

Improving Mathematics Teaching and Teacher Education through "Specification"

Annette Roskam, Anne Morris and James Hiebert

University of Delaware

Kara Suzuka, Jennifer Lewis, Yaa Cole, Hyman Bass, and Deborah Loewenberg Ball

University of Michigan

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What's your orientation?

With whom do you agree most?

Jill: I would really appreciate specified materials like these because teaching is so hard and this would help me teach well.

Judy: I would be reluctant to work with specifications like these, because they do not draw enough on what I know and know to do, and also does not allow room for me to adapt to the learners with whom I am working.

Alex: I would like to work with specified materials because they would make it so much easier to make decisions about how to improve my teaching as I would be able to study closely how particular aspects actually work.

Lucy: I would like to do it if everyone else was using them too so that we could learn from each other.

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The environment of U.S. mathematics education

- U.S. students are not learning as much as they could, or should
- Many causes, but instruction is key
- Students' access to high-quality instruction is uneven and inequitable
- We lack a reliable system of supplying such instruction
- Historically, two principal approaches to addressing these problems

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Improving teaching by recruiting PEOPLE

- Suppose Colin Powell tires of giving $100,000-a-pop speeches and wants to teach high school social studies. Suppose Meryl Streep has a hankering to teach drama. Alas, they would be "unqualified" for a public school. Elite private schools would snap them up, of course, but public schools that are begging for teachers would have to turn them away because they don't have teacher certification. That's an absurd snarl in our education bureaucracy. Let's relax the barriers so people can enter teaching more easily, either right out of college or later as a mid-career switch. Sure, there are lots of other problems in the U.S. education system. But this is one of the easiest to solve.

(Nicholas Kristof, April 30, 2007; *New York Times* op-ed)

- Teaching should be open to anyone with a pulse and a college degree—and teachers should be judged after they have started their jobs, not before.

(Malcolm Gladwell, December 15, 2008; *New Yorker* essay)

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Improving teaching by "teacher proofing" the WORK

- Individualized instruction (e.g., IPI)
- Basal textbooks and scripted teachers' guides
- Curriculum pacing guides
- Computer-assisted instruction (CAI)

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Improving teaching by learning to do the work

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Effective teaching as unnatural and intricate work

1. Teaching requires teachers to do things that are more than commonsense.
2. Teaching involves some moves or orientations that are unnatural in regular adult life.
3. Teaching practice is intricate, requiring a complex combination of knowledge, skill, timing, and relational work.

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Core teaching practices
(examples)

- Anticipating and probing students' thinking
- Diagnosing student difficulties
- Getting to the point in a lesson
- Posing good questions, choosing strategic examples
- Explaining content in terms that students can understand
- Assessing and keeping track of student progress
- Creating and managing a respectful learning environment
- Interacting with students' parents and caregivers
- Treating students with respect

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The case for detail

1. Teaching is intricate work: inefficient to develop by chance and through individual experience
2. Detailing the work of teaching can give credibility to the level of skill involved in doing it
3. Much about teaching is predictable
4. The scale of the occupation: many people must teach; need to provide equitable opportunities to learn: how to do this reliably

This session: Costs and benefits of detailing practice and ways of doing this

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Framing questions

1. In what forms does knowledge for teaching get specified?
2. What knowledge gets specified for teaching?
3. Who creates these specifications? Who uses them? Who revises them?
4. Why specify practice?

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Four examples

1. Michigan MMPG (preservice teachers being helped to learn teaching)
2. Delaware (teacher educators teaching preservice teachers)
3. Michigan mod4 (teacher developers working with teachers)
4. Japanese lesson study (classroom teachers working on their teaching)

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Example 1: MMPG

Preservice teachers being helped to learn teaching

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Specification of teaching practice for preservice teachers

Focus on practice in four domains

- Planning mathematics lessons
- Assessing students' mathematical knowledge, skills, and dispositions
- Representing mathematical ideas
- Leading a whole-class discussion about mathematics

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In what forms does the knowledge get specified?

