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(HOW) CAN EXPLICITNESS ABOUT MATHEMATICAL PRACTICES SUPPORT EQUITABLE INSTRUCTION?



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THE CORE CHALLENGE OF TEACHING

- Teaching complex mathematical knowledge and skill *(i.e., not just procedural technique)*
- Making complex mathematical knowledge and skill accessible to all students, and enabling all students to be successful with complex mathematics *(i.e., not just some students)*

What does this involve?

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UNPACKING—AND MEETING—THE INSTRUCTIONAL CHALLENGE

1. Why is explicitness crucial for strong mathematics learning and teaching?
2. A close look at one example: Identifying the conditions of a problem
3. How are explicitness and equitable practice related?
4. Examples of explicitness in teaching mathematical practices
5. Summary: What do we mean by "explicitness" in teaching and what does this imply for teachers' professional development?

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COMMON CORE: MATHEMATICAL PRACTICES

MP1. Make sense of problems and persevere in solving them.

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

MP4. Model with mathematics.

MP5. Use appropriate tools strategically.

MP6. Attend to precision.

MP7. Look for and make use of structure.

MP8. Look for and express regularity in repeated reasoning.

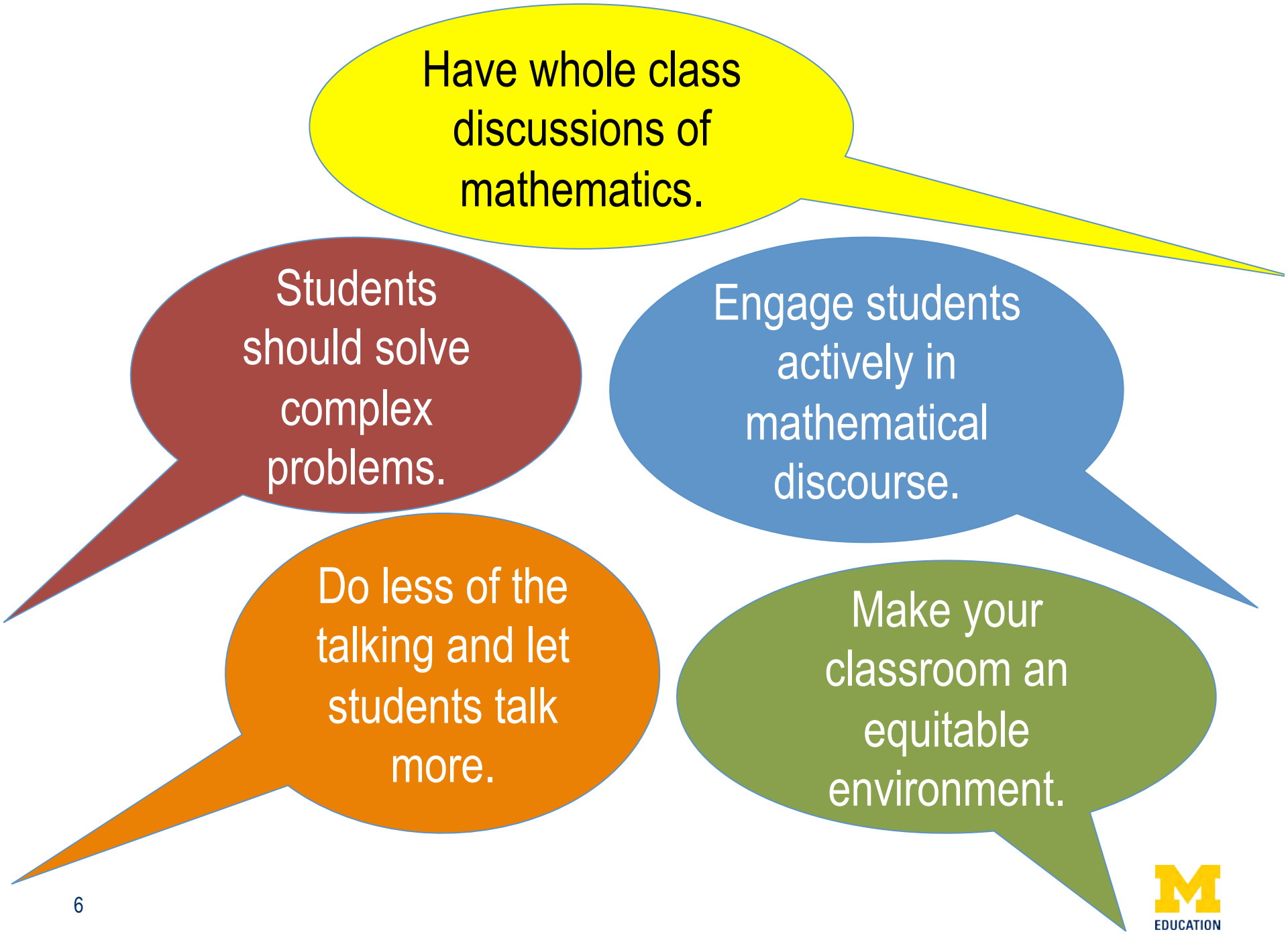
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OUR PREMISE: REQUESTING ≠ TEACHING

