2015 Meeting of the National Council of Teachers of Mathematics

April 17, 2015 • Boston, MA

Deborah Loewenberg Ball

WITH RESPECT FOR TEACHING: MAKING THE PRACTICE OF MATHEMATICAL INSTRUCTION EXPLICIT



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A PROFESSIONAL IMPERATIVE: RAISING UP TEACHING AS THE #1 POLICY ISSUE OF OUR TIME

1. 50,000,000 children and youth in our nation's schools, 3.7 million teachers.
2. 51% of youth are of color; 15% of teachers.
3. Schools under-serve students from traditionally marginalized populations: communities of color, communities of poverty. The demographic gap and broader policy environments reinforce this.
4. Curriculum doesn't teach; teachers do.
5. Despite the evidence of the power of effective teaching, overwhelming lack of public respect for the work of teaching.

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WE CAN WORK TO CHANGE THIS!

What do we do to make the public respect our work?

We have to start with respecting it more ourselves.

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My first two years of teaching, I was really a wreck.

You can't really teach someone to teach.

I like what she did, but it is not my style and it wouldn't work for me.

Even if some of this is what you experienced, talking this way in public doesn't elevate our profession.

Teaching is an art, not a science.

Everything I know, I learned on the job.

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THE CORE CHALLENGE OF TEACHING

- Teaching complex mathematical knowledge and skill *(i.e., not just procedural technique)*
- Making complex mathematical knowledge and skill accessible to all students, and enabling all students to be successful with complex mathematics *(i.e., not just some students)*

What does this involve?

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UNPACKING—AND MEETING—THE INSTRUCTIONAL CHALLENGE

1. Why is explicitness crucial for strong mathematics learning and teaching?
2. A close look at one example: Identifying the conditions of a problem.
3. How are explicitness and equitable practice related?
4. Examples of explicitness in teaching mathematical practices.
5. Summary: What do we mean by "explicitness" in teaching and how can we keep learning better how to make complex learning accessible to all of our students?

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REQUESTING ≠ TEACHING

- For students, simply being **requested**—asked or assigned—to solve rich challenging problems is **not teaching** them to do complex mathematical work.
- For teachers, simply being **requested**—through standards, assessments, or curricula—to teach complex mathematical ideas to all their students is **not supporting** us to learn to do such instruction.

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WHAT IS THIS PROBLEM ASKING, AND WHAT IS INVOLVED IN TRYING TO SOLVE IT?

How many different three-digit numbers can you make using the digits 4, 5, and 6, using each digit only once?

Show all the three-digit numbers that you found.

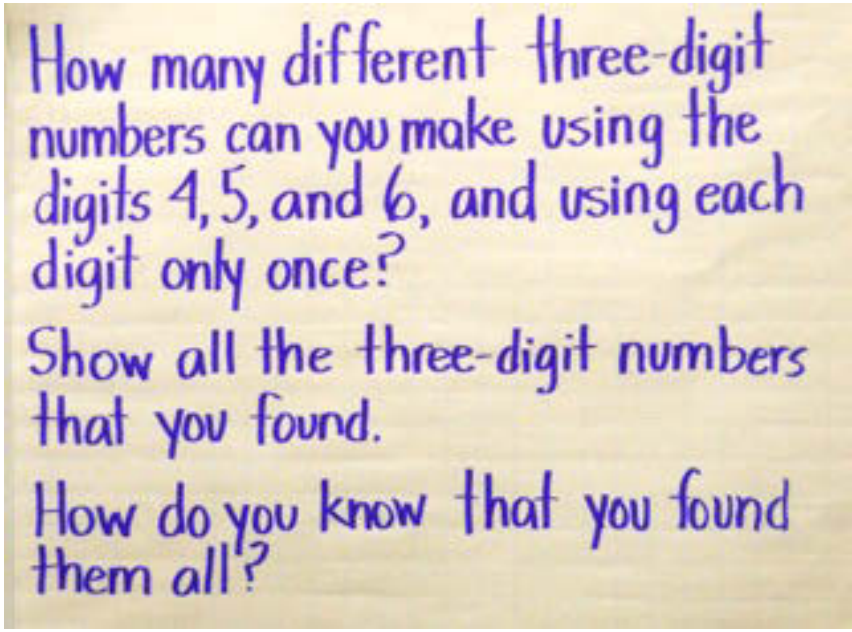
How do you know that you found them all?

