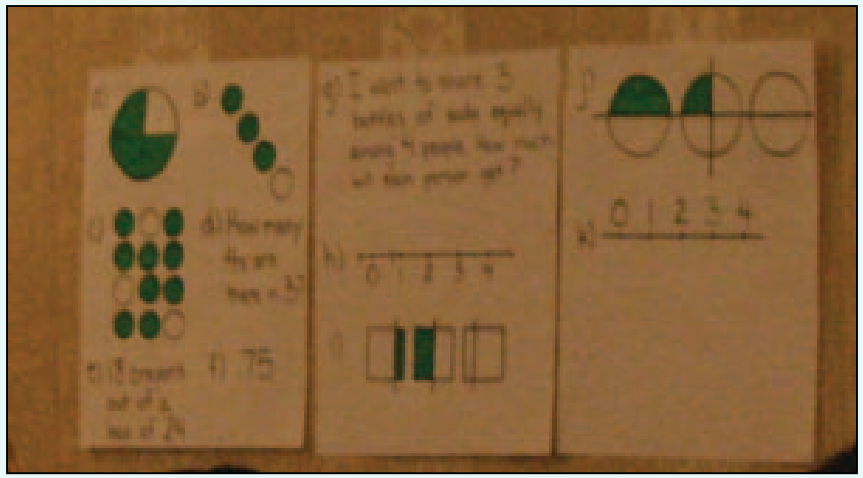
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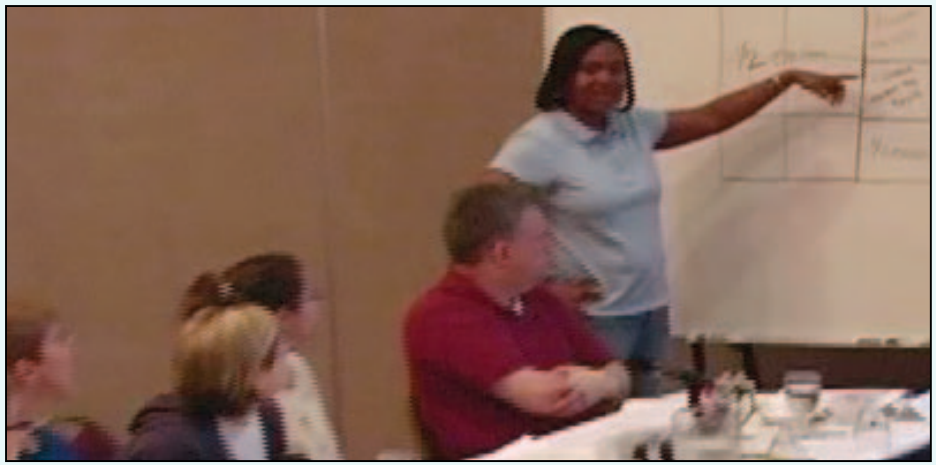
Linking Teacher Knowledge and Student Achievement: New Results

- Questionnaire consisting of 30 items
- Model: Student Terra Nova gains predicted by:
  - Student descriptors (family SES, absence rate)
  - Teacher characteristics (math methods/content, content knowledge)
- Teacher content knowledge significant
  - Small effect (< 1/10 standard deviation): 2 - 3 weeks of instruction
  - But student SES is also about the same size effect on achievement
    - (Hill,Rowan, and Ball, in press)

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2. Developing Mathematical Knowledge for Teaching: Exploring the Design and Teaching of Explicit Opportunities for Teachers' Learning





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Three Kinds of Learning of Mathematics

- Learning mathematics as a student
- Learning mathematics as a teacher
- Learning to teach mathematics

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Mathematical Content and Applications for the Teaching of Elementary School Mathematics

- A mathematics content course for prospective elementary school teachers
- Focused on **knowing and using mathematics for teaching**
  - ❑ Content
    - Mathematical topic: fractions
    - Mathematical practices: Using representations, using mathematical language, **making and evaluating mathematical explanations**
  - ❑ Applications
    - Using mathematics for tasks of teaching: Interpret and evaluate student work, discuss the mathematics in play in classroom lessons, analyze curriculum materials

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Making and Evaluating Mathematical Explanations

