The Role of Mathematical Language in Learning and Succeeding in Mathematics

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Session Outline

1. Our claim about mathematical language
2. Conceptual framework for mathematical language
3. Investigations of mathematical language within instruction and assessment
4. Strategies for teaching mathematical language

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Our Claim

Attending to

the **acquisition of mathematical language** is key to building **students' capacities in mathematics**,

and supporting their ability to **succeed on mathematics assessments**.

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What Do We Mean By "Language" in Mathematics?

Isn't math just numbers, symbols, formulas, graphs, and calculations, mostly bereft of text?

$x^2 - y^2 = (x + y)(x - y)$

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{4a}$$

NO!

1. Mathematics uses three forms of linguistic tools: Words, symbols, and diagrams.
2. These are <u>all</u> forms of text.
3. Doing mathematics requires reading, writing, speaking, and interpreting these.

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What is Important About Language in Mathematics?

- Necessary for mathematical communication
- Foundational to mathematical reasoning

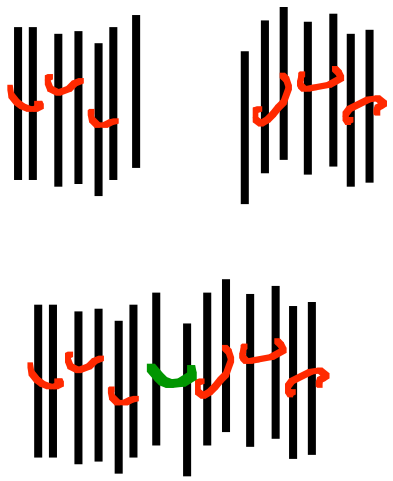
  An odd number plus an odd number equals an even number.

- Precision matters

  What is an even number?

- Words, symbols, and diagrams must map onto one another.

$$(n2 + 1) + (j2 + 1) = (n + j + 1)2$$



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Words: More Than "Story Problems"

- Mathematics uses words also used in other settings.

Examples:

  - Big, odd, even, prime, rational, radical, square
  - Right, similar, circle, regular, acute, obtuse

- The mathematical uses of these words are specialized.
- Definitions are used to specify technical uses of terms within mathematics.

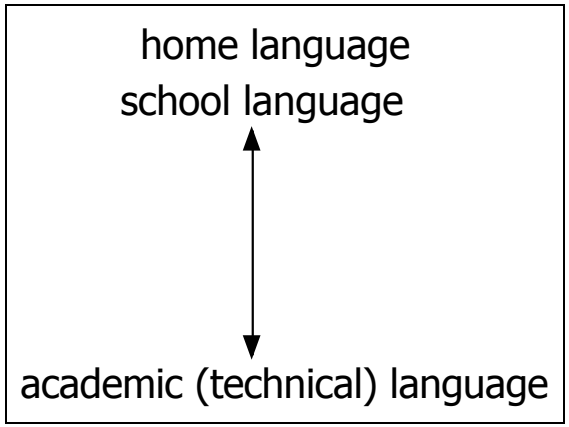
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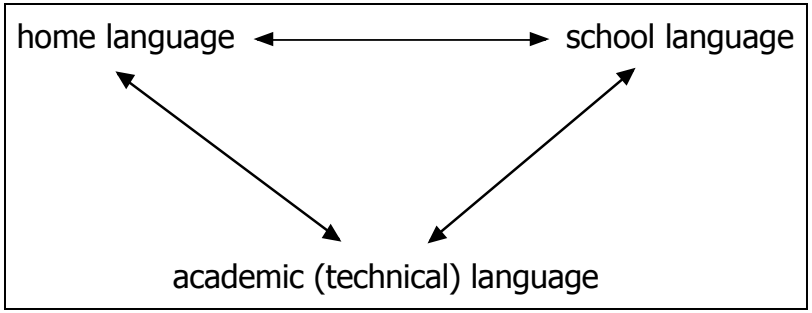
What Are These "Other Settings"? Learning Mathematics Requires Different Kinds of Linguistic Transitions

Language

- Vocabulary, grammar, syntax, talk (how language is "delivered")
- Discourse

Learners must navigate among everyday school English, their home language, and technical uses in mathematics.





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Investigation #1: Language in Instruction

CONTEXT:

- Grade 4 lesson episode; multi-lingual and multi-cultural class
- mid-May
- Culture of reasoning and discussion
- Task:

CLIP 1: What are the possible numbers that the spinner could land on when spinning it?