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CLASSROOM VIDEO CONTEXT

- 29 fifth graders; 15 girls, 14 boys; 77% African American, 13% White, 10% Latino/a
- Most have not experienced success in school mathematics or other areas
- Work on mathematical topics and practices named in the Common Core



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VIEWING FOCUS

MP.1 Making sense of and interpreting mathematics problems

- What is being made explicit about the mathematical work?
- What is left for students to figure out?

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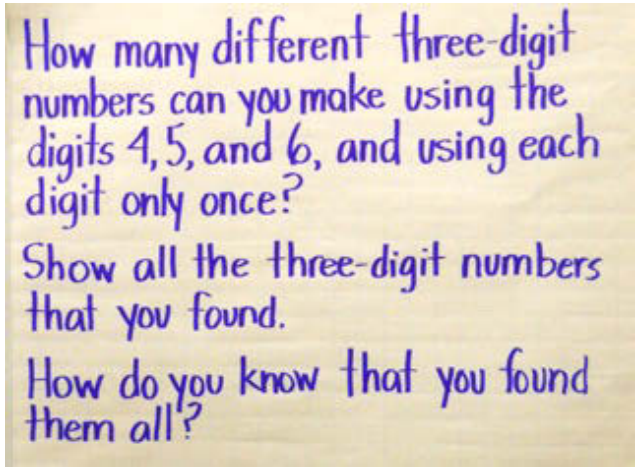
WHAT IS INVOLVED IN "MAKING SENSE OF AND INTERPRETING A PROBLEM"? (MP. 1)



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IDENTIFYING AND USING PROBLEM "CONDITIONS"



Conditions of the Problem

1. Use each digit only once.
2. Use 4, 5, and 6.
3. Make a 3-digit number.

MP.1 Making sense of and interpreting mathematics problems

- What is being made explicit about the mathematical work?
- What is left for students to figure out?

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UNPACKING "EXPLICITNESS" IN MAKING COMPLEX MATHEMATICAL WORK ACCESSIBLE

1. The "why" of explicitness in teaching mathematics for equity and access
2. The "what": Judgment in what to make explicit
3. The "how" of making complex mathematics explicit

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THE CORE QUESTION

What is involved in making success with complex mathematical work accessible for all students—without doing the work *for* them?

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WHY DOES "EXPLICITNESS" HAVE AN UNCOMFORTABLE PLACE IN MATHEMATICS EDUCATION?

1. Dominant patterns of telling in U.S. teaching that often reduces cognitive demand (e.g., Stein, Silver, and Henningsen)
2. The development of constructivism and theories of mathematics learning (e.g., Cobb, Steffe, Simon, Von Glasersfeld)
3. Widely-shared views of mathematics as activity, as a human construction, and a curricular aim of having students "do authentic mathematics" (e.g., Lampert, Schoenfeld)

