Assessing Enacted Mathematics Teaching Practice

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with discussant

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National Council of Teachers of Mathematics Research Presession

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Overview

1. Assessing beginning teachers' skill at eliciting and interpreting children's mathematical thinking
2. Comparing eliciting practices happening in different contexts
3. Noticing mathematical knowledge for teaching (MKT) in eliciting and interpreting
4. Another perspective on this work

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1. Assessing beginning teachers' skill at eliciting and interpreting children's mathematical thinking

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Assessment in teacher education

- Call for teacher education that focuses more directly on specific practices of teaching (Ball & Forzani, 2009; Grossman et al., 2009; Lampert & Graziani, 2009)
- Current assessment in teacher education
- Common assessments:
  - Develop elaborate lesson plans or units
  - Write detailed reflections on teaching or observations of students
  - Compile a portfolio that includes artifacts such as student work
  - Observations in practicum and student teaching
- Methods offer information about how beginning teachers think about teaching
- Limited evidence of beginning teachers' skill with enactment of specific teaching practices
- Need for assessments that evaluate the enactment of practice – "assessments of enacted practice"

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Assessments of enacted practice

- Based on actual performance, appropriate to the practice being assessed
- Entail, capture, and enable the appraisal of the doing of teaching
- Conducted in a variety of settings, depending on the practice: Real classrooms, in "performance centers", through simulations; some live and some scored through records of practice

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Context for our work

- Redesigning teacher education at U-M
  - High-leverage teaching practices
  - Content knowledge for teaching
  - Professional ethics
- Designing assessments of enacted practice for math methods courses that include a variety of:
  - High-leverage practices
  - Contexts
  - Forms
  - Purposes

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Examples of assessments of enacted practice

- Leading a mathematics discussion
- Setting up a mathematics task
- Eliciting and interpreting student thinking about mathematics
- Giving a mathematical explanation

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Benefits of assessing enactment

- Focuses design and enactment of learning opportunities on the doing of teaching
- Conveys that teaching practice "counts"
- Strengthens the connection to student learning by focusing on high-leverage practices and assessing beginning teachers' skills with those practices

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Challenges of assessing enactment

- Articulating the specific practices to be assessed
- Developing assessment tasks that can elicit the intended practice(s)
- Ensuring fairness with respect to what is being appraised and the teaching contexts in which it is appraised
- Constructing criteria that address the complexities in teaching and evaluating practice
- Creating assessment tasks, tools, and contexts that are efficient and sustainable
- Understanding whether performance in a particular instance of teaching practice will generalize to other performances

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Challenges of assessing enactment in "real" contexts

- Ensuring fairness with respect to:
  - What is being appraised
  - The teaching contexts in which it is appraised
- Creating assessment tasks, tools, and contexts that are:
  - Targeted and tailored to key mathematics
  - Reliably used by a range of assessors
  - Usable at scale

How might these challenges be managed in the design of assessments of practice?

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Assessing enactment in simulations

Assessments of enacted practice:

- Based on actual performance, appropriate to the practice being assessed
- Entail, capture, and enable the appraisal of the doing of teaching

Simulations can be used that:

- Occur outside of a "real" context of practice, which allows for standardization
- Fix content, student responses, and/or resources that are variable in "real" contexts
- Include essential conditions for enacting the practice being assessed

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Eliciting and interpreting assessment

Context

- **Focus:** Eliciting and interpreting student thinking with particular mathematics content
- **Timing:** Happening mid-program; after significant course-work focused on eliciting and interpreting student thinking

Assessment overview

A teaching intern:

- Interacts with a "standardized student" about a sample of student work
- Responds to a series of follow-up questions to ascertain:
  - Her/His interpretation of the student's thinking
  - Her/His hypothesis about how the student would perform on a similar task

Shaughnessy, Sleep, Boerst, & Ball 2011

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Setting the stage for eliciting and interpreting

The teaching intern:

1. **Prepares for an interaction with a standardized student about one piece of student work**

Your goal is to elicit and probe to find out both what the "student" did to produce the answer as well as why she performed the particular steps.

$$9\frac{2}{10} - 6\frac{7}{10} = 2\frac{5}{10}$$

Correct answer, unclear basis

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How is evidence of eliciting obtained?