- For students, simply being **requested**—asked or assigned—to solve rich challenging problems is **not teaching** them to do complex mathematical work.
- For teachers, simply being **requested**—through standards, assessments, or curricula—to teach complex mathematical ideas to all their students is **not supporting** them to learn to do such instruction.

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FOR STUDENTS AND FOR TEACHERS: A NEED FOR EXPLICITNESS

- Making complex mathematical work accessible to all students requires explicitness in instruction.
- Making teaching that can explicitly support students to do complex mathematical work doable by all teachers requires explicitness in professional development.

What might "explicitness in instruction" mean and involve, and why is it a matter of equity?

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WHAT IS THIS PROBLEM ASKING, AND WHAT IS INVOLVED IN TRYING TO SOLVE IT?

How many different three-digit numbers can you make using the digits 4, 5, and 6, using each digit only once?

Show all the three-digit numbers that you found.

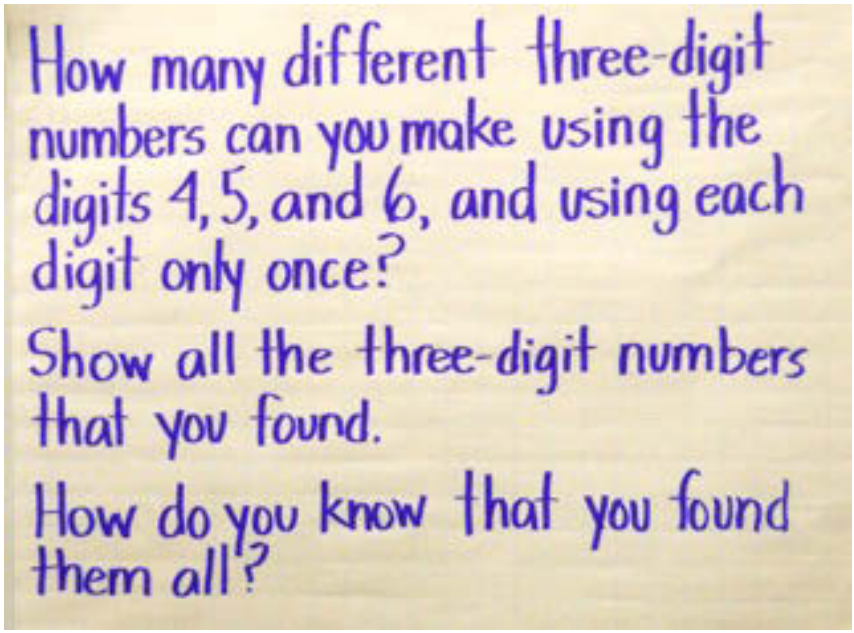
How do you know that you found them all?

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CLASSROOM VIDEO CONTEXT

- 29 fifth graders; 15 girls, 14 boys; 77% African American, 13% White, 10% Latino/a
- Most have not experienced success in school mathematics or other areas
- Work on mathematical topics and practices named in the Common Core



