



Teaching for Complex Mathematical Outcomes While Also Filling in Gaps

Lindsey Mann, Susanna Owens, and Deborah Loewenberg Ball

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Today's session

1. Framing the teaching problem: How to fill in gaps while also teaching for complex learning
2. Teaching for a robust view of mathematical skillfulness
3. Two instructional levers:
   - Designing and using particular kinds of tasks
   - Explicit teaching
4. Developing practice, developing practitioners

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Problem

- In the United States, many students perform poorly in mathematics, particularly members of groups who are historically not well served by schools (e.g., Kilpatrick, et al., 2001)
- Expectations for students' learning are increasing
- Gaps arise over time, and new goals may only further exacerbate these
- Common instructional approaches often either leave gaps or even increase them

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Teaching responsively and responsibly is challenging

- Individual students are not uniform: each student has mathematical strengths and weaknesses
- It is not easy to determine what students can do and what they "need"
- The curriculum and expectations for students shapes "struggles"
- Teachers teach "batches" of students, whose current mathematical capabilities vary

(Lampert, 2001)

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Risks of "remediation"

"Remediation" for students who are not performing well is often carried out separately and fundamentally differently from instruction that challenges and stretches students mathematically.

(see Jayanthi & Gersten, 2011; Fuchs & Fuchs, 2001; Boaler, 2002)

Remedial instruction often:

- Inadequately leverages student strengths
- Narrow focus undermines the development of mathematical skillfulness
- Slows students down, widening the gaps rather than filling them

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Need for "acceleration"

- Goals for student learning are increasing
- Although students may lack specific skills or understanding, they can also manage more complex work

FORTUNATELY, HOWEVER:

- A sixth-grade student is not the same as a second grader

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What does it mean to be "mathematically skillful"?

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The 8's problem

Write as many 8's as you like. Insert plus signs (+) so that the value of the expression is equal to 1000.

Note: You can string 8's together, such as 888.

- How many different equations can you write following this rule?
- What is your approach?
- Can you find all of them? How do you know?

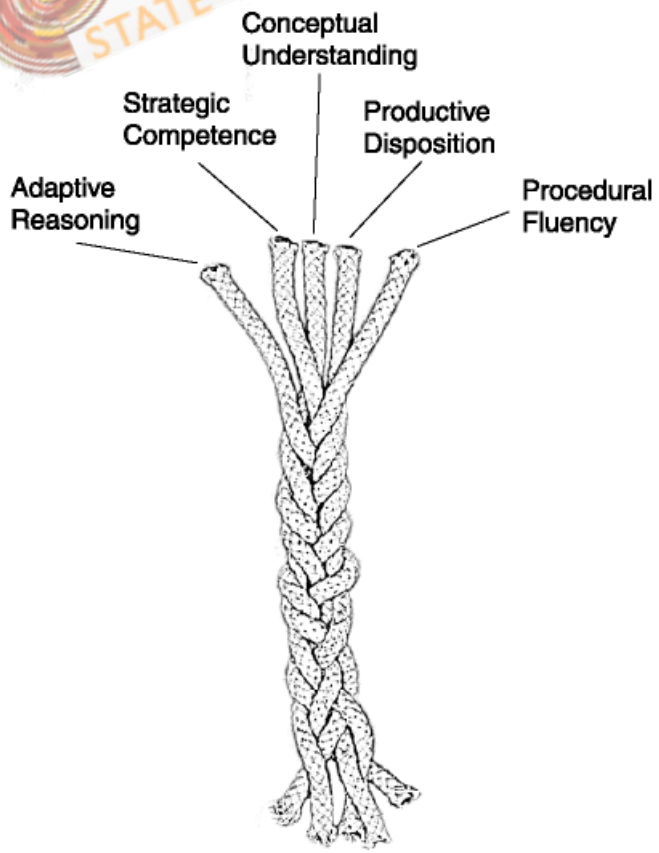
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Solutions to the 8's problem

number of 888's	number of 88's	number of 8's	number of terms
0	0	125	125
1	0	14	15
0	1	114	115
1	1	3	5
	2	103	105
	3	92	95
	4	81	85
	5	70	75
	6	59	65
	7	48	55
	8	37	45
	9	26	35
	10	15	25
	11	4	15

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Strands of mathematical proficiency



- ***Conceptual understanding*** - comprehension of mathematical concepts, operations, and relations
- ***Procedural fluency*** - skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- ***Strategic competence*** - ability to formulate, represent, and solve mathematical problems
- ***Adaptive reasoning*** - capacity for logical thought, reflection, explanation, and justification
- ***Productive disposition*** - habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

Kilpatrick, J., J. Swafford, and B. Findell. (2001). *Adding It Up: How Children Learn Mathematics*. Washington, DC: National Academy Press.

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THE COMMON CORE:

What is entailed in being mathematically skillful?