- Session plan(s)
- Discussion planning frameworks
- Commentaries to support enactment and instructors'/facilitators' understanding of important underlying ideas
- Published reports
- Records of practice collected from the session

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- Instructional activities (with rationale that links activity with learning goal)
- Script of teacher/instructor/facilitator talk
- Expected learner responses to the activities
- Suggested instructional responses and moves
- Hypotheses that propose future revisions
- (General) teaching practices accompanied by rationales and considerations
- Descriptions of competent practice
- "Cautious points," "Evaluation points" during the session
- Visual representation of mathematical ideas

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Planning frameworks

1. Setting up the task
    - How will you present the task to students?
    - What materials will you and the students need?
    - If needed, how will you familiarize students with representations used in the task?
    - What should students do if they finish the task?

2. Monitoring student work
    - What will you look for or listen for to determine if students are engaging with the task?
    - What will you do to learn about and keep track of what students do as they engage with the task (e.g. what strategies students are using, errors they are making, etc.) that might inform how you lead the discussion?
    - What questions or variations will you pose if students finish quickly or have trouble getting started?

3. Launching the discussion
    - How will you start the discussion of students' solutions and strategies? (i.e., What specific question, example, or idea will you use?)

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"Tool boxes" of teacher moves

When setting up the task with the students:

Establish the work environment

- Make sure students know where to sit
- Do introductions (if needed)
- Give clear, explicit instructions
- Establish expectations for work
  - Explain materials students will use and how they should record their work
  - Establish who students will work with (individually, with partner, in group, etc.)
  - Explain what will be done with the work that is finished (e.g., Tell students they will be asked to explain their thinking.)
  - Establish time frame (e.g., how long students will have to work, that they will be discussing their solutions, etc.)
  - Explain what students should do if they finish
  - How students signal when they need help or want to ask a question

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“Tool boxes” of teacher moves

6. Extending students’ current thinking, and assessing how far they can be stretched

- *What happens if you plug in different numbers?*
- *How would you explain that to a ___ grader?*
- *Are any other answers possible?*
- *What changes if you (add, subtract, multiply, divide) instead of (add, subtract, multiply, divide)?*
- *How else can you express that number?*
- *Can you give an example?*
- *What happens if you change the sign [of the number]?*
- *Can you think of a situation where that wouldn’t be true?*
- *Would it still be true if ____?*
- *What would happen if ____?*
- *Is that always true?*
- *How can we test it?*
- *Can we come up with a general rule?*
- *What other ways can we represent the solution?*
- *Can you show us (with a picture)?*
- *Can you solve it a different way?*
- *Can you use this same method to solve ____?*
- *What would happen if the numbers were changed to _____?*
- *What if the problem was like this instead: [give slight variation of problem]?*
- *If someone said [wrong answer], how would you respond?*
- *Can you think of another problem that could be solved with this method?*

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Discussion question

- What are the advantages and disadvantages of specifying an interactive performance such as leading a discussion?

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What is not specified, and why?

- Depends on context
  - Easier to specify teaching the more particular the instructional setting and task
  - Different types of and purposes for discussions impact what is and can be specified
- Depends on location in cycle of work on that practice
  - Continuum of specification from more to less
  - Gradually add on and integrate new practices
  - Gradually add nuance and complexity to already detailed practices

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How are specifications created? How are they used? How are they revised? By whom?

- Assessing changes based on students' learning
- Insert new ideas by applying theory and by asking new instructors
- Structures, organizations, and names typically provided by teacher educators
- Generate instructional designs (applying and elaborating upon existing theories of teacher knowledge and upon current practices)
- Document uses of instructional designs/specifications by others, analyze the adaptations made, and revise based on these findings
- Obtain feedback from people with different backgrounds/expertise and create new or revised instructional designs/specifications based on this
- Knowledge is collectively produced by participants who create, use, revise, (and often publish) the specifications

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In what ways is specification potentially problematic?

- Might encourage preservice teachers to plow through protocols or use teaching moves without attending to students or mathematics
- Might encourage preservice teachers to focus on "What am I supposed to do next?" rather than on "What are students supposed to be learning?" and "What is my purpose in doing this?"
- Can be hard to pull general practices out of the specifics
- Can be hard to reintegrate into more holistic tasks of teaching
- Can be mismatched to context, task, and instructional purpose

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Example 2: University of Delaware

Specifying the teaching of pre-service mathematics courses: A long-term project on improving teaching

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Context

K-8 mathematics teacher preparation program:

- 3 mathematics content courses
- 2 mathematics methods courses
- Student teaching

Several sections of the courses are offered each semester.

Several instructors teach each course each semester.