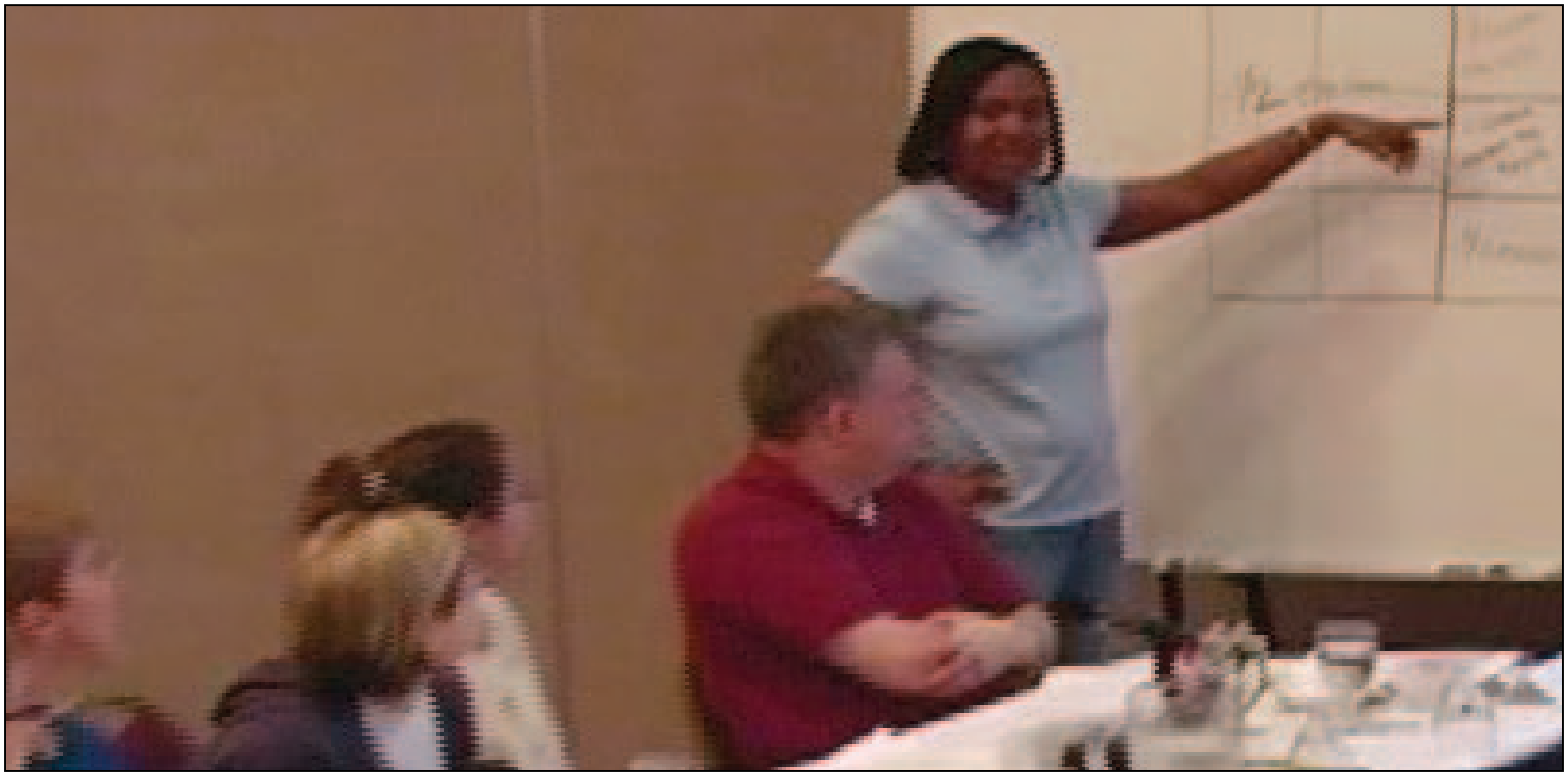
- The forms of expression that mathematically justify and help others understand why a mathematical claim is true
- Purposes and central features of mathematical explanation
- The social nature of explanation
- Providing adequate explanations
- Listening to and appraising others' explanations

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Cookie Jar Problem

There was a jar of cookies on the table. Kira was hungry because she hadn't had breakfast, so she ate half the cookies. Then Steve came along and noticed the cookies. He thought they looked good, so he ate a third of what was left in the jar. Niki came by and decided to take a fourth of the remaining cookies with her to her next class. Then Kayla came dashing up and took a cookie to munch on. When Pam looked at the cookie jar, she saw that there were two cookies left. "How many cookies were there in the jar to begin with?" she asked Kira.