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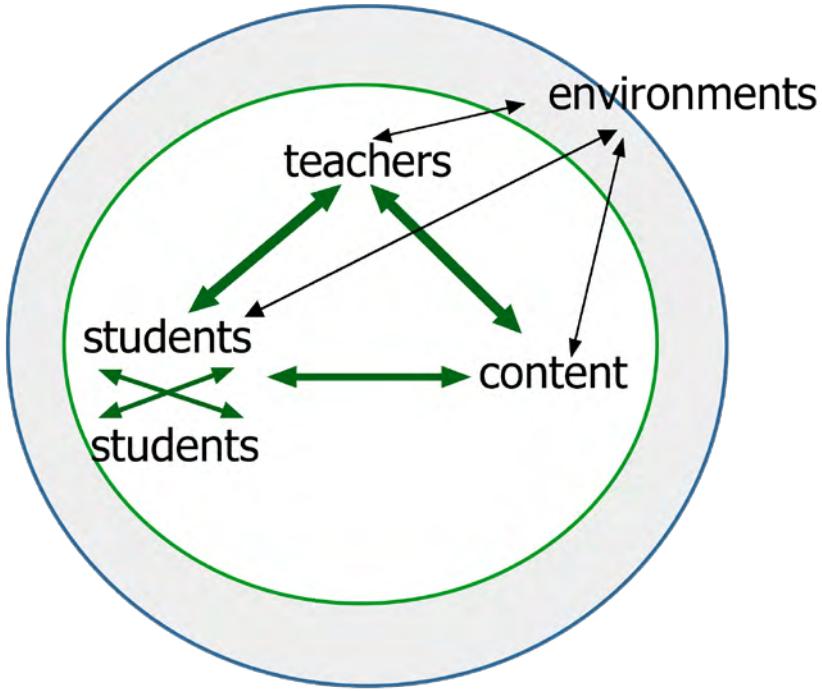
WHY MIGHT A FEAR OF EXPLICITNESS BE INEQUITABLE

1. Structure matters for creating spaces in school where students can experiment with possibilities
2. The role of instruction in intervening on broader societal and cultural views of what mathematics is and who is good at it
3. **Requesting** is not the same as **teaching**; when rich mathematical tasks and situations are used and students are left to puzzle about them on their own, likely will privilege those who have had opportunities with #1 and #2

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ATTENDING DELIBERATELY TO EQUITY INSIDE OF INSTRUCTION



- Inequity is reproduced inside of instructional practice
- Teachers can have leverage at strategic points in their work
- Breaking this cycle depends on joining concerns for equity with the daily and minute-to-minute work of teaching

(Cohen, Raudenbush, & Ball, 2003)

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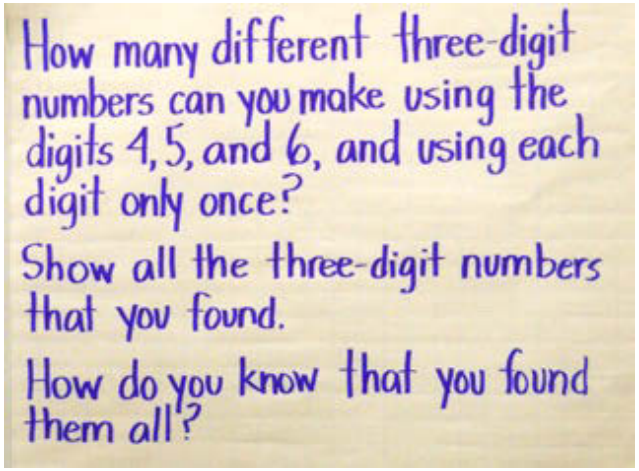
THE ROLE OF CONDITIONS OF A PROBLEM

- Identifying conditions can help in making sense of and interpreting a problem (MP.1)
- Using the conditions can help in persevering in solving a difficult problem (MP.1)
- Conditions are useful in constructing a mathematical argument (MP.3)
- Referring to the conditions is useful in critiquing an argument (MP.3)

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CONSTRUCTING A MATHEMATICAL ARGUMENT



Conditions of the Problem

1. Use each digit only once.
2. Use 4, 5, and 6.
3. Make a 3-digit number.

How could one explain that 456 is a solution to this problem?