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VIEWING FOCUS

MP.1 Making sense of and interpreting mathematics problems

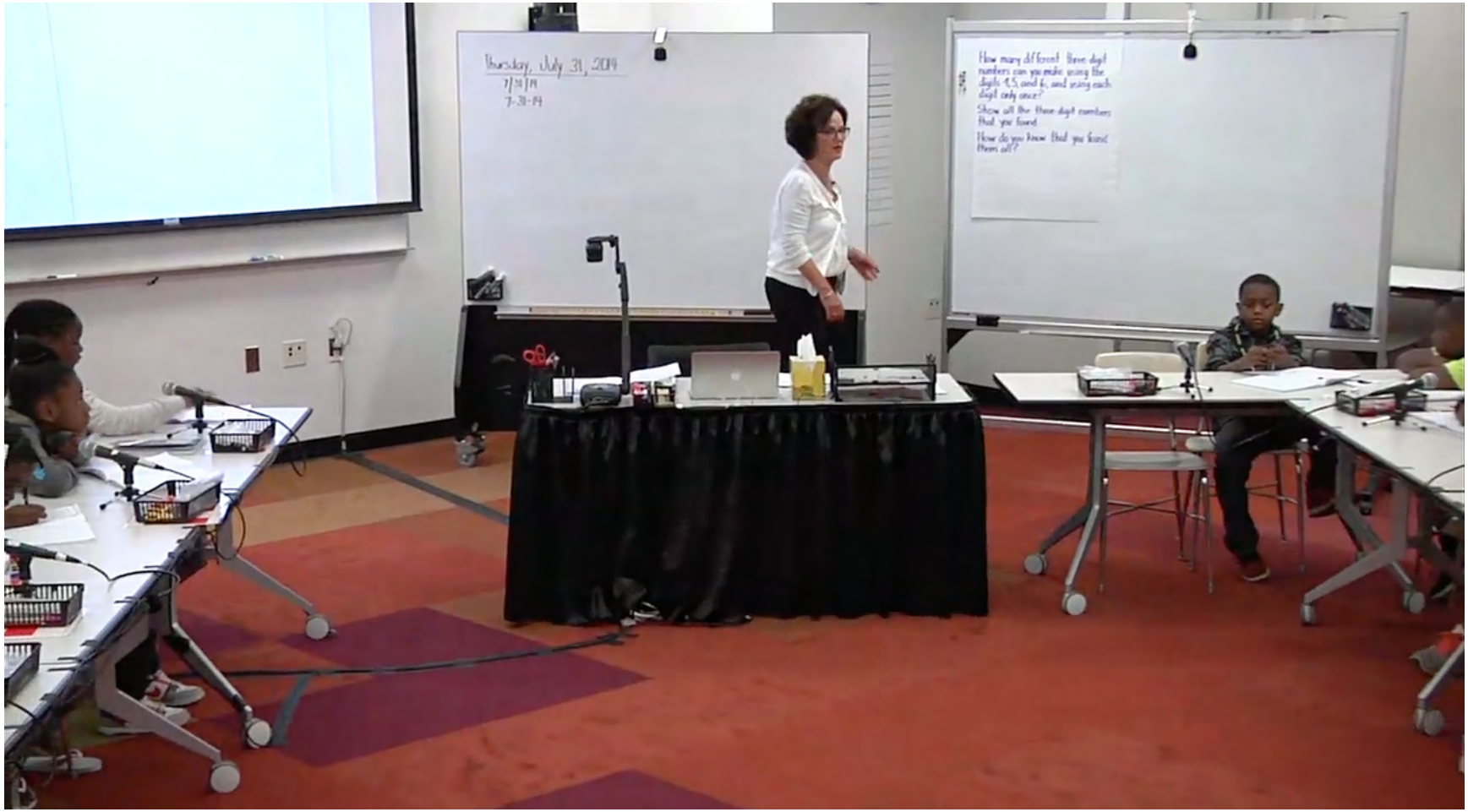
- What is being made explicit about the mathematical work?
- What is left for students to figure out?

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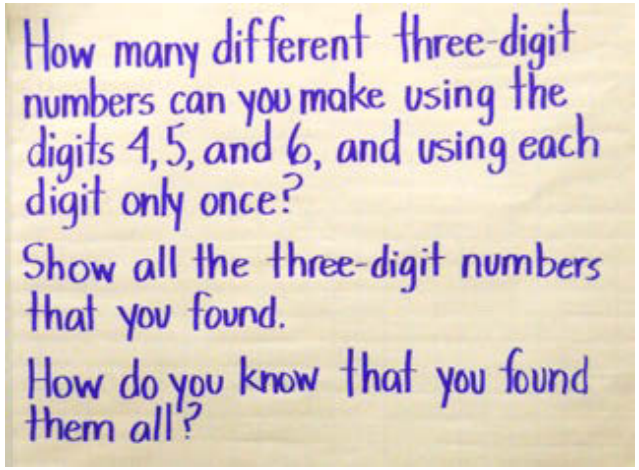
WHAT IS INVOLVED IN "MAKING SENSE OF AND INTERPRETING A PROBLEM"? (MP. 1)



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IDENTIFYING AND USING PROBLEM "CONDITIONS"



Conditions of the Problem

1. Use each digit only once.
2. Use 4, 5, and 6.
3. Make a 3-digit number.

MP.1 Making sense of and interpreting mathematics problems

- What is being made explicit about the mathematical work?
- What is left for students to figure out?