- 7 faculty
- Approximately 10 doctoral students

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Example 3: mod4 Project

Teacher educators and professional developers working with teachers

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mod4 Project

- Developing materials for preservice teacher education and inservice professional development
- Focused on helping elementary teachers develop mathematical knowledge for teaching (MKT)
- Treats practice as the content, context for, and method of learning
- Tasks and supports for teachers educators and professional developers (facilitators) to use in their work with teachers

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What have we tried to specify in this use of video?

- Preliminary setup
- Goals for the work
- Contextual setup of the video
- Preliminary viewing task
- Viewing focus setup and video viewing task
- Post-viewing reflection
- Post-viewing discussion
- Transition to session wrap-up

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Preliminary viewing task

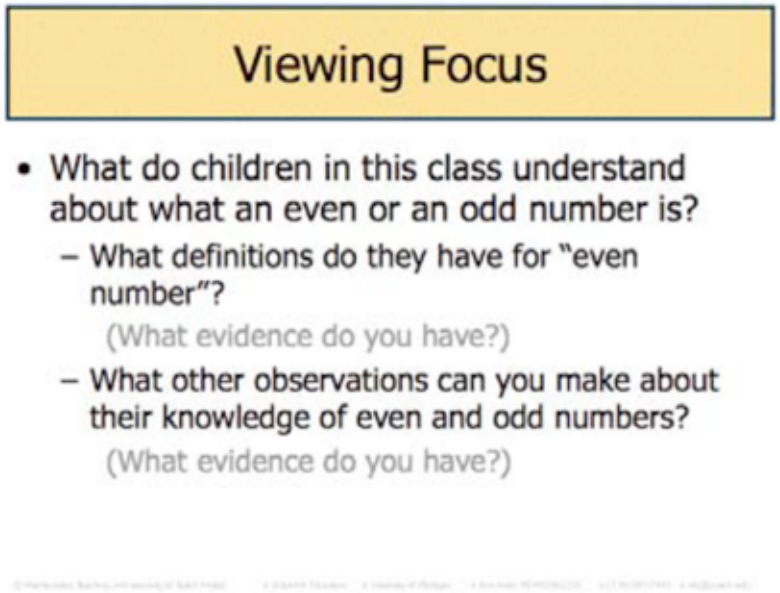
Detailed description of activity	Comments & other resources
<u>Optional preliminary fifteen-minute activity before introducing the viewing focus (i.e. the activity below):</u> If you have extra time available (approximately fifteen minutes), you might want to do a preliminary activity before introducing the viewing focus for the video. (See the notes on the right for the rationale and suggested work.)	*This video can provoke strong reactions among viewers who sometimes find the teaching quite exciting or rather troubling. It can be helpful to allow time for general reactions to be aired so that these first impressions are given some attention. Often this preliminary activity enables viewers to then turn to the viewing focus more directly. You may want to –* • *Delay presenting the* Viewing Focus *and say that you will be showing the video through once before turning to the real focus of the work. Point out that people often have strong reactions to the teaching shown in the video and you want to provide them a little time to make general comments before returning to you work on definitions.* • *Show the video all the way through and spend a few minutes collecting comments. When you are ready to move on, you might want to mention that the teaching practice captured in the video makes the children's reasoning and ideas more visible and available for examination than might otherwise be the case. (This can be said regardless of how people feel about students' interactions and the teaching.) You can then say that the group will move on to watching for and discussing the children's ideas and reasoning.*

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Viewing focus set up and video viewing task

Setting up theViewing focus

- While watching this video, try to figure out what the children understand about what an even or an odd number is.
- What definitions – or implicit definitions – of "even number" do you hear?
- Take notes on the transcript; be prepared to give specific references to important moments in the video that shed light on the children's ideas about even and odd numbers.
- There will be time immediately after watching the video to look over and add to your notes.