The teaching intern:

1. Prepares for an interaction with a standardized student about one piece of student work
2. **Probes the standardized student's thinking to understand the steps she took as well as why she performed the particular steps**

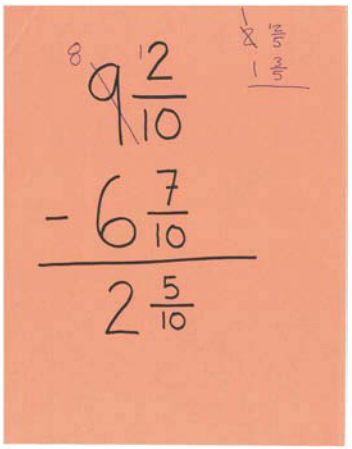
A Standardized Student

Developed response guidelines focused on:

- What the student was thinking
- General orientations towards responses
- Responses to anticipated questions

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An eliciting performance





What aspects of eliciting are observable in the performance?

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What is being assessed about the eliciting?

Evaluate whether the teaching intern:

- Launched the interaction with a question that was neutral, open, and focused on student thinking
- Elicited the specific steps of the student's process
- Attended to the student's ideas in follow-up questions
- Used appropriate tone and manner

$$\begin{array}{r} 9\frac{2}{10} \\ -\ 6\frac{7}{10} \\ \hline 2\frac{5}{10} \end{array}$$

Not assessed:

- Established an environment in which the student was comfortable sharing his/her thinking
- Monitored the rest of the class while working with an individual student

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How is evidence of interpretation obtained?

The teaching intern:

1. Prepares for an interaction with a standardized student about one piece of student work
2. Probes the standardized student's thinking to understand the steps she took as well as why she performed the particular steps
3. **Responds to questions about her/his interpretation of the student's thinking, including predicting the student's response on a similar task**

Questions

a) Briefly describe what was learned about the student's thinking

b) Predict how the student would solve a similar problem

$$\begin{array}{r} 7\frac{1}{9} \\ -\ 2\frac{4}{9} \\ \hline \end{array}$$

c) Explain prediction using evidence from the interview

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2. Comparing eliciting practices happening in different contexts

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Eliciting practices in different contexts



What do we learn about the nature of teaching interns' skill at eliciting student thinking through assessments that simulate teaching practice?



What do we learn about the nature of teaching interns' skill at eliciting student thinking through interviews in their field placement?

How does their performance correspond with eliciting students' mathematical thinking in classroom contexts?

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Exploratory investigation

- Two contexts:
  - Field placement: Interview with a upper elementary school student
  - Assessment: Interview with an "upper elementary" standardized student
- Similar mathematics content:
  - Field placement: Core ideas about fractions
  - Assessment: Subtraction of mixed numbers
- Approximately same point in time
  - End of first year in a 2-year teacher education program

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Field-based interview

- Course assignment to interview a third, fourth, or fifth-grader in field placement classroom
  - In this case, a fourth grade student
- Interview tasks were provided
- No written student work on the interview was available to be analyzed prior to the interview
- Some scaffolding was provided for follow up questions

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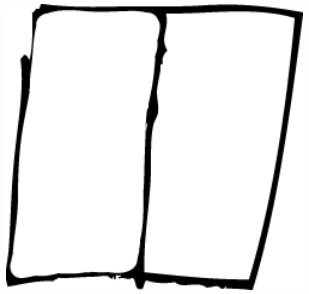
The focal task

Question #2: An example of a fraction
Can you write an example of a fraction? What is that fraction?
Probing a student's answer: *Ask the student to draw a picture to represent the fraction.* *If the student drew a picture rather than writing a fraction, ask the student to write a fraction that corresponds with the picture.* To extend the student's current thinking and to assess how far the student can stretch his or her thinking: *Ask the student for a fraction that is greater than the fraction that they wrote.* *Ask the student for a fraction that is less than the fraction that they wrote.* *Other questions to ask and/or probes:*

