For copies of these slides, and further information, please go to:
http://www-personal.umich.edu/~dball

Learning Mathematics for Teaching

Deborah Loewenberg Ball
University of Michigan

National Council of Supervisors of Mathematics Annual Meeting 2003
San Antonio, TX

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Overview

1. Clarifying the issue of teachers' mathematical knowledge
2. Seeing teaching as mathematical work
3. Developing a practice-based approach to learning mathematics for teaching

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

1. Clarifying the Issue of Teachers' Mathematical Knowledge

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Shifting the Focus — From Knowing to Knowing and Using

- Many criticisms of teachers for not knowing mathematics
- Evidence: teachers' scores, research on teacher knowledge, anecdotes
- Remedy: increase mathematics requirements for teachers, add mathematics content to professional development

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

A Closer Look:
Knowing Multiplication

$$\begin{array}{r} 35 \\ \times\ 25 \\ \hline \end{array}$$

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Which of these students is using a method that could be used to multiply any two whole numbers?[1]

Student A	Student B	Student C
35 x25 125 +75 875	35 x25 175 +700 875	35 x25 25 150 100 +600 875

[1]Measures copyright 2001, Study of Instructional Improvement (SII)/Consortium for Policy Research in Education (CPRE). Not for reproduction or use without written consent of SII. Measures development supported by NSF grant REC-9979873, and by a subcontract to CPRE on Department of Education (DOE), Office of Educational Research and Improvement (OERI) award #R308A960003.

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

What is the actual issue that we are trying to address?

The quality of mathematics teaching and learning

Teachers' knowledge of mathematics and their ability to use it in their teaching

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

What is Necessary Mathematics Knowledge for Teaching?

1. Examine teaching, and identify the mathematical work that teaching entails
2. Analyze what mathematical knowledge — topics and skills — is needed to do that work
3. Analyze also the qualities of that knowledge — how it must be understood and known to be serviceable for the work

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

2. Seeing Teaching as Mathematical Work

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

What Mathematical Problems Arise in Teaching?

Examples

- Analyzing errors
- Giving and evaluating explanations
- Appraising unexpected claims, solutions, and methods
- Choosing and using representations
- Investigating correspondences among representations and solutions
- Choosing and using definitions
- Interpreting and responding to students' ideas

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

What Mathematical Problems Arise in Teaching?

Examples

- **Analyzing errors**
- Giving and evaluating explanations
- **Appraising unexpected claims, solutions, and methods**
- Choosing and using representations
- Investigating correspondences among representations and solutions
- **Choosing and using definitions**
- Interpreting and responding to students' ideas

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

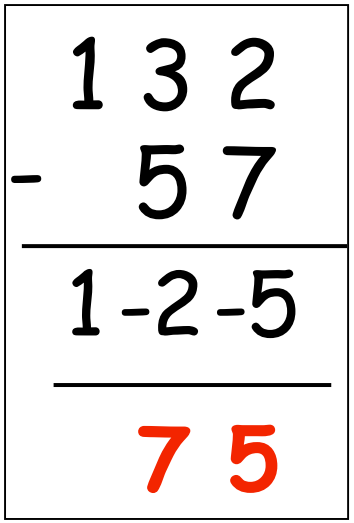
Analyzing Errors

$$\begin{array}{r} 132 \\ -\ 57 \\ \hline 125 \end{array}$$

What method is producing this answer?

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Analyzing Mathematical Procedures



Will this method work to subtract any two whole numbers?

What are the mathematical advantages and pitfalls of this method?

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Defining Concepts and Terms

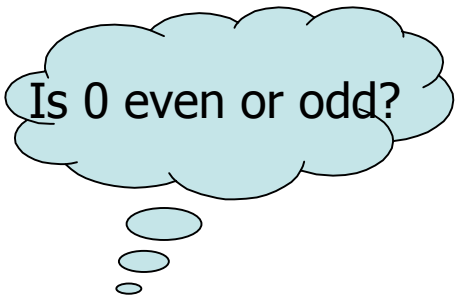


© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Examining textbook definitions

How well does any of these help to answer the question?