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Anna asks: “How did you decide that the one cookie eaten by Kayla got its own square?”

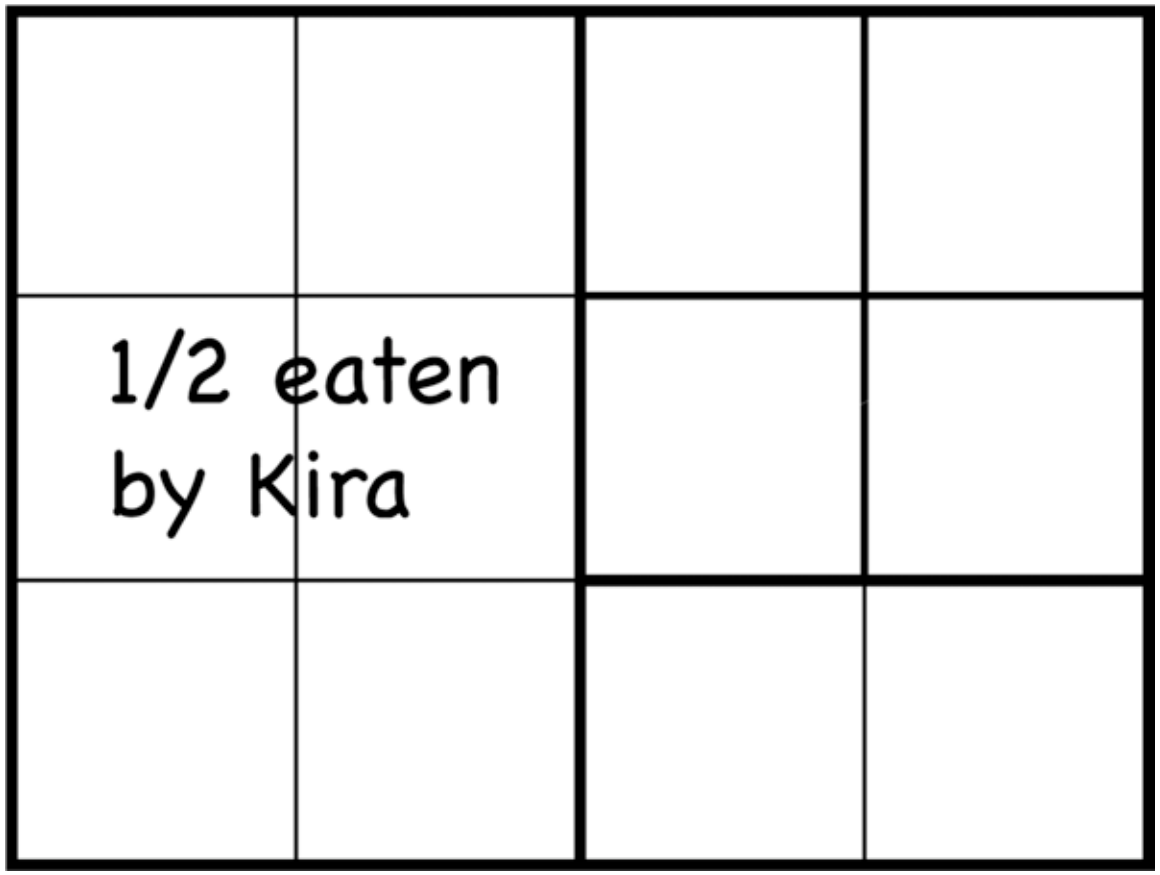
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Why <u>does</u> one square equal one cookie?