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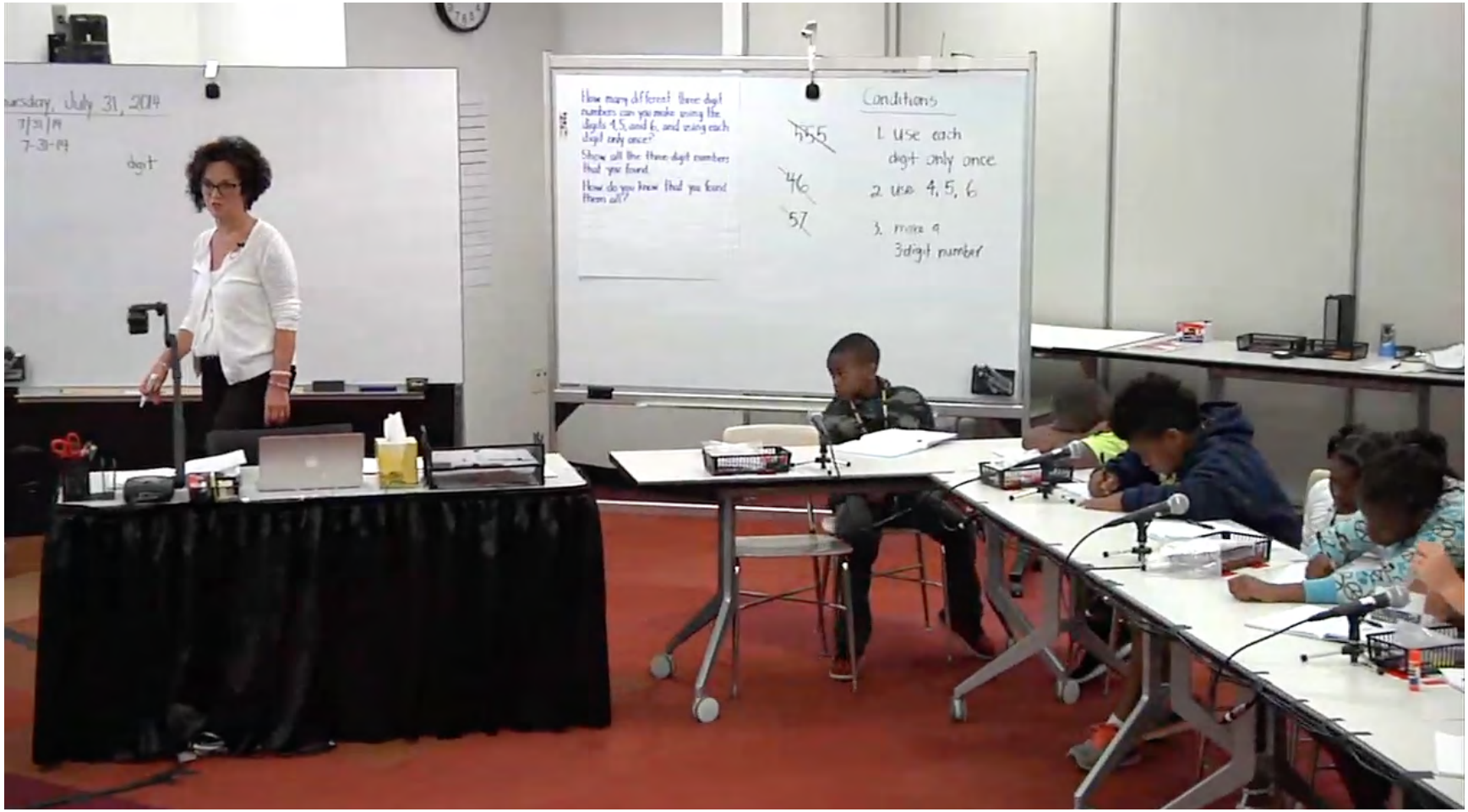
VIEWING FOCUS

- What do you notice about Toni's explanation?
- What is she using that was made explicit?
- What is she having to figure out herself?

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TONI'S EXPLANATION



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VIEWING FOCUS

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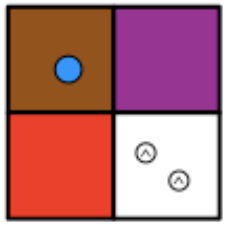
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ASSESSING STUDENTS' LEARNING OF MATHEMATICAL ARGUMENT (MP.3), USING CONDITIONS

3. A student is working on this problem:

Use <u>exactly</u> one positive checker and one negative checker to make the number 6 on the minicomputer.