1. An even number is a number of the form 2k, where k is an integer.
2. An even number is a natural number that is divisible by 2.
3. An even number is any multiple of 2.
4. An even number is a number that has 0, 2, 4, 6, or 8 in the ones place.

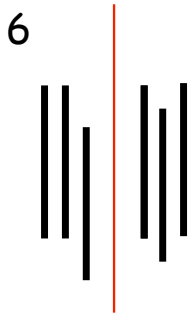


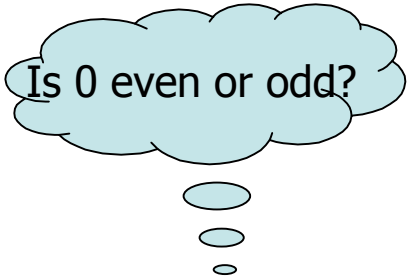
© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Considering students' definitions of even number

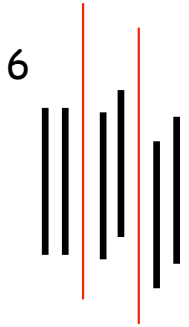
How well does any of these help to answer the question?

1. An even number is a number you can split in two equal parts without having to break anything in half.

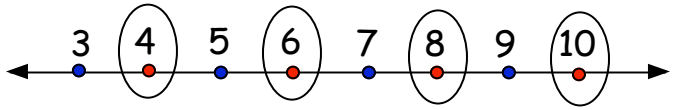




2. An even number is a number that when you group it in twos, there are none left.



3. The even numbers are every other number on the number line, like 2, 4, 6, and so on.

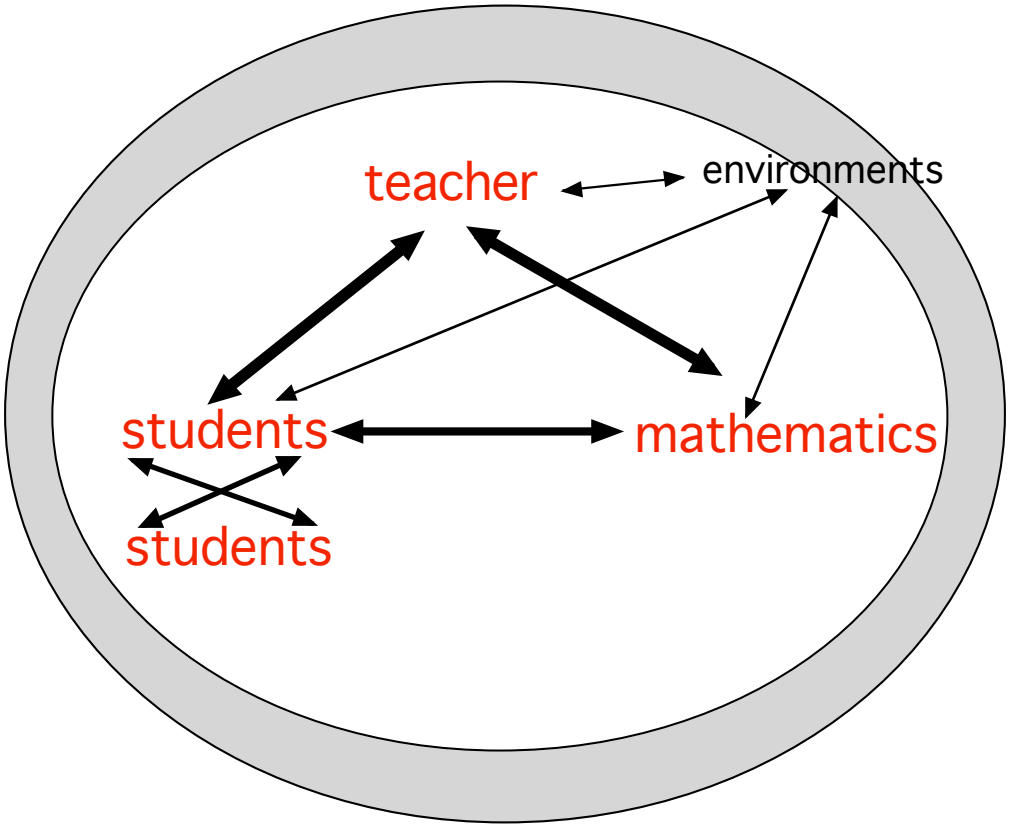


© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

What is a mathematically appropriate and usable definition of "even number" for third graders?

