Enabling Mathematics Achievement by All Students: The Work of Teaching

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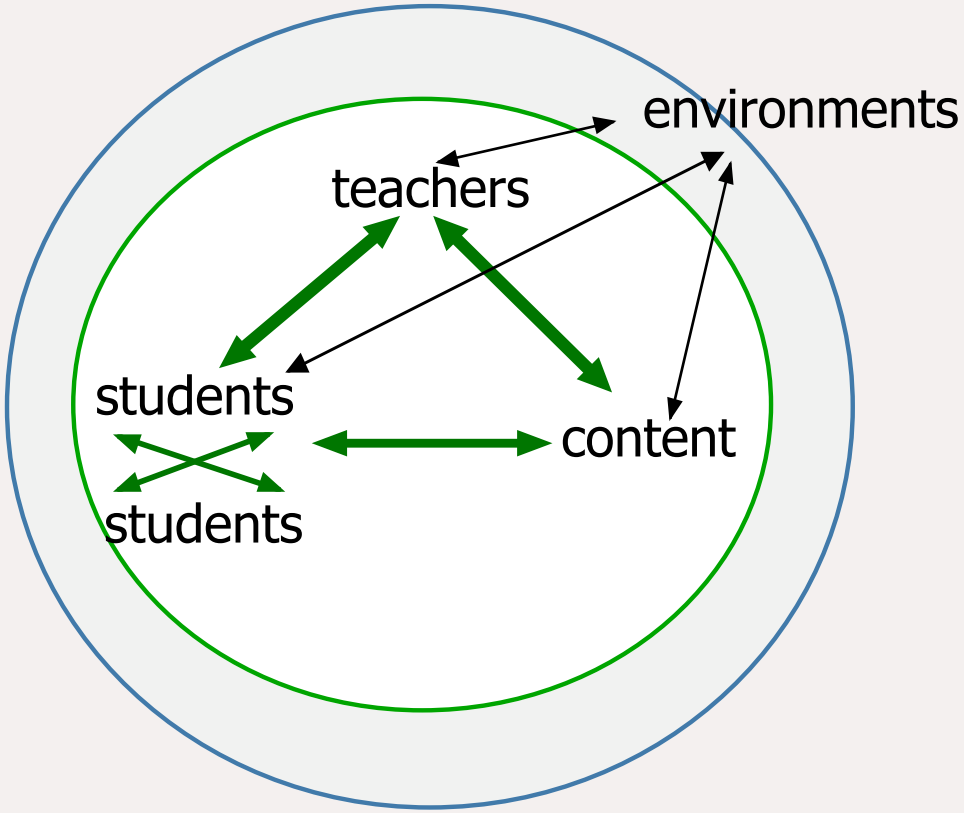
Research Presession National Council of Teachers of Mathematics

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Teaching mathematics as unnatural and intricate work



- Coordinating, over time, and with groups of students to accomplish specific goals
- Using resources
- Managing and using environments

Cohen, Raudenbush, & Ball (2003); Lampert (2001); Lee (2007).

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Four key hypotheses about teaching all students

1. Expecting and enabling complex mathematical work of each student
2. Coordinating mathematical, school, and students' language
3. Using contexts with sensitivity to students' funds of knowledge and to the integrity of the mathematics
4. Creating a respectful mathematically-focused learning environment

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The Elementary Mathematics Laboratory

- A live laboratory for the design and study of teaching
  - Direct program for children in local school district
  - Live setting for the study of teaching, learning, and mathematics by teachers, researchers, teacher education students, mathematicians, university faculty, visitors from the public
  - Source of unusual and valuable records of practice

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The 2007 laboratory class

- 27 students, entering fifth grade, mostly not very successful in school math; from a working class community, under-resourced district, falling achievement
- 2½ hours of instruction per day
- Mathematical content: Fractions (definitions, representations), proving an impossible combinatorics problem
- Mathematical skills: explaining, representing, proving

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A glimpse of the classroom



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The observers in the laboratory classroom



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Our focus today

1. Expecting complex mathematical work of every student
2. Coordinating mathematical, school, and students' language
3. Using contexts with sensitivity to students' funds of knowledge and to the integrity of the mathematics
4. Creating a respectful mathematically-focused learning environment