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UNPACKING "EXPLICITNESS" IN MAKING COMPLEX MATHEMATICAL WORK ACCESSIBLE

1. The "why" of explicitness in teaching mathematics for equity and access
2. The "what": Judgment in what to make explicit
3. The "how" of making complex mathematics explicit

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OUR QUESTION

What is involved in making success with complex mathematical work accessible by all students without doing the work for them?

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WHY DOES "EXPLICITNESS" HAVE AN UNCOMFORTABLE PLACE IN MATHEMATICS EDUCATION?

1. Dominant patterns of telling in U.S. teaching that often reduces cognitive demand (e.g., Stein, Silver, and Henningsen)
2. The development of constructivism and theories of mathematics learning (e.g., Cobb, Steffe, Simon, Von Glasersfeld)
3. Widely-shared views of mathematics as activity, as a human construction, and a curricular aim of having students "do authentic mathematics"

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WHY MIGHT A FEAR OF EXPLICITNESS BE INEQUITABLE

1. Structure matters for creating spaces in school where students can experiment with possibilities
2. The role of instruction in intervening on broader societal and cultural views of what mathematics is and who is good at it
3. **Requesting** is not the same as **teaching**; when rich mathematical tasks and situations are used and students are left to puzzle about them on their own, likely will privilege those who have had opportunities with #1 and #2

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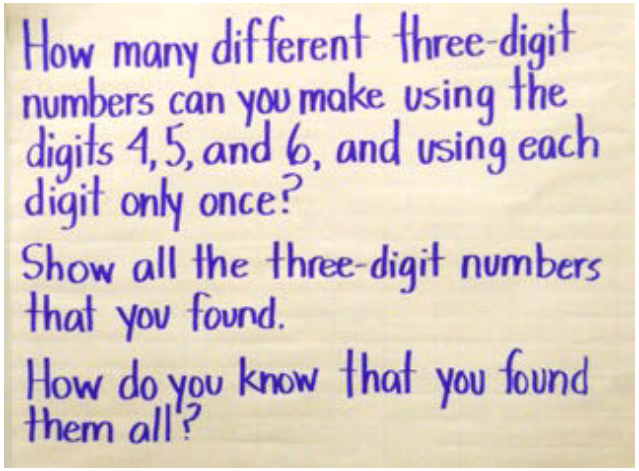
THE ROLE OF CONDITIONS OF A PROBLEM

- Identifying conditions can help in making sense of and interpreting a problem (MP.1)
- Using the conditions can help in persevering in solving a difficult problem (MP.1)
- Conditions are useful in constructing a mathematical argument (MP.3)
- Referring to the conditions is useful in critiquing an argument (MP.3)

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CONSTRUCTING A MATHEMATICAL ARGUMENT



Conditions of the Problem

1. Use each digit only once.
2. Use 4, 5, and 6.
3. Make a 3-digit number.

How could one explain that 456 is a solution to this problem?