CLIP 2: What are the possible outcomes or sums that we could get by adding two spins together?

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Focus Questions

1. WORDS

What mathematical words do you notice in the video segment?

- Used in other settings/unique to mathematics
- Used precisely/used generally

2. TEACHER'S ROLE

What do you see the teacher doing to support students' use of mathematical language?

- Explicitly
- Implicitly
- Nothing

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Investigation #2: Language in Instruction

CONTEXT:

- Grade 3 lesson episode; multi-lingual and multi-cultural class
- mid-May
- Culture of reasoning and discussion
- Task:

Which is more -- 4/4 or 4/8?

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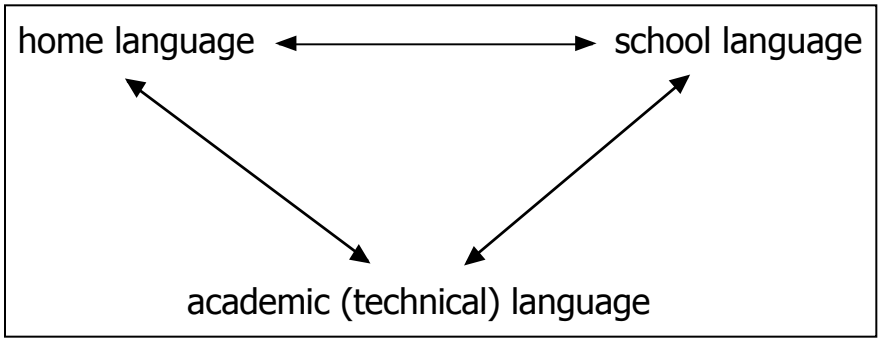
Investigation #3: Language in Assessments

- Succeeding on assessments requires reading.
- But it also requires skills and knowledge of language beyond reading.
- What are the language demands of interpreting and answering assessment items?

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Focal Questions

- What are the language demands of interpreting and answering the item?
- Which of these demands are mathematical? (content language requirements)
- Which of these demands are academic? (school language requirements)
- Which of these demands reflect confusions in the item's language? (item flaws)



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Fifth Grade, Fall 2005 State Assessment

#17

What is the value of this expression?

420 ÷ 4

A) 15
B) 100
C) 105
D) 150

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Same Assessment, item 24

#24

Which tool can be used to measure the dimensions of a shoe box?

A) 100-gram balance
B) 12-inch ruler
C) digital thermometer
D) 8-ounce measuring cup

1. What are the language demands of interpreting and answering the item?
2. Which of these demands are mathematical? (content language requirements)
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Same Assessment, item 46

#46

Which of the following is true of equilateral triangles?

A) One of the angles measures 90 degrees.

B) All of the angles measure 60 degrees.

C) Two of the angles measure less than 60 degrees.

D) Two of the angles measure greater than 60 degrees.

1. What are the language demands of interpreting and answering the item?
2. Which of these demands are mathematical? (content language requirements)
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Strategies for Teaching Mathematical Language

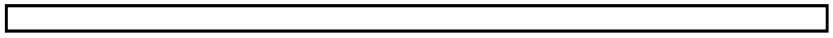
- Introduction in context; natural use; highlighting
- Explicitly pointing out special use of a term in mathematics
- Defining a mathematical term directly; degree of explicitness dependent on the nature of the term and who the student is
- Developing definitions together (even number example last year)

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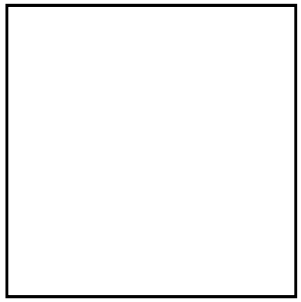
What is a rectangle?

Which of these shapes is a rectangle?