This is one solution that the student makes:



Is this answer correct? YES NO

Explain why or why not. _____

_____

_____

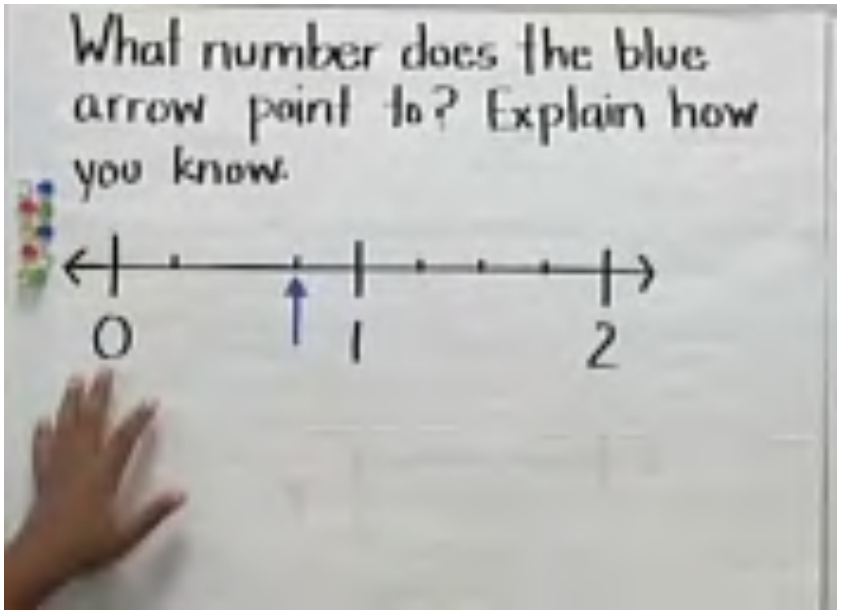
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EXPLAINING FRACTIONS

What number does the blue arrow point to? Explain how you know.

What is a **good mathematical explanation** for what number the blue arrow points to?



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GRADE 3: NUMBER AND OPERATIONS–FRACTIONS

Develop understanding of fractions as numbers.

CCSS.MATH.CONTENT.3.NF.A.1

Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.

CCSS.MATH.CONTENT.3.NF.A.2

Understand a fraction as a number on the number line; represent fractions on a number line diagram.

CCSS.MATH.CONTENT.3.NF.A.2.A

Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.

CCSS.MATH.CONTENT.3.NF.A.2.B

Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.

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(USABLE) CCSS DEFINITION OF A FRACTION, DEVELOPED BY THE STUDENTS AND TEACHER

1. Figure out what the whole is.
2. Make sure that the whole is divided into equal parts. If not, make equal parts.
3. Count how many equal parts there are. We call the number of equal parts *d*.
4. Write $\frac{1}{d}$ to show one of the equal parts. This is a unit fraction.
5. If more than 1 of those parts is shaded, count them (*n*) and write $\frac{n}{d}$.
6. If $n = d$, then you have the whole.

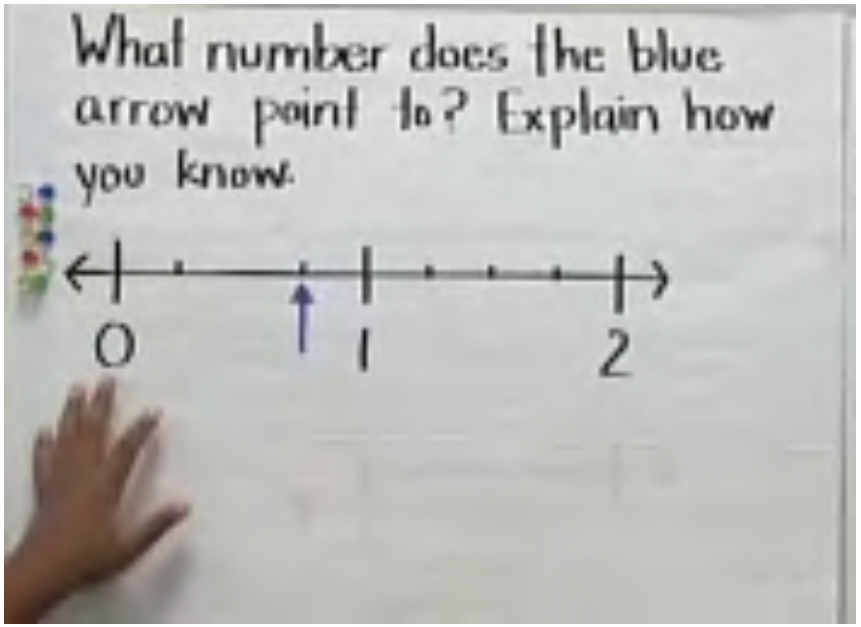
Two important points:

- d cannot be 0 ($d \neq 0$)
- n and d are whole numbers for now

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WHAT IS BEING TAUGHT?
WHAT IS BEING MADE EXPLICIT ABOUT COMPLEX MATHEMATICAL WORK?



MATHEMATICAL KNOWLEDGE AND SKILL

- Understanding a fraction as a number
- Locating fractions on the number line
- Explaining an answer
- Using a definition to build a mathematical explanation

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VIEWING FOCUS

Using a definition of a fraction to name and explain a specific fraction on the number line.

- What is being made explicit about the mathematical work?
- What is left for students to figure out?

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ASHTON'S EXPLANATION



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VIEWING FOCUS

Using a definition of a fraction name and explain a specific fraction on the number line.

- What is being made explicit about the mathematical work?
- What is left for students to figure out?

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THE ROLE OF A DEFINITION, AND ITS USE(FULNESS) IN ELEMENTARY MATHEMATICS

- Definitions support precision in use of terms and ideas.
- Definitions support reasoning and explanation, including constructing and critiquing arguments.
- To do these things, definitions must be accessible and usable. This involves:
  - Built with shared knowledge and language
  - Accurate even if formulated in elementary terms

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1. UNPACKING AND MAKING CONTENT VISIBLE

- Using a task that highlights key concept of **equal** parts
  - The fractions chosen and the particular diagram (number line)
  - Focus on the idea, not the complexity of the numbers
  - Uses student familiarity with concept image of fractions
- Displaying problem on large poster to coordinate student talk with the idea of equal parts, and to make it possible to see and to follow what is involved in giving a complete explanation

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2. PROVIDING LANGUAGE AND TOOLS FOR EXPLAINING

- Supplying language to support shift from everyday informal language to mathematical language: equal (not “even” or no reference to size); spaces (not lines)
- The working definition of fractions, with the elements that define a fraction and that must be attended to in explaining a fraction

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3. SUPPORTING EXPLANATORY TALK AND WRITING

- Displaying large poster with problem, matches problem students have in their notebooks
- Orienting students to one another
- Valuing the following of others' arguments
- Naming specific talk moves (e.g., talk to class, follow, repeat, agree, disagree, comment)
- Asking meta-questions ("What did Ashton do well in explaining? With fractions?")

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EXPLICIT TEACHING ≠ DIRECT INSTRUCTION: INVESTIGATING SIMILARITIES AND DIFFERENCES

DIRECT INSTRUCTION

1. Breaks down practice or knowledge into small constituent parts
   - Presentation of information, rules, and examples
   - Tasks only address information from the presentation and the task is uncomplicated
   - Lessons are composed of 4–10 exercises (presentations with task series) with only 10% new material
2. Teacher's role to demonstrate, students follow, shift to independent practice
3. Seeks to reduce complexity

EXPLICIT TEACHING

1. Unpacks practice or knowledge to make it open to learners
2. Teacher's role to make elements visible, provide language, supports
3. Seeks to make complex practice accessible

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EXPLICIT TEACHING OF MATHEMATICS

- Naming, labeling, writing about important aspects of mathematical ideas, concepts, and procedures
- Naming, highlighting, scaffolding specific mathematical practices
- Naming and supporting qualities of productive mathematical habits and mindset
- Co-sponsoring student work and presentations