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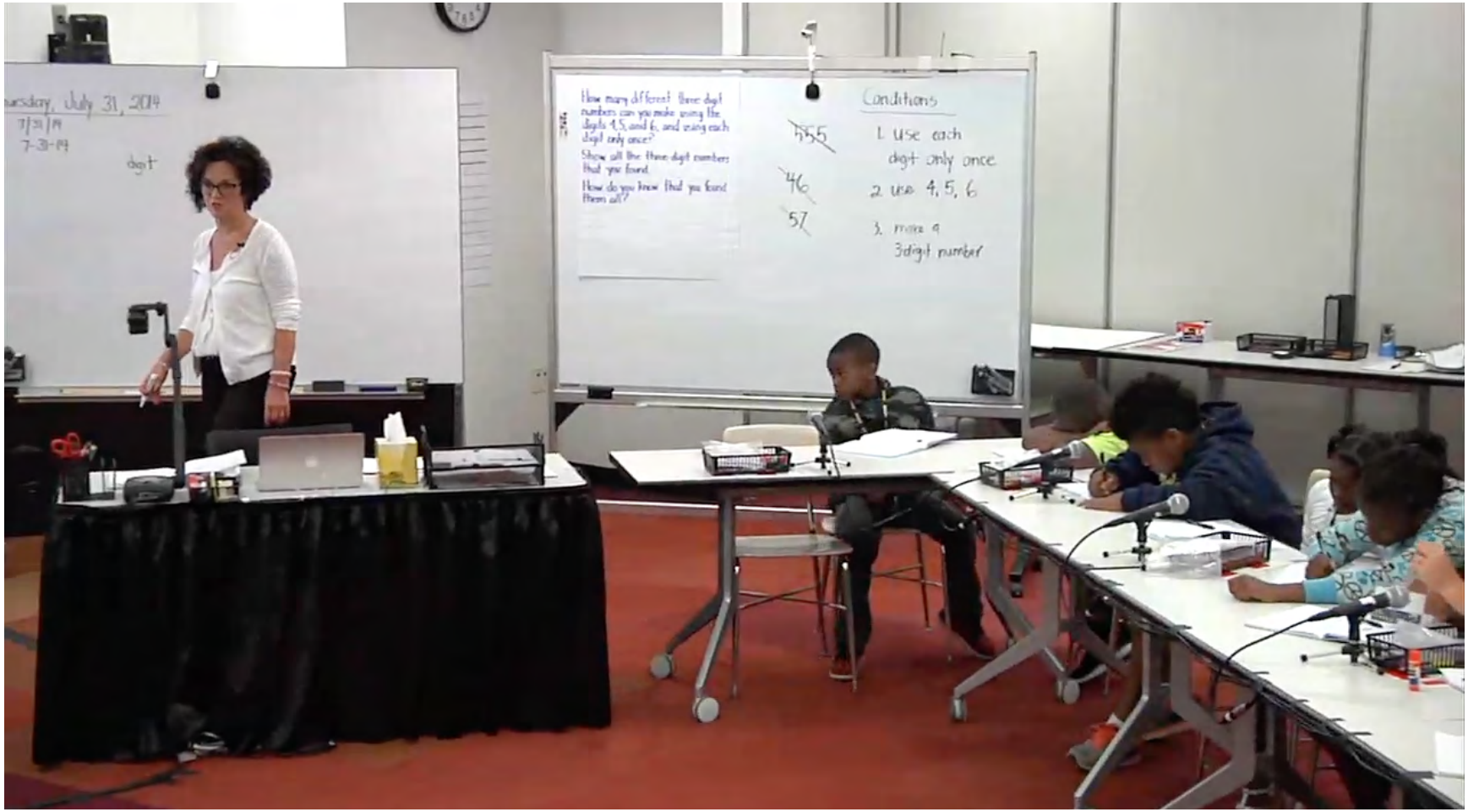
VIEWING FOCUS

- What do you notice about Toni's explanation?
- What is she using that was made explicit?
- What is she having to figure out herself?

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TONI'S EXPLANATION



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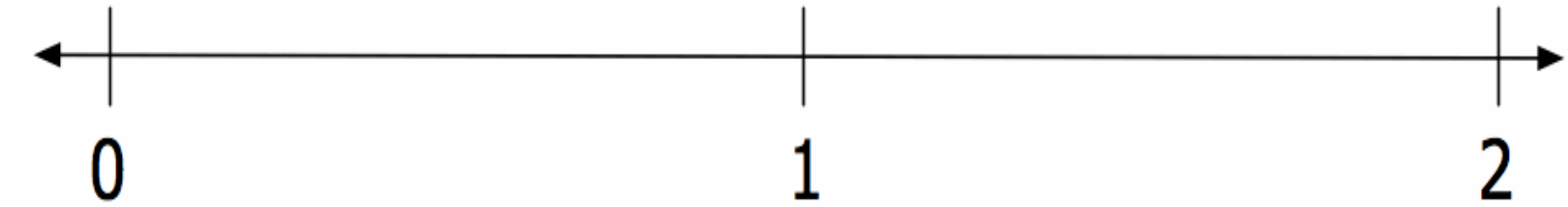
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EXPLAINING FRACTIONS

Mark ¾ on the number line below.

Explain how you know where is located (refer to the whole and equal parts in your explanation).



What is a good mathematical explanation for why ¾ goes where it does on the number line?

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VIEWING FOCUS

Using a definition of a fraction to name and explain a specific fraction on the number line.