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Knowing and using mathematics in teaching



© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

3. Developing a Practice-Based Approach to Learning Mathematics for Teaching

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

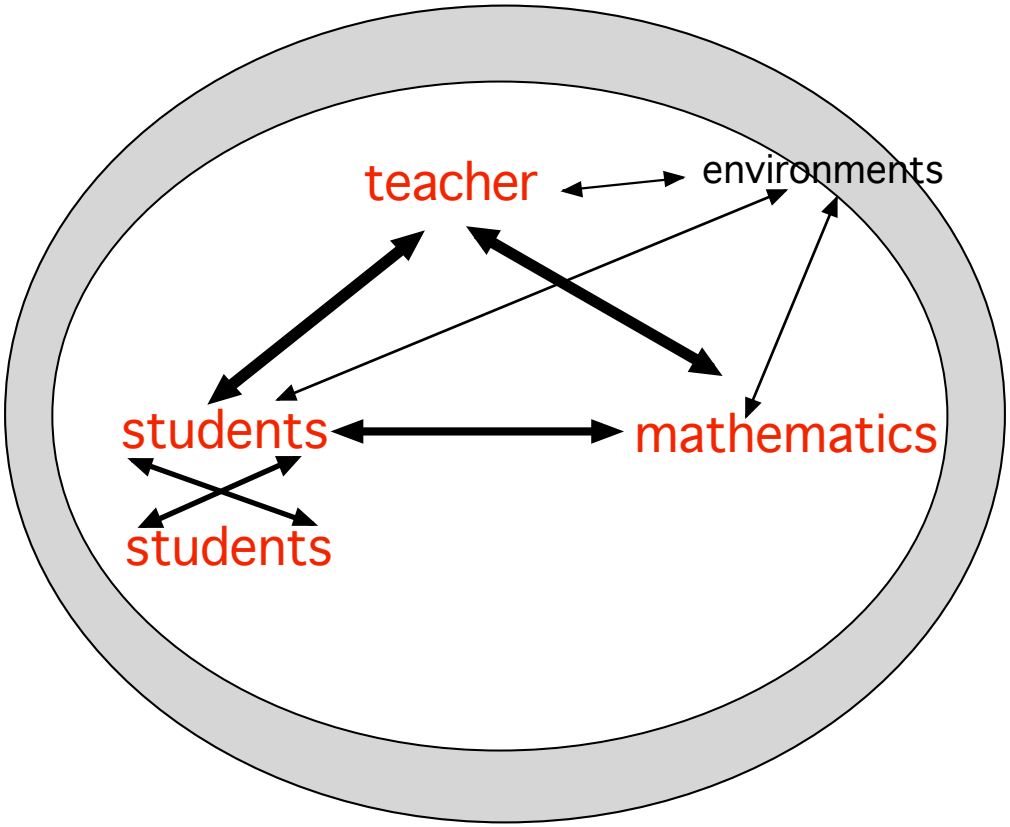
What Mathematical Problems Arise in Teaching?

Examples

- Analyzing errors
- **Giving and evaluating explanations**
- Appraising unexpected claims, solutions, and methods
- **Choosing and using representations**
- **Investigating correspondences among representations and solutions**
- Choosing and using definitions
- **Interpreting and responding to students' ideas**