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1. Expecting complex mathematical work of every student

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Enacting high expectations in mathematics instruction

- Broaden what it means to be successful in math class
- Selection of tasks and problems
- Make mathematical practices explicit
- Support students' mathematical work both publicly and privately
  - Listen carefully to students' talk
  - Notice and improve ambiguous talk
  - Ask mathematical questions

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Expecting and enabling complex mathematical work by each student

- What does it mean, in practice, to "hold high expectations"?
- Creating a learning community:
  - Establishing group expectations
  - Establishing individual expectations
- EML 2007 Contract

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EML 2007 student contract

Student Contract

I understand that my ability to succeed in the Elementary Mathematics Laboratory class depends on my commitments and effort this summer. The following are the commitments asked of me to participate in the special class:

1. I will attend class every day.
2. I will keep neat detailed records of my mathematical thinking in my math notebook.
3. I will complete my homework carefully each night.
4. I will seek help or advice from my teacher or parents/caregivers if I have questions or concerns.
5. I will give my best effort to all the work I complete for this class.

I have read, understand and will follow through with the statements in this contract to the best of my ability.

Print student's full name: _____

Student signature:_____ Date:_____

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Teacher Contract with

Student's name

I understand that your ability to succeed in the Elementary Mathematics Laboratory class depends on my commitments and effort this summer. The following are the commitments I make to you as your teacher in this special class:

1. I will attend our class every day.
2. I will come to class prepared to teach interesting and appropriate material to you.
3. I will expect you to do your very best work all the time and I will help you learn to do so.
4. I will make sure that you understand the mathematics and problems we work on.
5. I will answer any questions or concerns that you may have about this class or our work.
6. I will keep careful records of my mathematical thinking and ideas.

I have read, understand and will follow through with the statements in this contract to the best of my ability.

Print teacher's full name: _____

Teacher's signature:_____ Date:_____

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Cautions about "high expectations"

- Helping students to do hard work
- Scaffolding complex work appropriately
- Making wise judgments about what to leave open and what needs to be made explicit
- Commenting on mathematics, and mathematical productions, not features of students
- Supporting error as a fruitful site for mathematical work, and teach students to use error productively

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2. Coordinating mathematical, school, and home language

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What do we mean by "language" in mathematics?

Isn't math just numbers, symbols, formulas, graphs, and calculations, mostly bereft of text?

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{4a}$$

$x^2 - y^2 = (x + y)(x - y)$

NO!

1. Mathematics uses three forms of linguistic tools: Words, symbols, and diagrams.
2. These are <u>all</u> forms of text.
3. Doing mathematics requires reading, writing, speaking, and interpreting these.

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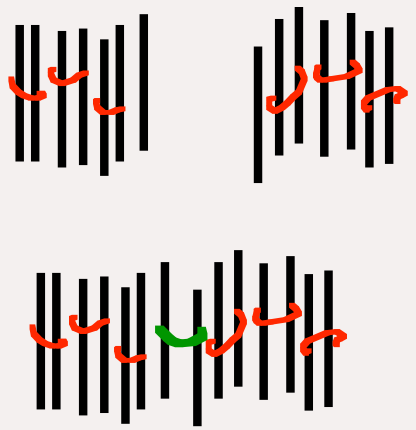
What is important about language in mathematics?

- Necessary for mathematical communication
- Foundational to mathematical reasoning

  Fractions are written as a/b where a and b are whole numbers, and b is not 0. The number a is the numerator, the number b is the denominator.
- Precision matters

  What <u>is</u> a fraction?
- Words, symbols, and diagrams must map onto one another.

$$(n2 + 1) + (j2 + 1) = (n + j + 1)2$$



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Words: More than story problems

- Mathematics uses words also used in other settings.

Examples:

  - **Big, odd, even, prime, rational, radical, square**
  - **Right, similar, circle, regular, acute, obtuse**

- The mathematical uses of these words are specialized.
- Definitions are used to specify technical uses of terms within mathematics.