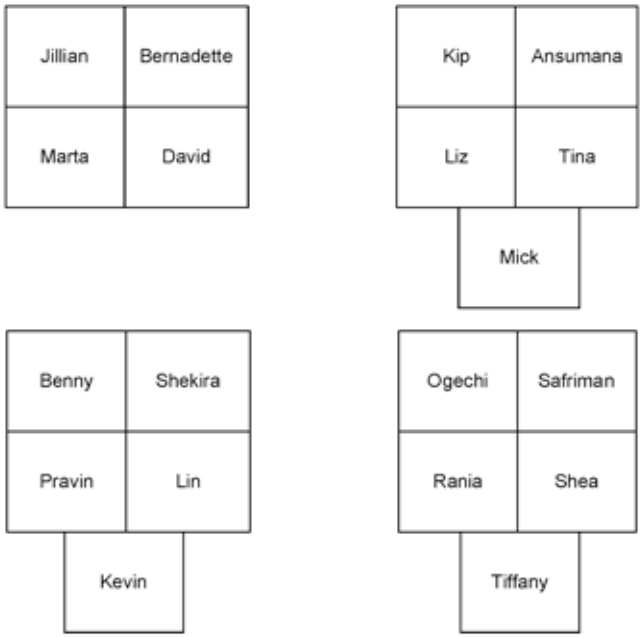


mod4

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Supporting materials for video viewing task:

Seating Arrangement



January 19, 1990. *On this day, the teacher began the class by asking students for comments on the meeting they had had the previous day where they talked about even and odd numbers with students from another class. A few minutes into the discussion, a boy named Benny made the observation that even numbers can be "made" from two other even numbers--like 4+4 or 6+6. This video segment opens with the teacher asking if anyone has other comments. She calls on Shea who has his hand raised. Shea doesn't have a comment about the meeting but he has noticed something special about the number six which he claims can be an odd and even number.*

1:05:31

1	Teacher:	More comments about the meeting? I'd really like to
2		hear from as many people as possible what comments
3		you had or reactions you had to being in that meeting
4		yesterday. Shea?
5	Shea:	Um, I don't have anything about the meeting yesterday,
6		but I was just thinking about six, that it's a. . . I'm just
7		thinking. I'm just thinking it can be an odd number, too,
8		'cause there could be two, four, six, and two, three twos,
9		that'd make six...
10	Teacher:	Uh-huh . . .
11	Shea:	And two *threes*, that it could be an odd and an *even*
12		number. Both! *Three* things to make it and there could
13		be *two* things to make it.
14	Teacher:	And the two things that you put together to make it were
15		odd, right? Three and three are each *odd*?
16	Shea:	Uh huh, and the other, the twos were even.
17	Teacher:	So you're kind of--I think Benny said then that he wasn't
18		talking about *every* even number, right, Benny? Were
19		you saying that? Some of the even numbers, like six,
20		are made up of two odds, like you just suggested.
21		Other people's comments?

1:06:36

22	Teacher:	Tina?

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Post-viewing discussion

Detailed description of activity	Comments & other resources
Taking comments **Grounded in evidence:** As teachers share their comments, consistently press them to identify where in the transcript or video they saw or heard whatever it was that led to their claim. This is an important form of rigor for making observations; it also lessens the risk of having personal beliefs/positions dominate the discourse. **Generating notes/making comments hearable:** Write brief notes in the public space about people's observations (or have one of the teachers help you); use the note-taking process as an opportunity to restate ideas clearly and concisely (have the person who made the comment assist with this if it is helpful to you/them). Also, record the transcript location (line numbers) that served as the basis for each comment. **Making connections:** As the notes are generated, ask teachers to look for connections or contradictions among the comments; you may want to briefly comment about interesting or helpful connections you see or press people to resolve apparent contradictions, refine their statements, etc. if it seems important.	*There is a notes template available for taking discussion notes that can be projected from a computer.* *See notes sample to see how the template can be used and the types of comments that often arise during this discussion.*
Examining children's definitions Three significant definitions (or ways of thinking about even numbers) that the children used in the episode should surface from this discussion: • **Fair share** – a number is even if it can be made of two equal groups with none left over, using only whole numbers) (Shea) • **Pairs** – a number is even if it can be grouped in pairs, with nothing left over) (Ogechi) • **Alternating** – even and odd numbers alternate on the number line, starting with zero being even) (Tina) If one or more of these definitions does not surface, you might consider: • Calling specifically on someone you noticed had picked up on the missing definition(s) during the individual reflection time or pair discussions • Asking about the definition or understanding of *even number* that one of the specific children used (e.g., How was Tina determining whether 6 was even or odd in lines 28-33 of the transcript?) • Asking the group if there is evidence that indicates the children had another way to tell whether a number—like 1, 2, 3, 4, or 10—is even or odd?	*You could also ask about definitions of or assumptions about odd numbers.* *If the teachers seem to cover this ground with some confidence, you might ask them to consider how they might try to reconcile the definitions. For example,* • *Are the children's definitions compatible? How could you show this?* • *If the definitions are all correct, then how could you prove that the fair share and pairs definitions are equivalent – in other words, that a number is even by one definition if and only if it is even by the other?* • *Which of these would it be easiest to show is equivalent to the alternating definition?*

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Why specify practice?