CONCEPTS AND PROCEDURES	MATHEMATICAL PRACTICES
• Understanding of operations (+, x) • Fluency with whole number computational skills (+, x) • Multi-digit computation • Understanding of mathematical structure	• Making sense of and interpreting problems • Representing mathematical ideas in strategic ways • Persevering with complex mathematical problems • Constructing mathematical arguments • Look for and use regularity in mathematical reasoning • Look for and make use of mathematical structure

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Elementary Math Lab (EML)

- Ten-day class for sixth-grade students identified by their teachers as "struggling" in mathematics
- Aims to blend remediation and acceleration
- Carefully documented – video, student work, teacher lesson plans

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Our working conjecture

- Remediation and acceleration can happen in the context of whole group work in a diverse class
- Two features of instruction work together to provide opportunities for simultaneous remediation and acceleration
  - **Tasks** provide a broad space to work on mathematics
  - **Explicit teaching** is used to help students do skillful mathematical work

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Task design

1. Targeted focus on ideas and skills that are "missing" for some students
2. Also includes major focus on challenging content
3. Layered design
4. Elements of novelty
   - The "housing" of the problem
   - The kind of question (e.g., reverse, critique)

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True and false equations

Mark true or false, and explain.

$4 + 2 = 2 + 4$

$3 - 5 = 2$

$8 = 5 + 3$

$4 + 6 = 5 + 5$

$2/2 = 4/4$

$r + y + w = n$

$y = g + r$

$0/3 = 0/5$

Make each equation true.

$9 + 3 = ___ + 5$

$8 + 3 = ___ + 7$

$__ + 8 = 15 - 5$

$___ = 11 + 4$

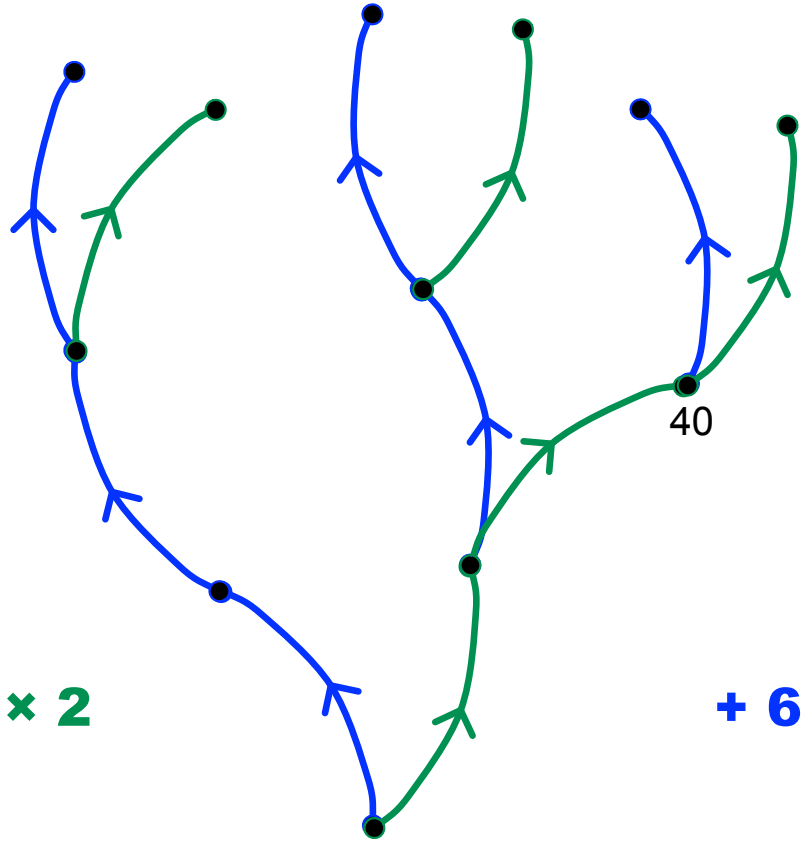
$0 = __ - 7$

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Arrow roads

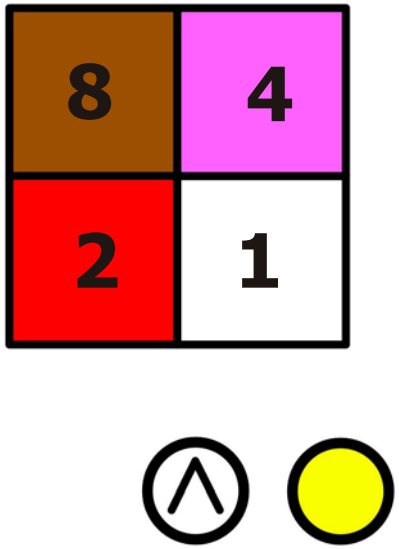
(adapted from CSMP)

Label the dots.
44 and 52 are in this picture. Circle their dots.



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Minicomputer problems



Using exactly one positive and one negative checker, find <u>all</u> the numbers that can be represented on this Minicomputer board.

Prove your answer and explain why.