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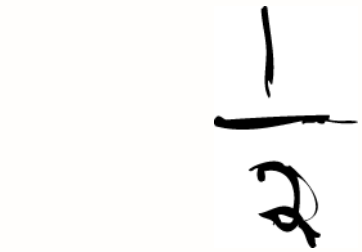


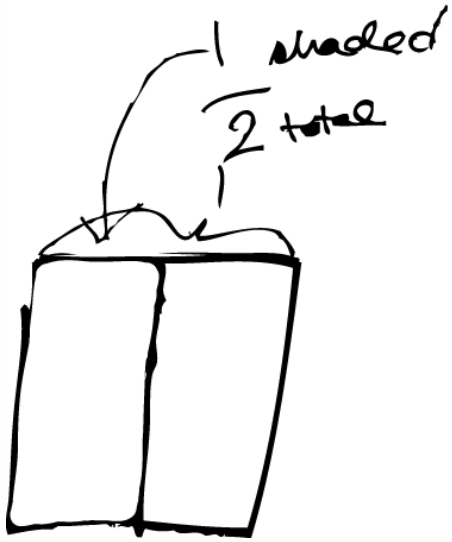


What do you notice about the intern's skill at eliciting student thinking?

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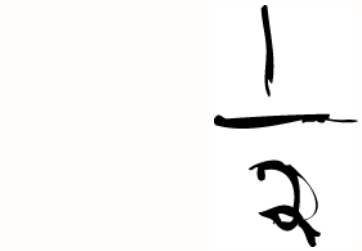




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Four lenses for analysis

Content, student thinking, and teaching practices are at the core of our program and so we selected the following attributes for analysis:

1. Mathematical focus of the questions
2. Types of questions posed, a focus on process or understanding
3. Attention to what the student has said or written/ drawn
4. Use of tone and manner

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1. Mathematical focus of the questions

- Do the follow up questions target focal mathematical ideas?

Field-based interview

Focus on targeting focal mathematical ideas:

- Asks for an example of a fraction
- Asks the child to draw a picture to represent the fraction
- Asks the child to map between the picture and the symbolic notation

Simulation assessment

Focus on targeting focal mathematical ideas:

- Can you tell me why you put a 1 in front of the 2, what does that 1 mean?

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1. Mathematical focus of th

- Do the follow up questions target focal mathe

Focus on focal mathematical ideas in both contexts

Field-based interview

Focus on targeting focal mathematical ideas:

- Asks for an example of a fraction
- Asks the child to draw a picture to represent the fraction
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Simulation assessment

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2. Types of questions posed

- Do the follow up questions target the "process" that the student used to produce the answer?
- Do the follow up questions target "understanding"?

Field-based interview

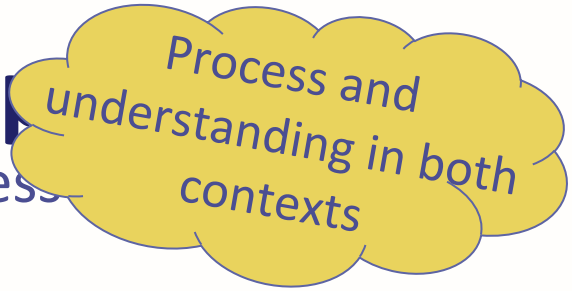
- Focus on process:
  Representing ½ by cutting the square in half
- Focus on understanding:
  - Each side is a half?
  - Where is the 2 represented?
  - Where is the 1 represented?

Simulation assessment

- Initial focus on the process:
  - What would be your first step in solving it?
  - Can you tell me how you borrowed?
- Focus on understanding
  - Can you tell me why you put a 1 in front of the 2, what does that 1 mean?

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Field-based interview

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Simulation assessment

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3. Attention to what the student has said or written/drawn

- Do the follow up questions build on what the student, written or drawn?

Field-based interview

- Consistently draws upon what the student has said or written/drawn in follow up questions
- Fills in ideas and asks for agreement
  - Okay, so the two in the problem tells us how many pieces that paper is cut into?