Specification allows for several things when grounding teacher learning in practice –

For teachers:

- Holds some of the complexity teaching "still" so that focused work can be done on MKT
- Enables MKT, embedded within practice, to be articulated, studied, and rehearsed

For facilitators:

- Provides detailed supports to help facilitators attend to and focus teachers' work on the special *mathematical* knowledge and skills needed for *teaching*

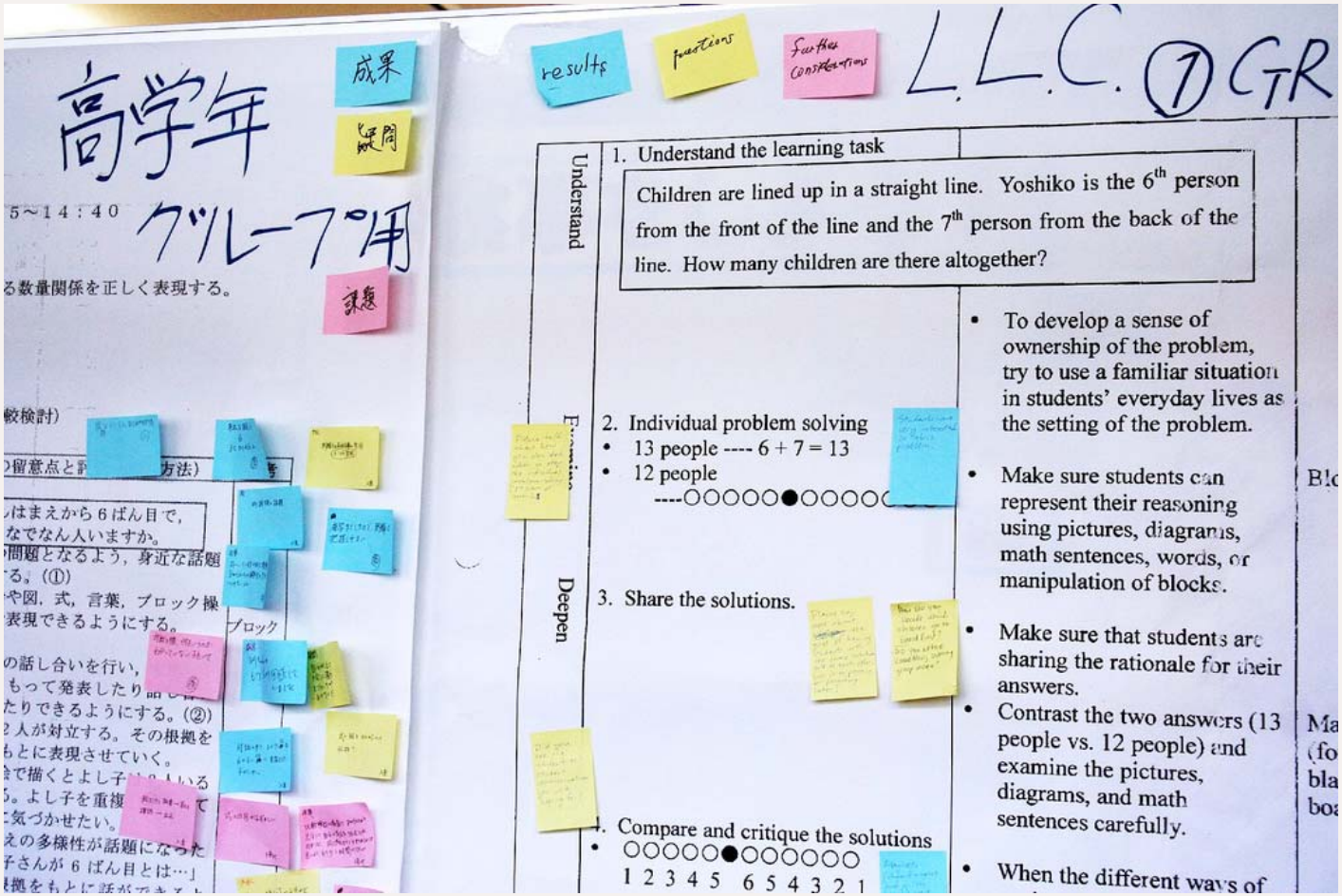
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Example 4: Japanese lesson study