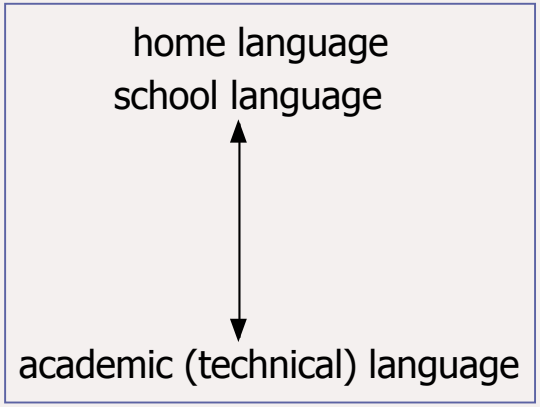
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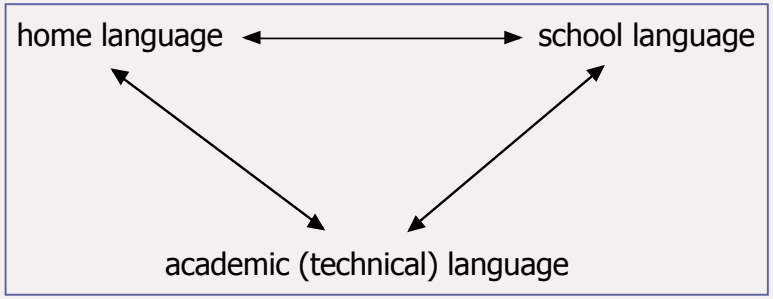
What are these "other settings"? Learning mathematics requires making linguistic transitions

Language

- Vocabulary, grammar, syntax, talk (how language is "delivered")
- Discourse

Learners must navigate among everyday school English, their home language, and technical uses in mathematics.





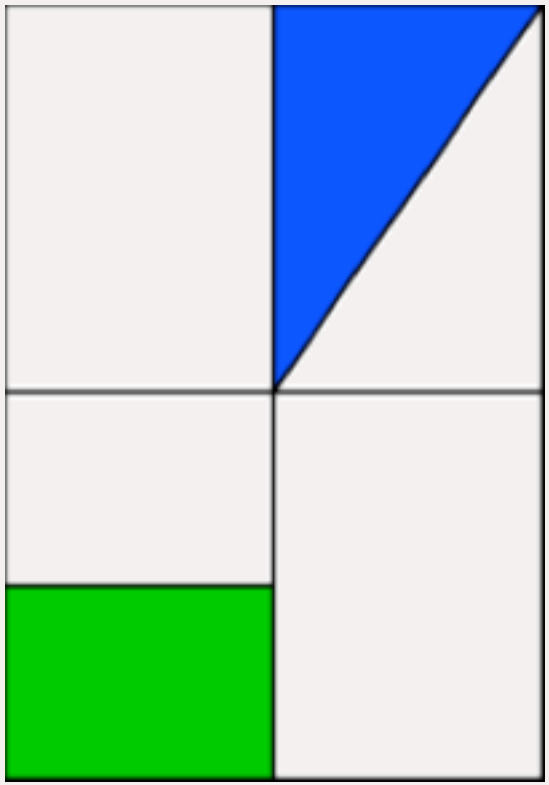
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Video clip

- Day 7, discussion of warm up problem
- Focused attention on equal parts
- Developing working ideas about fractions
  1. Identify the whole
  2. Make equal parts
  3. Count how many equal parts out of the whole

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Setting up the problem



What fraction of the big rectangle is shaded blue?

What fraction of the big rectangle is shaded green?