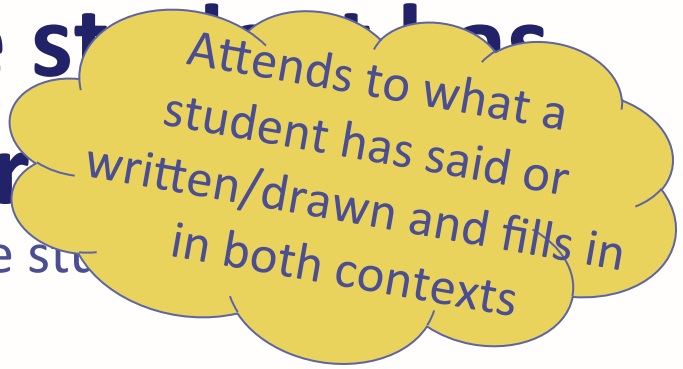
Simulation assessment

- Consistently draws upon what the student said, wrote or drew in follow up questions
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4. Use of tone and manner

- Is the teaching intern's tone neutral and non-evaluative?
- Does the teaching intern demonstrate interest in how the student is thinking and encourage the student to share his/her reasoning?

Field-based interview

- Tone is mostly neutral with some evaluative language: "Beautiful", "okay"
- Demonstrating interest: "Why do you think that this is bigger than one-half if we're talking you want something bigger than this one-half that we originally had?"
- Encouraging: "It's okay. We can – We have time. Go ahead – If you want to think or another fraction.."

Simulation assessment

- Neutral tone with no evaluative language
- Demonstrates interest by asking questions about what the child has said or done
- Little evidence that the teaching intern is being encouraging

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4. Use of tone and m

Some similarities and some differences in use of tone and manner

- Is the teaching intern's tone neutral and n
- Does the teaching intern demonstrate interest
  student is thinking and encourage the student to share his/her reasoning?

Field-based interview

- Tone is mostly neutral with some evaluative language: "Beautiful", "okay"
- Demonstrating interest: "Why do you think that this is bigger than one-half if we're talking you want something bigger than this one-half that we originally had?"
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Simulation assessment

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Summary of case analysis

- Similar patterns:
  - Mathematical focus of the questions
  - Types of question
  - Attention to what the student has said or written/ drawn
- Different patterns
  - Use of tone and manner

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3. Noticing Mathematical Knowledge for Teaching in eliciting and interpreting

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Developing informative and efficient assessments

Since content is integral to enacted teaching practice, there are opportunities to assess mathematics within assessments of enacted practice

Advantages
- Mathematics in use
- Efficiency and timeliness

Challenges
- Impact of context
- Simultaneously assessing more attributes

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Noticing Mathematical Knowledge for Teaching in enacted practice

The mathematical knowledge, practices, and habits of mind entailed by the work of teaching, such as:

- Analyzing, selecting/designing, and solving special kinds of mathematical problems
- Engaging in specialized mathematical reasoning

"It's the use of mathematics in practice that matters"

(Ball et al., 2011)

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Identifying high-leverage mathematics content

① Considerations related to mathematics teaching

- Mathematical ideas and practices that are central to the discipline
- Mathematics with which students typically struggle
- Prominent in elementary curriculum and standards
- Viewed as essential by parents, administrators, and colleagues
- Is unlikely to be learned simply from experience teaching

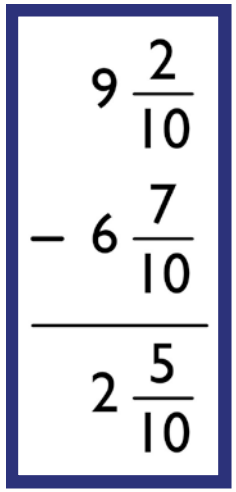
② Considerations related to initial teacher education