(Ball, Mann, Shaughnessy, & Bass, 2015; Bass & Ball, 2014; Mann, Owens, & Ball, 2013)

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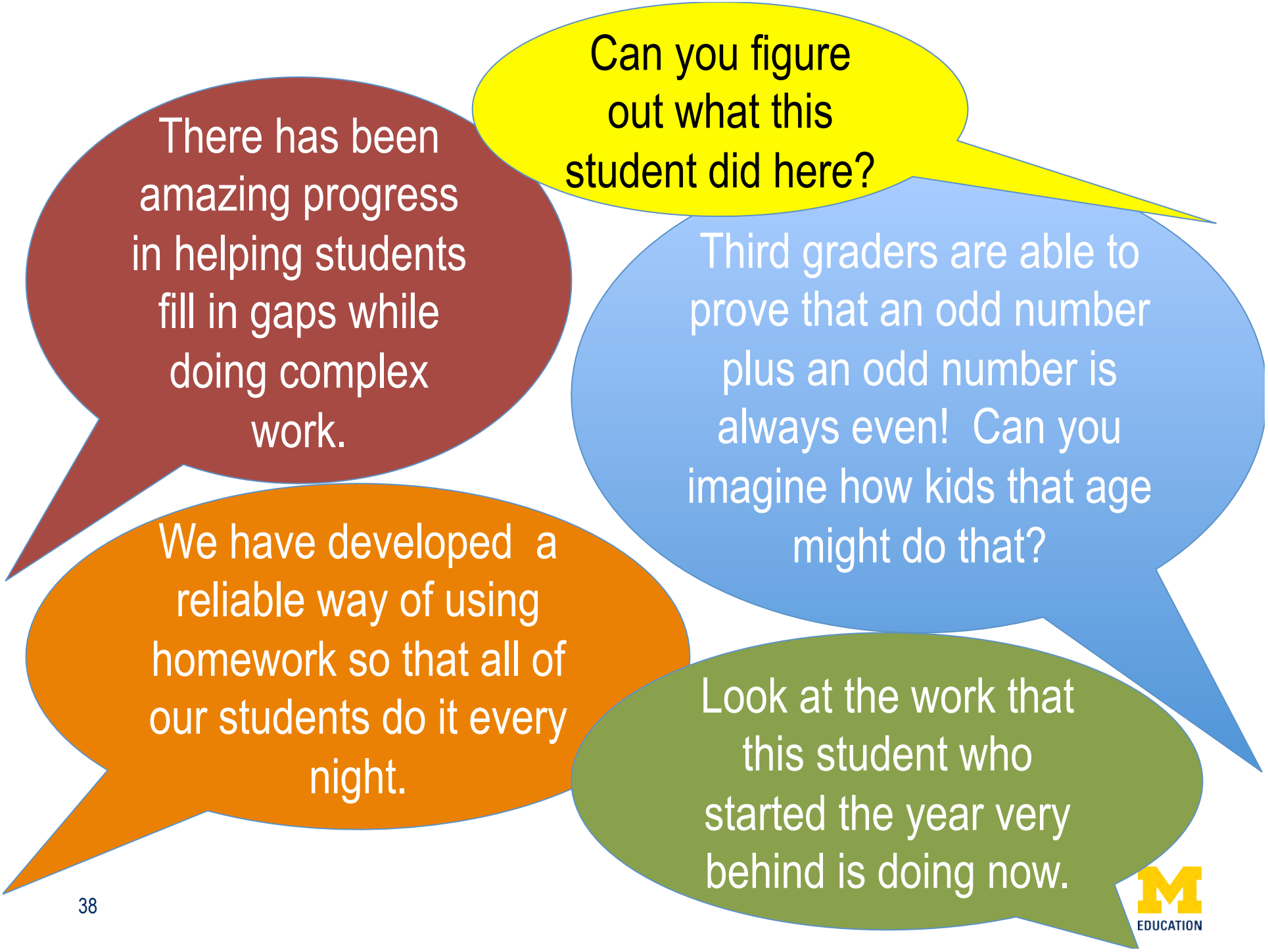
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REPRESENTING TEACHING AS SKILLFUL WORK THAT DEMANDS AND DESERVES PROFESSIONAL PREPARATION AND ONGOING DEVELOPMENT