Classroom teachers working on their teaching

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The Specification of Visual Representations in Mathematics Instruction: An Example from Lesson Study



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An image of lesson study



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- Learning goals
- Instructional activities (with rationale that links activity with learning goal)
- Script of teacher/instructor/facilitator talk
- Expected learner responses to the activities
- Suggested instructional responses and moves
- Hypotheses that propose future revisions
- (General) teaching practices accompanied by rationales and considerations
- Descriptions of competent practice
- "Cautious points," "Evaluation points" during the session
- **Visual representation of mathematical ideas**

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Specification of learning goals

- Interest, desire, attitude
- Mathematical thinking
- Representations and procedures
- Knowledge and understanding

- Current state of the students
- History of the topical idea for the class
- Development of the topic across grades

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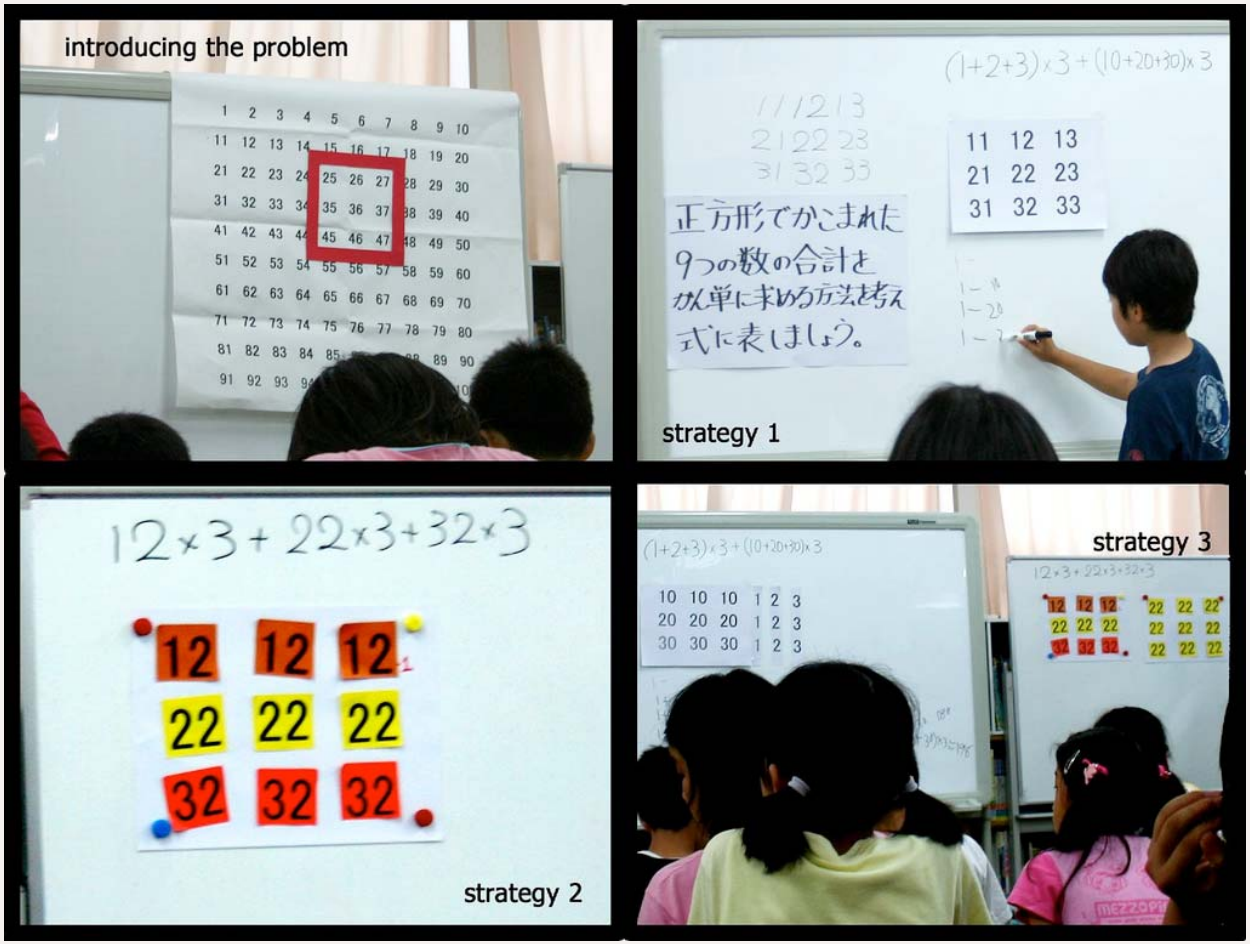
Goals of the unit	Current state of the students
⊙ Students will understand the division algorithm for dividing 2- and 3-digit numbers by 1-digit numbers and be able to use it appropriately. [Interest, Desire, Attitude] • Students will try to use their previous study of division to think about how to divide 2- and 3-digit numbers by 1-digit number. [Mathematical Thinking] • Students will be able to clearly explain that the division algorithm proceeds from the tens digit and then the ones when you are divide 2-digit numbers by 1-digit numbers. • Students will be able to clearly explain that the division algorithm for dividing 3-digit numbers by 1-digit numbers works in the same way as was learned previously (2-digit ÷ 1-digit). [Representations, Procedures] • Students will be able to accurately calculate 2- and 3-digit numbers ÷ 1-digit numbers. • Students will be able to mentally calculate 2- and 3-digit numbers ÷ 1-digit numbers with 2-digit quotients. [Knowledge, Understanding] • Students will understand how to divide 2- and 3-digit numbers by 1-digit numbers using the division algorithm. • Students will understand that they can use division to determine "times as much". • Students will understand that one math sentence may be used even when situations involve both multiplication and division or two consecutive divisions.	According to the results of the readiness test, virtually all students can accurately calculate division problems that use the basic multiplication facts once, which was studied in Grade 3. Although a few students missed remainders on some problems, all problems were correctly answered by at least 97% of the students. There were 2 problems that involve the ideas to be studied in Grade 4: (1) 40 ÷ 2, and (2) 600 ÷ 3. Although it seems like students should be able to anticipate the answers, the success rates were (1) 85% and (2) 70%. Moreover, for both problems, 9% of the students left them blank. Given the success rates on the items involving previously learned topics, we would like to plan a lesson that will take advantage of their ability to read and interpret math sentences and further extend this ability. However, in the prior unit on circles and spheres, some students had difficulty using compasses appropriately, indicating that some children still lack fine motor skills. Moreover, some students' understanding may still be rather superficial, and their computational skills are more advanced than their ability to think and reason. Therefore, we want to provide opportunities for students to enjoy solving problems by thinking carefully and manipulating objects.