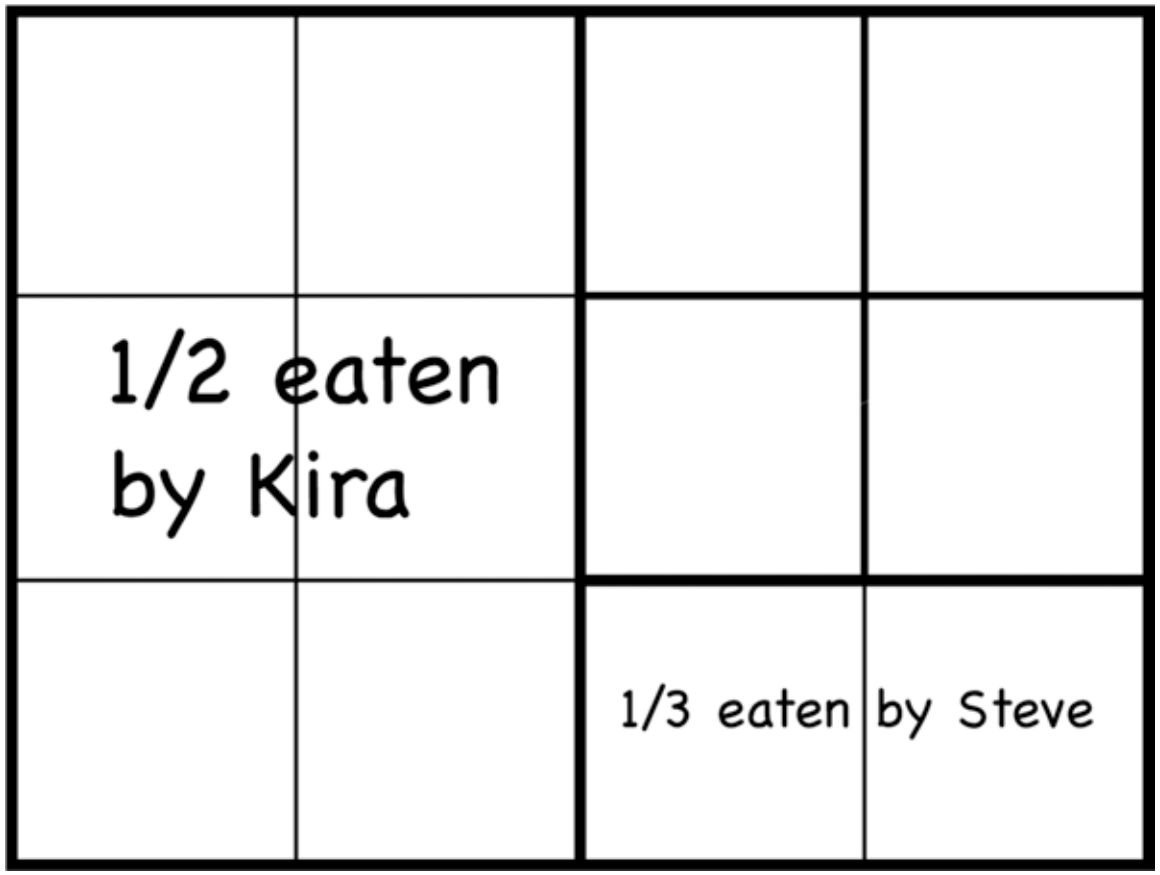
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Kira ate half the cookies



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Steve ate a third of what was left



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Niki took a fourth of the remaining cookies with her



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Since Niki took a fourth of the remaining cookies, there must be three-fourths left



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Kayla took a cookie to munch on



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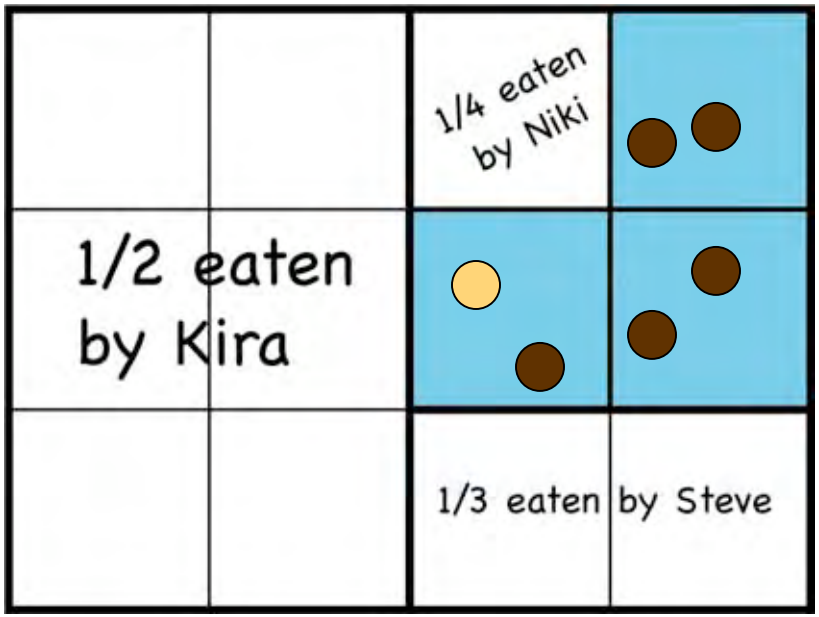
Why does one square equal one cookie?