- Share examples that display the complexity of the work (and that "regular" people can't do)
- Stand up for the importance of rigorous licensure for beginning to teach
- Help to develop shared knowledge useful for initial and continuing professional training

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INTERPRETING LEARNERS' THINKING: ANALYZING ERRORS

What mathematical steps could have produced this answer?

(a)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 405 \\ 108 \\ \hline 1485 \end{array}$$

(b)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 225 \\ 100 \\ \hline 325 \end{array}$$

(c)

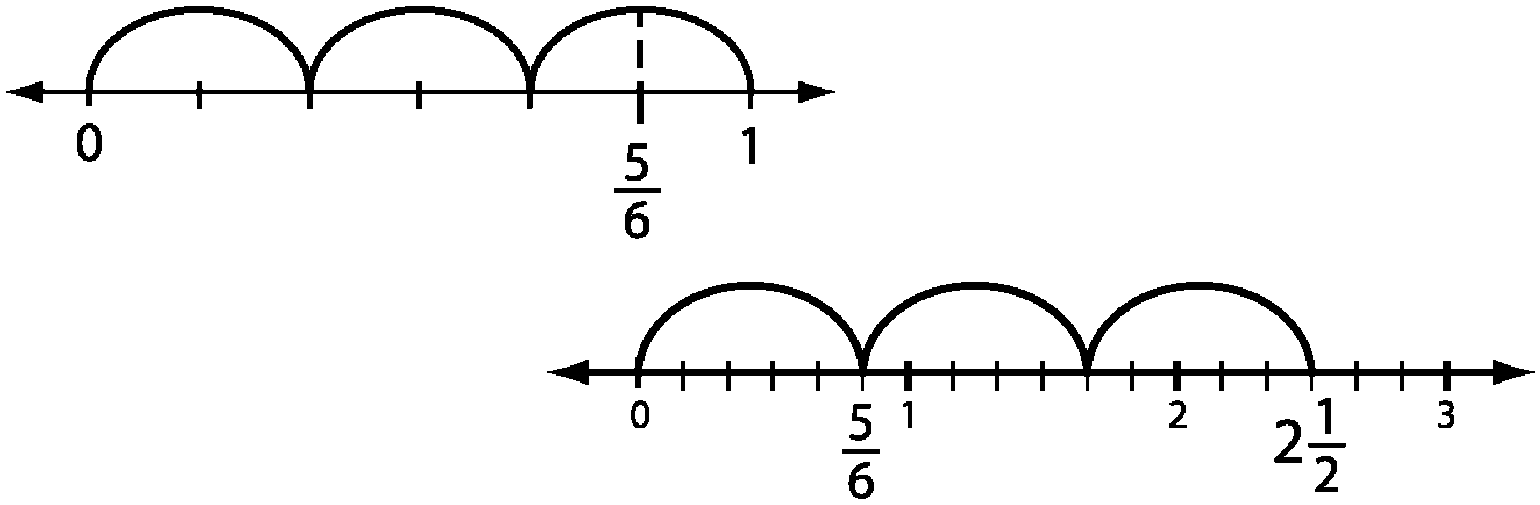
$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 1250 \\ 25 \\ \hline 1275 \end{array}$$

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USING REPRESENTATIONS

What is the quotient of $\frac{5}{6} \div \frac{1}{3}$?



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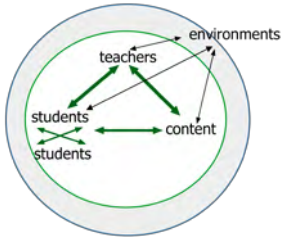
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LET'S WORK TOGETHER TO ELEVATE PUBLIC RESPECT FOR TEACHING.

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Graphic on slide 17:
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