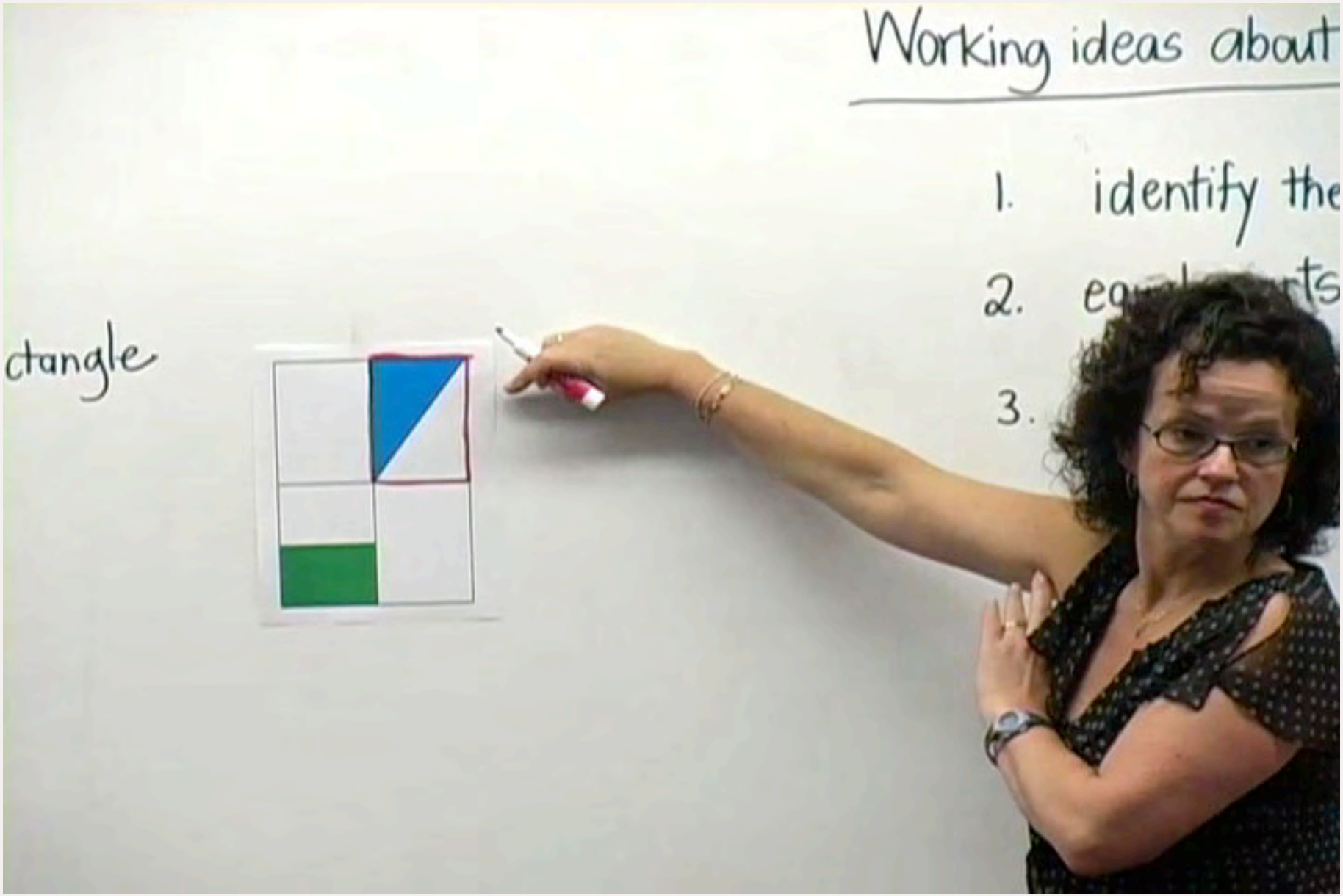
What fraction of the big rectangle is shaded altogether?

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Viewing focus

- What is the teacher doing to expect **complex mathematical work** from students? Where are missed opportunities to expect such work?
- How is the teacher managing the **language** learning opportunities? Where are missed opportunities?

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The work of teaching in this clip

EXPECTING COMPLEX MATHEMATICAL WORK

1. Selecting tasks
2. Listening to and interpreting students' responses (Mamadou)
3. Relentless press on classroom mathematical practices
4. Supporting error as a fruitful site for mathematical work

ATTENDING TO LANGUAGE

1. Attending to ambiguity of "big rectangle"—central mathematical work
2. Deciding what to clarify, what to make more precise, what to leave in student's own language

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Next steps

Connect to teacher education—

- Analyze the mathematical knowledge, skills, and dispositions needed to do this work
- Analyze other qualities, orientations, and understandings used to do this work
- Use artifacts from the laboratory class as media for teachers' learning

Design 2008 work of the laboratory class

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Discussion and cross-talk

- On what aspects of the problem do we think we have made some progress?
- What are the key challenges?
- Are there key areas of commonality among us? Key differences? What is the significance of these?

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