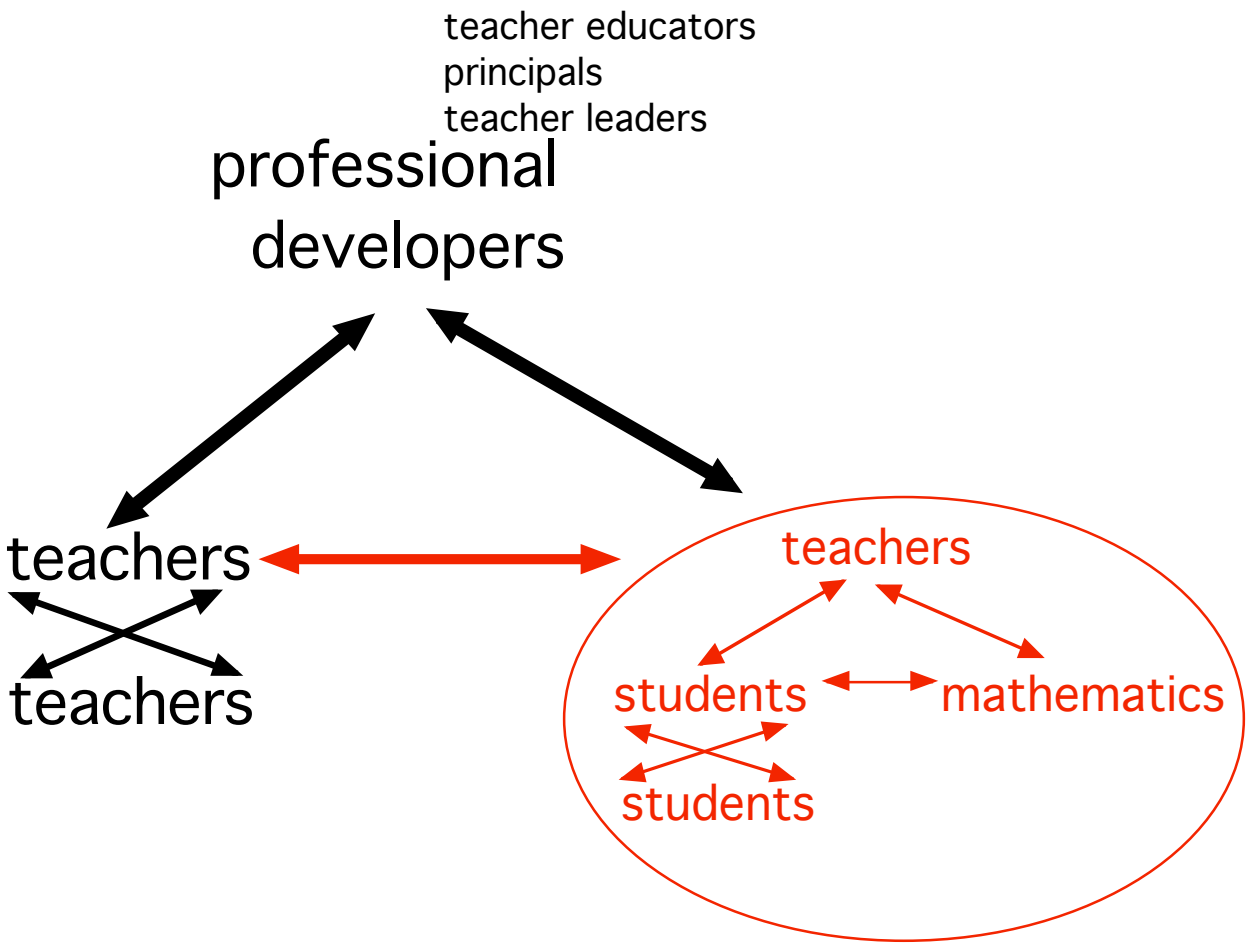
© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Knowing and using mathematics in teaching



© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Learning in and from practice



© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Example:

Using Records of Practice to Study Mathematics for Teaching

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Video Clip from 3rd Grade Lesson on Fractions

- Early May (ten days into fractions unit)
- Diverse classroom, many English language learners
- Classroom in which norms of taking mathematics seriously, attending to other students' ideas with care, and working together on mathematics have been a goal all year

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Mathematics problems

Yesterday:



How many crayons in --

$\frac{1}{2}$ box $\quad \frac{2}{3}$ box $\quad 1\frac{1}{2}$ boxes

$\frac{1}{4}$ box $\quad \frac{3}{4}$ box

Today:

How many in three-quarters of a dozen?