- What is being made explicit about the mathematical work?
- What is left for students to figure out?

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ASHTON'S EXPLANATION



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VIEWING FOCUS

Using a definition of a fraction name and explain a specific fraction on the number line.

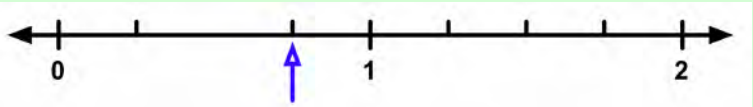
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WHAT IS BEING TAUGHT? WHAT IS BEING MADE EXPLICIT ABOUT COMPLEX MATHEMATICAL WORK?

What number does the blue arrow point to?



Explain how you know.

MATHEMATICAL KNOWLEDGE AND SKILL

- Understanding a fraction as a number
- Locating fractions on the number line
- Explaining an answer
- Using a definition to build a mathematical explanation

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THE ROLE OF A DEFINITION, AND ITS USE(FULNESS) IN ELEMENTARY MATHEMATICS

- Definitions support precision in use of terms and ideas.
- Definitions support reasoning and explanation, including constructing and critiquing arguments.
- To do these things, definitions must be accessible and usable. This involves:
  - Built with shared knowledge and language
  - Accurate even if formulated in elementary terms

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1. UNPACKING AND MAKING CONTENT VISIBLE

- Using a task that highlights key concept of **equal** parts
  - The fractions chosen and the particular diagram (number line)
  - Focus on the idea, not the complexity of the numbers
  - Uses student familiarity with concept image of fractions
- Displaying problem on large poster to coordinate student talk with the idea of equal parts, and to make it possible to see and to follow what is involved in giving a complete explanation

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2. PROVIDING LANGUAGE AND TOOLS FOR EXPLAINING

- Supplying language to support shift from everyday informal language to mathematical language: equal (not “even” or no reference to size); spaces (not lines)
- The working definition of fractions, with the elements that define a fraction and that must be attended to in explaining a fraction

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3. SUPPORTING EXPLANATORY TALK AND WRITING

- Displaying large poster with problem, matches problem students have in their notebooks
- Orienting students to one another
- Valuing the following of others' arguments
- Naming specific talk moves (e.g., talk to class, follow, repeat, agree, disagree, comment)
- Asking meta-questions ("What did Ashton do well in explaining? With fractions?")

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EXPLICIT TEACHING OF MATHEMATICS

- Naming, labeling, writing about important aspects of mathematical ideas, concepts, and procedures
- Naming, highlighting, scaffolding specific mathematical practices
- Naming and supporting qualities of productive mathematical habits and mindset

(Ball, Mann, Shaughnessy, & Bass, 2015; Bass & Ball, 2014; Mann, Owens, & Ball, 2013)

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EXPLICIT TEACHING ≠ DIRECT INSTRUCTION: INVESTIGATING SIMILARITIES AND DIFFERENCES

DIRECT INSTRUCTION

1. Breaks down practice or knowledge into small constituent parts
   - Presentation of information, rules, and examples
   - Tasks only address information from the presentation and the task is uncomplicated
   - Lessons are composed of 4 – 10 exercises (presentations with task series) with only 10% new material
2. Teacher's role to demonstrate, students follow, shift to independent practice
3. Seeks to reduce complexity

EXPLICIT TEACHING

1. Unpacks practice or knowledge to make it open to learners
2. Teacher's role to make elements visible, provide language, supports
3. Seeks to make complex practice accessible

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THE PARALLEL CORE RESPONSIBILITY OF PROFESSIONAL EDUCATORS

- Supporting teachers to teach complex mathematical knowledge and skill to all learners.
- Supporting all teachers.

If we do not do this, we leave teachers to struggle.

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MAKING THE TEACHING OF COMPLEX MATHEMATICAL WORK EXPLICIT FOR TEACHERS' LEARNING

- Unpacking ideas (e.g., conditions of a problem, elements of producing a mathematical explanation)
- Practicing explaining mathematics and labeling what is involved in doing and supporting it
- Studying enactments of practice and decomposing and labeling moves, ideas, visible and less visible aspects of the work
- Practice with students, video record, analyze

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RESOURCES FOR EXPLICIT TEACHING

- Mathematical language, professional language: focuses and provides control and focus
- Opportunities to analyze and to do
- Collective work: extends opportunities to learn, discussion highlights, challenges, extends

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