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Consider an alternate version of the problem:

Kayla took a cookie to munch on.
Pam saw that there were five cookies left.



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Students' Explanations

1. What do you notice about what the students are doing, about what teacher is doing, and about the content?
2. Are these explanations? What do they have, or lack?
3. How does what students are working on relate to learning mathematical knowledge for teaching?

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Launching Explicit Work on Mathematical Explanation

STUDENTS' WORK IN CLASS

- Are you convinced that the only solution to the Cookie Jar Problem is 12? If so, what has convinced you? If not, what is leaving you skeptical?

- Are there things you have noticed that you would call "features of good mathematical explanations"?

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Features of "Good" Mathematical Explanations for the Cookie Jar Problem

- Makes clear at the outset what is being explained, and why you start there, and carefully connects the explanation to the question or idea being explained
- Starts from the beginning, and traces the logical flow of the reasoning
- Should be logical and complete, makes conclusion clear and links back to original question or claim or problem
- Might number the steps if appropriate, or label parts of a diagram
- Strives to be as simple and clear as possible
- Defines terms as needed, uses available definitions as needed
- Uses representation(s) accurately (algebraic, geometric, etc.), and combining representations
- Links the language and diagrams clearly to the steps of the argument
- Shows what something means or why is true, and is convincing to the person to whom you are explaining
- Is calibrated to the context (considers the person to whom you are explaining, and what is already established as true and does not need more explanation)

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Exam: Question #1

- DIRECTIONS: This question will be completed in pairs. You and your partner will take turns being the presenter and the evaluator. After the presenter completes the explanation, the evaluator will appraise the explanation using the criteria we developed in class, and will provide feedback. Then you will switch roles.

- You are encouraged to prepare your explanations in advance, however you may not use any notes for this question. Although you will only explain one problem, you should do the other as well, so that you are prepared to listen closely to your partner.

- Paper and pens will be available for you to use during your explanation, as will copies of the problems and the list of criteria for good mathematical explanation. You should also refer to our list of criteria for good mathematical explanations, but do not use it rigidly (i.e., you need not comment on every item).

- For this question, you will be assessed on both your ability to provide and to evaluate a mathematical explanation. When you are listening to your partner's explanation, your role is to appraise the quality of the explanation. You should not be coaching your partner, and should listen neutrally so you do not distract or otherwise intervene in your partner's reasoning.

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Problems for Explanation Performance Task on Exam

- Amy has 5 yards of beautiful imported embroidered ribbon. For a special project, she decides to cut it equally into 8 strips.
- What fraction of a yard will each strip be?
- What fraction of all of the ribbon will each strip be?

- Chris bought 7 boxes of Girl Scout chocolate mint cookies. He loves cookies. In order to exercise self-control, he decides to eat an equal amount each day, and he wants his cookie supply to last for 20 days.
- What fraction of a box of cookies does he have for lunch each day?
- What fraction of his entire cookie supply does he eat each day?

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Conclusion: Questions and Challenges

- Tackling issues of coverage
  - Choosing and justifying mathematical content (topics and practices) for **in-depth work**
  - Choosing and justifying content for **brief treatment**
  - Choosing and justifying content **to omit**
- Understanding and developing mathematical fluency for teaching
- Developing tasks and pedagogy appropriate for MKT: Do we really think that our courses/workshops should be taught as teachers would teach their own students?
- Developing performance assessments: knowing and using mathematics (e.g., explaining, representing)
- Rethinking the intersection of content and methods
- The professional development of professional developers

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Slides will be available at

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