This lesson plan is originally written in Japanese and translated into English by the Global Education Resources for the Lesson Study Immersion Program in Japan, June 21- July 5, 2007.

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Example of specification: Visual representation of mathematical ideas



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How are specifications created? How are they used? How are they revised? By whom?

- Assessing changes based on students' learning
- Insert new ideas by applying theory and by asking new instructors
- Structures, organizations, and names typically provided by teacher educators
- Generate instructional designs (applying and elaborating upon existing theories of teacher knowledge and upon current practices)
- Document uses of instructional designs/specifications by others, analyze the adaptations made, and revise based on these findings
- Obtain feedback from people with different backgrounds/expertise and create new or revised instructional designs/specifications based on this
- Knowledge is collectively produced by participants who create, use, revise, (and often publish) the specifications

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Why specify practice?

- For children's learning:
  - To direct learning clearly and sequentially
  - To think through all possible solution paths and make them explicit, in order to be prepared for students' individual and group work, their ideas, and their turns in discussion
- For teachers' learning:
  - To make visible the thinking of the teacher and his planning team
  - To invite observers into the deliberations about teaching this content
  - To generate a record so that participants can compare what the planners anticipated and what actually transpired with children
  - To generate and maintain a sense of collective endeavor

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What's your orientation?

With whom do you agree most?

Jill: I would really appreciate materials like these because teaching is so hard and this would help me teach well.

Judy: I would be reluctant to work with materials like these, because they do not draw enough on what I know and know to do, and also does not allow room for me to adapt to the learners with whom I am working.

Alex: I would like to work with materials like these because they would make it so much easier to make decisions about how to improve my teaching as I would be able to study closely how particular aspects actually work.

Lucy: I would like to do it if everyone else was using them too so that we could learn from each other.

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