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Designing Opportunities to Learn Mathematics From the Study of a Video Clip

What mathematical issues do you notice in this video clip?

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

A video clip from a lesson on May 10, 1990 was shown here. See transcript.

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Designing Opportunities to Learn Mathematics From the Study of a Video Clip

Task #1

Shea uses a graham cracker to explain the meaning of three fourths to Rania. He also considered a dollar.

Analyze and compare each as a mathematical representation of the meaning of 3/4.

Task #2

Why do some of the students want to vote? Why do some object to voting?

What are the mathematical issues here?

Task #3

The teacher uses 1/2 of 6 to help Rania reason about the meaning of three fourths.

Analyze the mathematical considerations in this move. Is there another move that would be preferable mathematically?

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Designing Opportunities to Learn Mathematics From the Study of a Video Clip

Task #4

Yesterday the class successfully reasoned about fractions of 24. What mathematical reasons are there to work on three quarters of 12 in today's lesson?

What would be another question, and what does it offer mathematically?

Task #5

What problem or question would be useful to pose to the class next?

Task #6

What can you say about each of the following students' understanding of some mathematical issue discussed in this clip?

Kevin

Shea

Shekira

David

Lin

Rania

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Solving Problems in Learning Mathematics for Teaching

#1: Shea uses a graham cracker to explain the meaning of three fourths to Rania. He also considered a dollar. Analyze and compare each as a mathematical representation of the meaning of 3/4.	
#2: Why do some of the students want to vote? Why do some object to voting? What are the mathematical issues here?	
#3: The teacher uses 1/2 of 6 to help Rania reason about the meaning of three fourths. Analyze the mathematical considerations in this move. Is there another move that would be preferable mathematically?	
#4: Yesterday the class successfully reasoned about fractions of 24. What mathematical reasons are there to work on three quarters of 12 in today's lesson? What would be another question, and what does it offer mathematically?	
#5: What problem or question would be useful to pose to the class next?	
#6: What can you say about each of the following students' understanding of some mathematical issue discussed in this clip? (Kevin, Shea, Shekira, David, Lin, Rania)	

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Solving Problems in Learning Mathematics for Teaching

#1: Shea uses a graham cracker to explain the meaning of three fourths to Rania. He also considered a dollar. Analyze and compare each as a mathematical representation of the meaning of 3/4.	Comparing representations, investigating correspondences among representations
#2: Why do some of the students want to vote? Why do some object to voting? What are the mathematical issues here?	Interpreting student thinking; concepts of mathematical reasoning
#3: The teacher uses 1/2 of 6 to help Rania reason about the meaning of three fourths. Analyze the mathematical considerations in this move. Is there another move that would be preferable mathematically?	Posing a question to scaffold student thinking, choosing specific numerical examples
#4: Yesterday the class successfully reasoned about fractions of 24. What mathematical reasons are there to work on three quarters of 12 in today's lesson? What would be another question, and what does it offer mathematically?	Posing task to assess and develop students' thinking, choosing specific numerical examples
#5: What problem or question would be useful to pose to the class next?	Designing sequential tasks, choosing specific numerical examples
#6: What can you say about each of the following students' understanding of some mathematical issue discussed in this clip? (Kevin, Shea, Shekira, David, Lin, Rania)	Interpreting students' mathematical thinking and understanding; identifying and naming particular mathematics

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Conclusions

1. Knowledge needed for teaching is different from what is needed for other occupations or professions where mathematics is used (e.g., physics, mathematics, carpentry, tailoring, business).

2. Knowledge needed for teaching must be usable for the mathematical work that teachers do.

3. Mathematics teaching involves specialized mathematical problem solving.

4. Learning mathematics for teaching can be grounded in practice by designing opportunities for teachers to solve mathematical problems that arise recurrently in teaching.

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

For copies of these slides, and further information, please go to:
http://www-personal.umich.edu/~dball

© 2003 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission