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Deborah Loewenberg Ball

TEACHING INSTEAD OF REQUESTING: WHAT IS INVOLVED IN MAKING COMPLEX MATHEMATICAL WORK EXPLICIT?



SCHOOL OF
EDUCATION
UNIVERSITY OF MICHIGAN

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When your students don't learn as expected, what do you find are typically the reasons?

Jackson & Gibbons, 2014

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WHAT DO TEACHERS THINK WHEN STUDENTS STRUGGLE?

UNPRODUCTIVE

1. Students aren't trying hard enough.
2. Students lack mathematical ability.

"These kids are used to be[ing] spoon-fed and they'll sit there and say, 'I don't get it.'…[U]ntil you actually sit down and show them step [by] step how to do that problem, they don't get it. They don't know how to think."

Jackson & Gibbons, 2014

PRODUCTIVE

1. There might be something in the way this was taught.
2. I could support the work better.

"I normally look first at me to see or is there something in the lesson that I didn't emphasize well enough or…I may talk to the teacher they had last year and say 'When you went over this was this something that they struggled with?'"

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A CORE CHALLENGE OF TEACHING

- Teaching complex mathematical knowledge and skill
- Teaching all students

What does this involve?

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COMMON CORE: MATHEMATICAL PRACTICES

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP4. Model with mathematics.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

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Show that you have found all the solutions.

Explain your answer.

Come to the board and share your solution.

Keep track of your work.

We have a test on Friday. Be sure you study for it.

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Have whole class discussions of mathematics.

Students should solve complex problems.

Engage students actively in mathematical discourse.

Do less of the talking and let students talk more.

Make your classroom an equitable environment.

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THE PARALLEL CORE RESPONSIBILITY OF PROFESSIONAL EDUCATORS

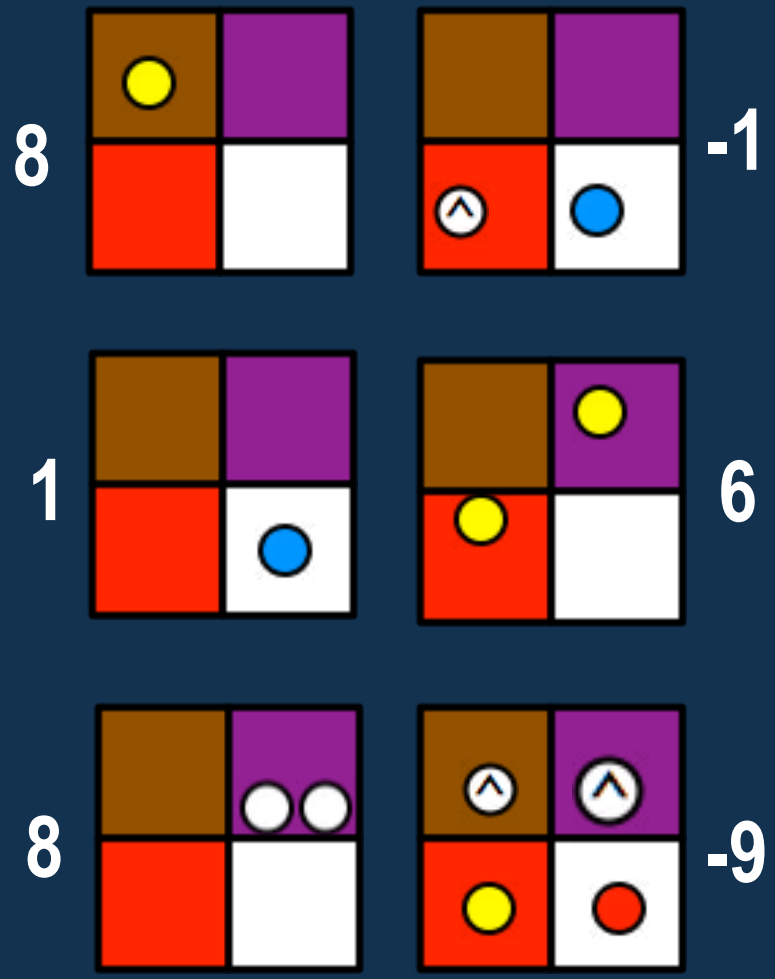
- Supporting teachers to teach complex mathematical knowledge and skill to all learners.
- Supporting all teachers.

If we do not do this, we leave teachers to struggle.

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THE MINICOMPUTER





(Papy Minicomputer)

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WHAT IS THIS PROBLEM ASKING, AND WHAT IS INVOLVED IN TRYING TO SOLVE IT?



Using exactly one positive and one negative checker, find all the numbers that can be represented on this minicomputer board.

Prove your answer and explain why.

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IDENTIFYING AND USING PROBLEM "CONDITIONS"



Using exactly one positive and one negative checker, find <u>all</u> the numbers that can be represented on this minicomputer board. Prove your answer and explain why.

CONDITIONS OF THE PROBLEM

1. Use exactly one positive and one negative checker.
2. Must be a number that can be made on the Minicomputer.

PROPOSED SOLUTIONS

3 *YES, because it can be made with a positive on the 4 and a negative on the 1. This (1) uses exactly one positive and one negative checker and (2) it can be made on the Minicomputer.*

9 *NO, because it cannot be made on the Minicomputer with exactly one positive and one negative checker on the Minicomputer.*

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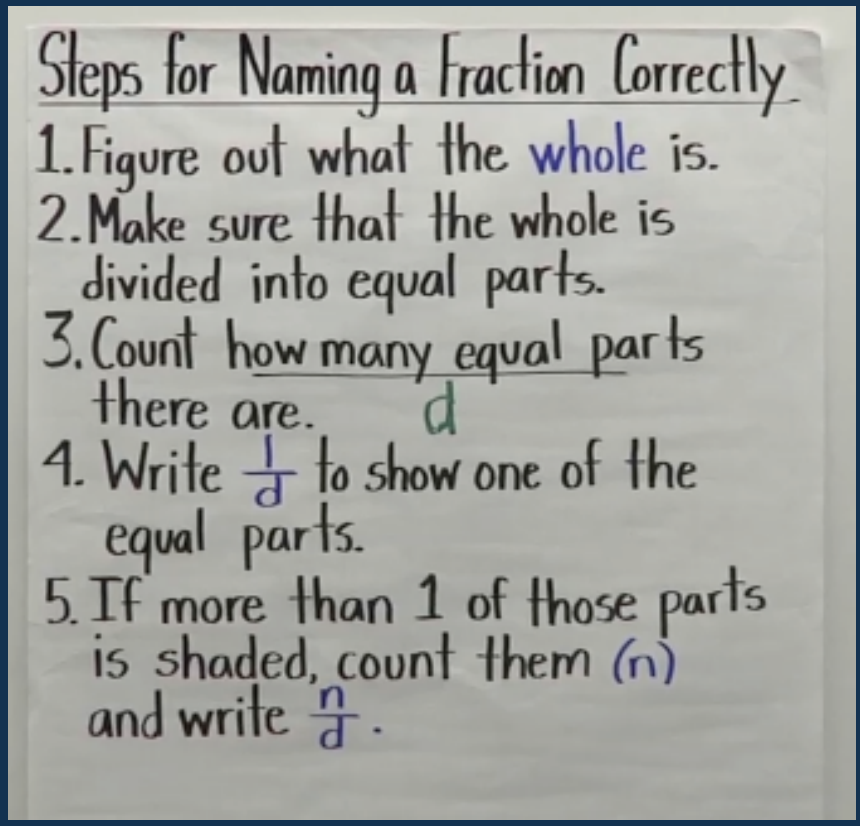
THE ROLE OF CONDITIONS OF A PROBLEM

- Identifying conditions can help in making sense of and interpreting a problem (MP.1)
- Using the conditions can help in persevering in solving a difficult problem (MP.1)
- Conditions are useful in constructing a mathematical argument (MP.3)
- Referring to the conditions is useful in critiquing an argument (MP.3)

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CLASSROOM VIDEO CONTEXT

1. 29 fifth graders; 13 girls, 16 boys; 66% African American, 16% Latino/a
2. Most have not experienced success in school mathematics or other areas
3. Work on fractions: identifying, defining, naming, comparing, representing



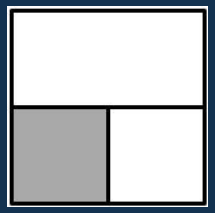
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EXPLAINING FRACTIONS

What fraction of the rectangle below is shaded gray?



What fraction of the rectangle below is shaded gray?

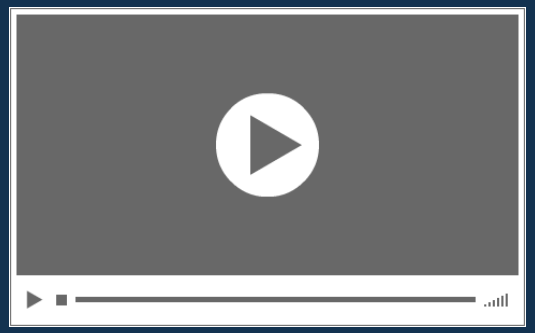


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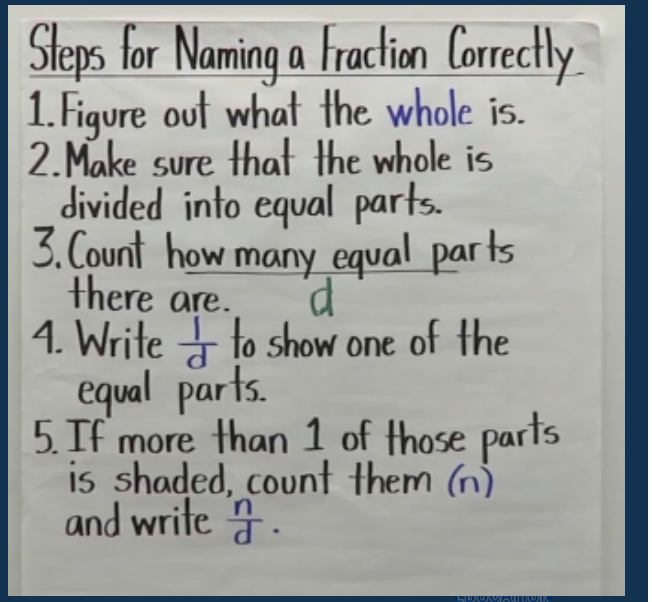
VIEWING FOCUS

1. What do you notice about the student's explanation?
   - The mathematical elements
   - The language and talk
   - The performance
2. What makes this a mathematical explanation?
3. What seems to be supporting the student's performance?

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ARIE'S EXPLANATON: AFTER SEVERAL DAYS OF CLASS



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VIEWING FOCUS

1. What do you notice about Arie's explanation?
   - The mathematical elements
   - The language and talk
   - The performance
2. What makes this a mathematical explanation?
3. What seems to be supporting Arie's performance?

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EXPLICIT TEACHING OF MATHEMATICS

- Naming, labeling, writing about important aspects of mathematical ideas, concepts, and procedures
- Naming, highlighting, scaffolding specific mathematical practices
- Naming and supporting qualities of productive mathematical habits and mindset

(Mann, Owens, & Ball, 2013)

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EXPLICIT TEACHING OF TEACHING PRACTICE

- Naming, labeling, writing about important considerations, knowledge, and practices
- Naming, highlighting, scaffolding specific instructional skills and practices, ways of reasoning and thinking
- Naming and supporting qualities of productive pedagogical habits and mindset

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EXPLICIT TEACHING ≠ DIRECT INSTRUCTION: INVESTIGATING SIMILARITIES AND DIFFERENCES

DIRECT INSTRUCTION

1. Breaks down practice or knowledge into small constituent parts
   - Presentation of information, rules, and examples
   - Tasks only address information from the presentation and the task is uncomplicated
   - Lessons are composed of 4 – 10 exercises (presentations with task series) with only 10% new material
2. Teacher's role to demonstrate, students follow, shift to independent practice
3. Seeks to reduce complexity

Engelmann & Colvin (2006)

EXPLICIT TEACHING

1. Unpacks practice or knowledge to make it open to learners
2. Teacher's role to make elements visible, provide language, supports
3. Seeks to make complex practice accessible

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MAKING COMPLEX PRACTICE LEARNABLE BY ALL BY MAKING IT EXPLICIT

① Helping students learn to explain mathematics

② Helping teachers develop practices of explicit teaching

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HELPING STUDENTS LEARN TO EXPLAIN MATHEMATICS

1. Unpacking and making the content visible
2. Providing language and tools
3. Supporting explanatory talk and writing

(Consider Arie's explanation after several days of class.)

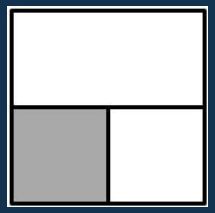
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EXPLAINING FRACTIONS

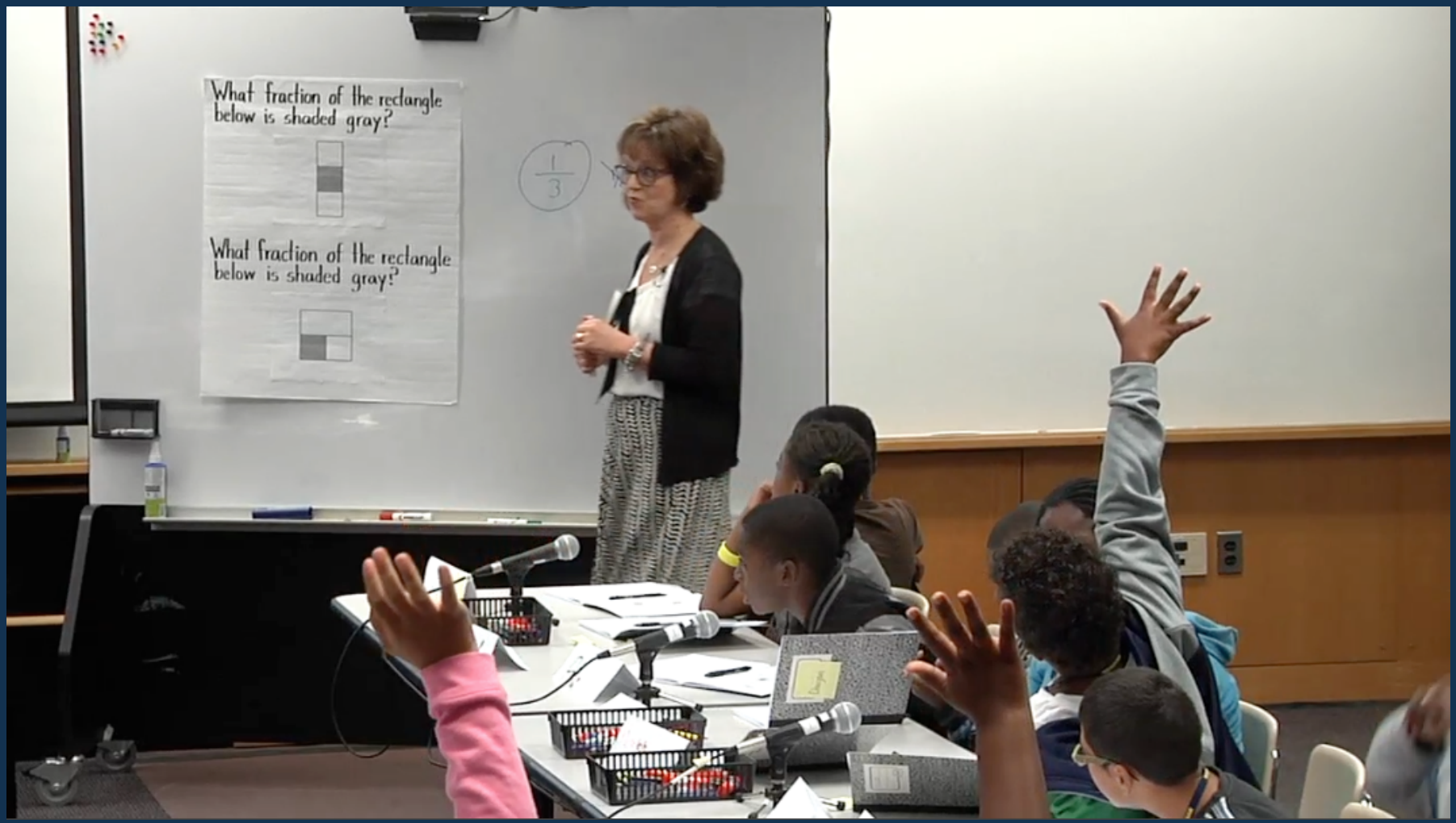
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DISCUSS

What did you notice about the teacher moves to:

1. Unpack and make the content visible
2. Provide language and tools for explaining
3. Support explanatory talk and writing

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1. UNPACKING AND MAKING CONTENT VISIBLE

- Using a task that highlights key concept of **equal** parts
  - The fractions chosen and the particular diagram (provokes key stumble)
  - Focus on the idea, not the complexity of the numbers
  - Uses student familiarity with concept image of fractions
- Displaying problem on large poster to coordinate student talk with the idea of equal parts

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2. PROVIDING LANGUAGE AND TOOLS FOR EXPLAINING

- Supplying language to support shift from everyday informal language to mathematical language (equal)
- Providing the "removable line"

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3. SUPPORTING EXPLANATORY TALK AND WRITING

- Displaying large poster with problem, matches problem students have in their notebooks
- Orienting students to one another
- Valuing the following of others' arguments
- Naming specific talk moves (e.g., talk to class, follow, repeat, agree, disagree, comment)

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MAKING THE TEACHING OF COMPLEX MATHEMATICAL WORK EXPLICIT FOR TEACHERS' LEARNING

- Unpacking ideas (e.g., conditions of a problem, elements of producing a mathematical explanation)
- Practicing explaining mathematics and labeling what is involved in doing and supporting it
- Studying enactments of practice and decomposing and labeling moves, ideas, visible and less visible aspects of the work
- Practice with students, video record, analyze

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RESOURCES FOR EXPLICIT TEACHING

- Mathematical language, professional language: focuses and provides control and focus
- Opportunities to analyze and to do
- Collective work: extends opportunities to learn, discussion highlights, challenges, extends

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FOLLOW UP

- Teaching and Learning Exploratory: tle.soe.umich.edu

- Elementary Mathematics Laboratory (July 28 – August 8, 2014): teachingworks.org/training/2014EML

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