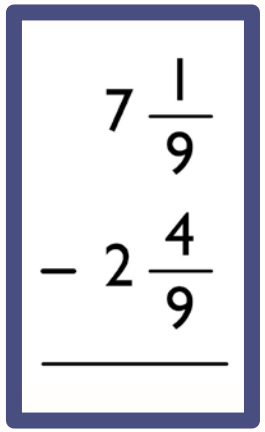
- Affords opportunity for "productive struggle" for beginning teachers
- Useful resources exist about patterns of student thinking
- Is possible for beginners to practice in school settings

(Shaughnessy, Boerst, Sleep, and Ball, 2012)

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Focusing on an instance of computation





- To see knowledge of place value, number, and operations in use
- To see skill in asking about and unpacking what is "hidden" in algorithms making them difficult to teach and prone to errors

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Opportunities to see MKT in use

During the simulation assessment of eliciting and interpreting there are opportunities to appraise interns' MKT, for instance when interns:

- Pose follow-up problems to elicit the standardized student's thinking
- Apply interpretations of the student's thinking in the context of a similar problem

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Posing a follow-up problem

- What follow up problem(s) could a teacher pose to gain a better sense of the student's method or thinking?
- What is the mathematical point of the follow-up problem?

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Single follow-up problems

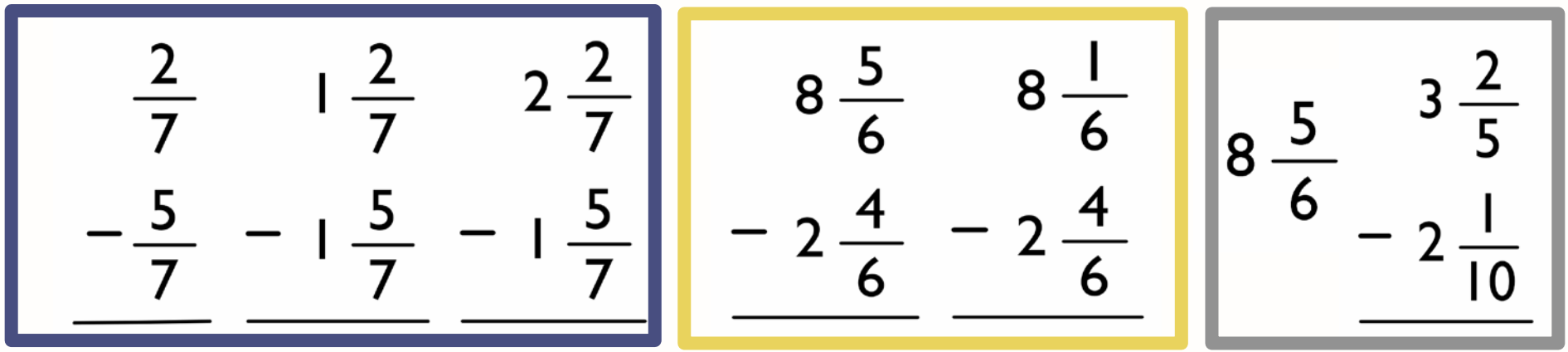
- What follow up problem(s) could a teacher pose to gain a better sense of the student's method or thinking?
- What is the mathematical point of the follow-up problem?

Targeted/Efficient	Less targeted/Inefficient
$9\frac{2}{8} - 6\frac{7}{8}$	$3\frac{2}{5} - 2\frac{1}{10}$ $\quad 9\frac{7}{10} - 6\frac{2}{10}$ $\quad \frac{2}{5} - \frac{3}{4}$ $\quad 34 - 26$