Papy Minicomputer

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Minicomputer problems

4 7 9 10 12 16 20 24 32 36 40 48

Figure out which of these numbers can be made on the minicomputer using exactly one of these checkers:



 = _____

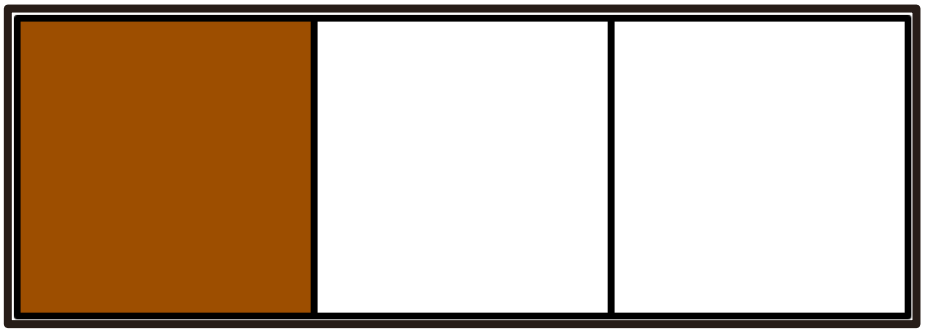
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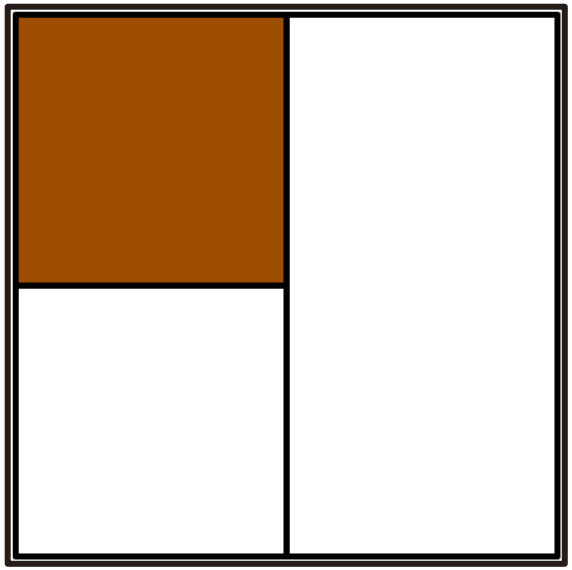
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 = _____

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How much of the rectangle is shaded brown?





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Task design: Brown rectangle

- Targeted focus on ideas and skills that are "missing" for some students
- Includes challenging content: concepts and practices
- Layered design
- Elements of novelty

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Explicit teaching: Making skilled work explicit

- Focusing on important aspects of the mathematical ideas, concepts, and procedures: naming, labeling, writing about
- Making practices visible: naming, highlighting, encouraging
- Commenting on qualities of productive mathematical habits and mindset

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What is being made explicit?

1. Mathematical reasoning: attention to specific details (e.g., the role of the line)
2. Mathematical language (e.g., equal parts," "the whole")
3. Attention to others' reasoning ("Can someone explain what Mackenzie just said?")
4. Naming fractions ("one-third")
5. Norms for communication (e.g., "louder")
6. How to decide what fraction is shaded in an unequally partitioned shape with or without drawing the line ("What is she doing in her mind?")
7. Understand and critique the reasoning of others ("Why do you think Mackenzie is arguing that it's 1/3?")

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Developing practice, developing practitioners

What will be involved in learning to teach in ways that fill in gaps while also working on complex mathematical learning goals?

1. Continue to develop this approach to "differentiation"
2. Develop a broader and more complex view of "mathematical skillfulness"
3. Practice identifying students' mathematical capabilities, including their skills, understandings, practices, and habits in specific contexts and content
4. Work on developing and using multi-dimensional tasks that open the range of learning possibilities
5. Focus on ways to make mathematics explicit that engage students successfully in mathematical work

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Practice in identifying students' mathematical competence

- Developing broader and more robust view of what it means to be "mathematically skillful"
- Studying and discussing students' work, including written work, actual mathematical talk, in light of a robust view of mathematical competence

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Work on mathematical tasks

- Seeing and discussing examples of multi-dimensional tasks: Identifying the mathematical potential, articulating the opportunities for work on gaps, mathematical practices, and new content
- Considering "novelty" and ways to create it equitably

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Making skilled work explicit

- Shift from a focus on distinguishing "teacher-centered" and "student-centered" instruction to a focus on making mathematical work explicit with and for students
- Articulating what there is to make explicit and considering alternative ways to do it
- Developing ways of naming and labeling specific mathematics and mathematical work

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Conclusion

1. Moving beyond the typical dualism of "remediation" and "acceleration" is an important equity agenda.
2. Particular kinds of tasks and explicitness can be very helpful.
3. Teachers would need opportunities to learn to use these, and to develop a broader view of mathematical capability.

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Dev-TEAM

"Reasoning and Explanation" module
Fall 2013 pilot

We are seeking teacher educators doing real-time professional development with practicing elementary teachers

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The Elementary Mathematics Laboratory (EML)

July 29 – August 9

- A program for entering fifth-grade students
- A lab for the study of mathematics teaching practice
- Professional development opportunities for educators

http://www.teachingworks.org/training/LaboratoryClasses

TeachingWorks, at the University of Michigan



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