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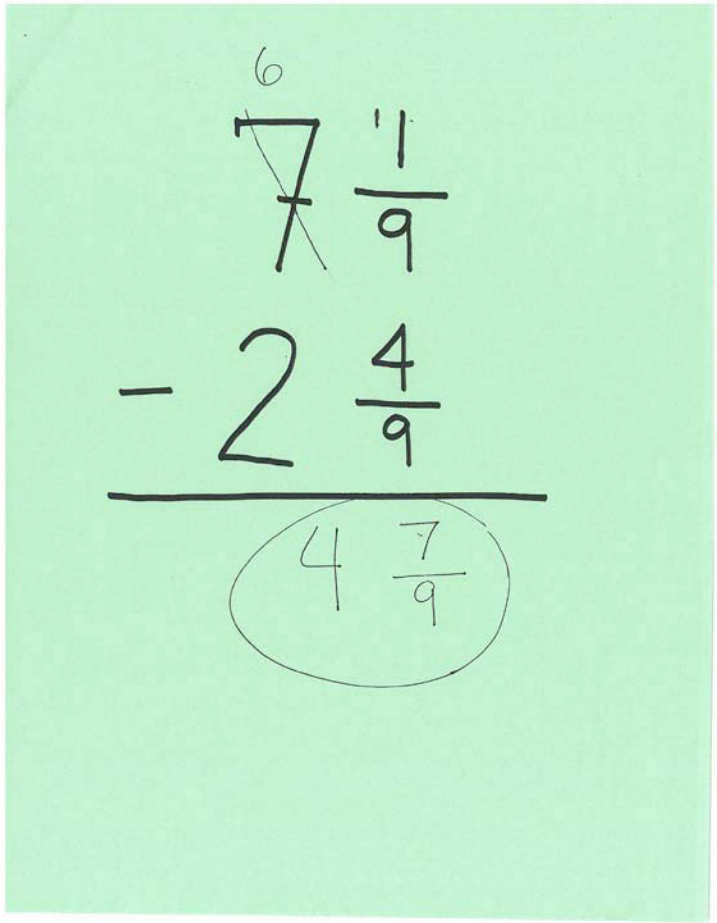
Problem sequences

- What follow up problem(s) could a teacher pose to gain a better sense of the student's method or thinking?
- What is the mathematical point of the follow-up problem?



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Another instance of MKT in use: Specific or more general cases

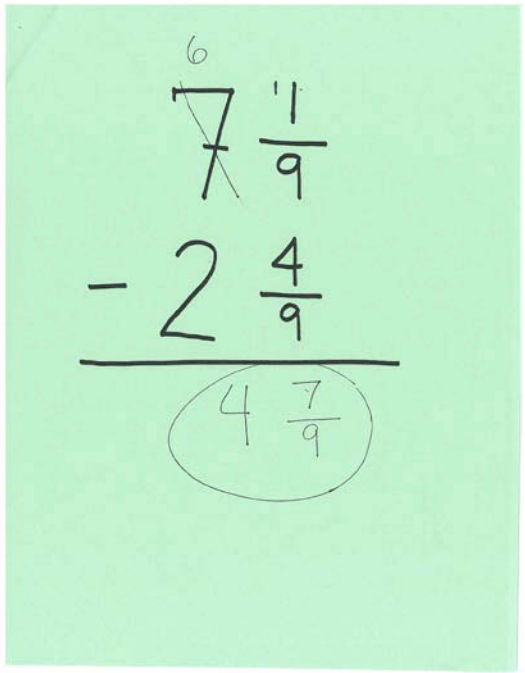


When applying the student's method to a new problem:

- Would the method produce the correct answer?
- Why or why not?

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Another instance of MKT in use: Specific or more general cases



- Is the answer correct?
  Yes No I don't know
- How do you know that the answer to this problem is incorrect?
  - Let me see/let me check.
  - No, it's not how I do it.
  - Yes, because it worked last time.

- You change the value of 7 1/9 doing it that way.
- If you take one from 7, you have 9/9. The way you write it here shows 10/9.

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Summary points

This assessment of practice supported assessment of MKT through:

- Analysis of the follow-up problems that differed in their connection to the mathematics of the student's method and thinking *(for some)*
- Appraisal of the interpretations of the student's thinking in the context of a similar problem *(for all)*

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Next steps

- Further unpacking of the MKT potential of the assessments
- Further comparison of performance on the assessment simulation and performance in "real" teaching contexts
- Refining the logistics associated with simulations of enacted practice
- Developing "assessment prototypes" as a way to generate versions that address different patterns of student thinking and mathematics topics/strands
- Harnessing the potential of assessments as opportunities for teacher educators' learning

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