A



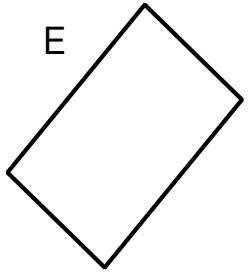
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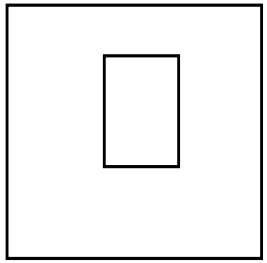
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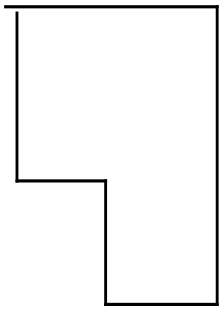
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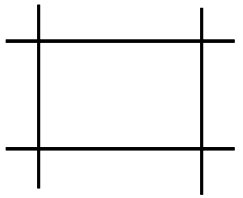
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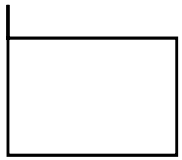
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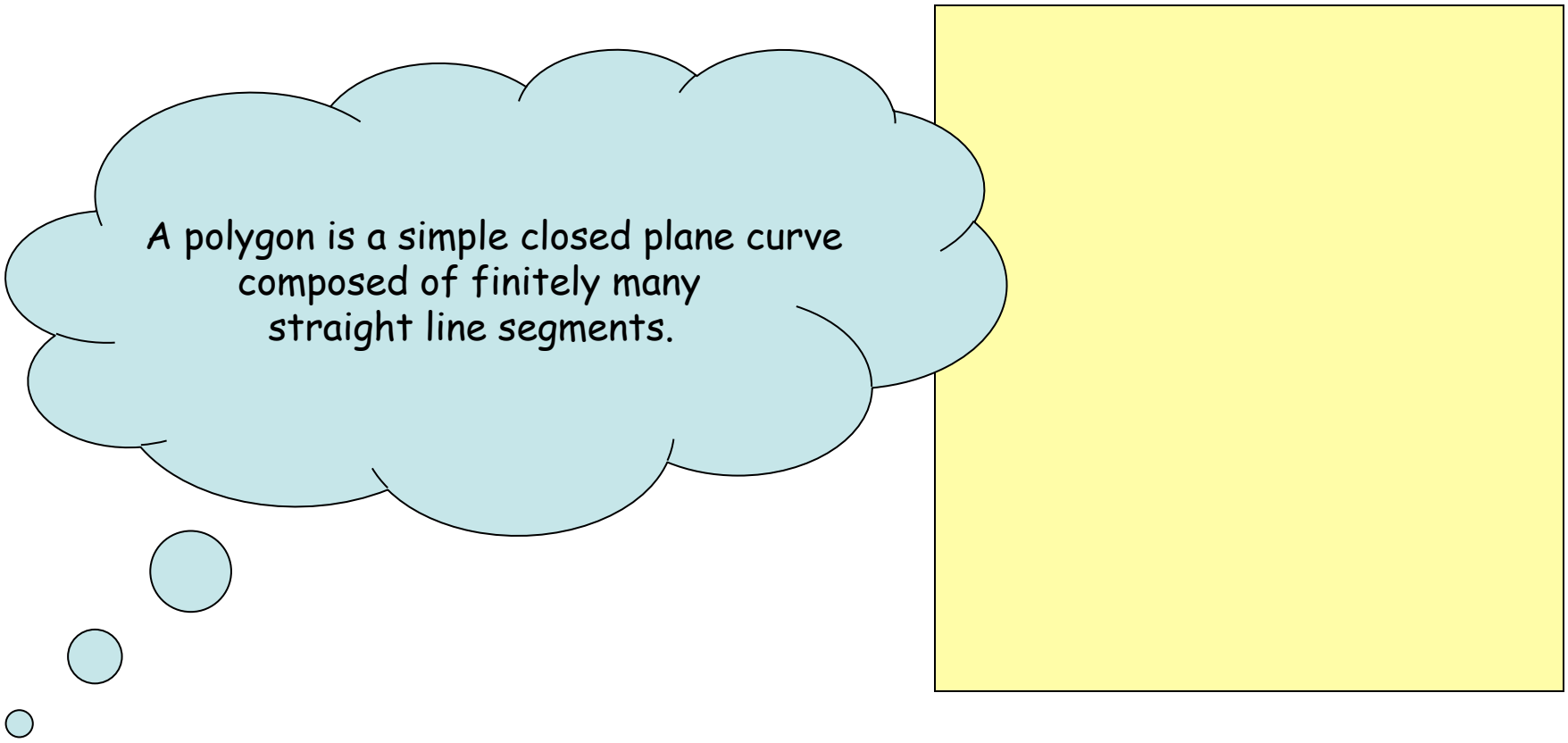


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Helping Students Develop a Definition of "Polygon"

Which of these shapes are polygons?

A polygon is a simple closed plane curve composed of finitely many straight line segments.



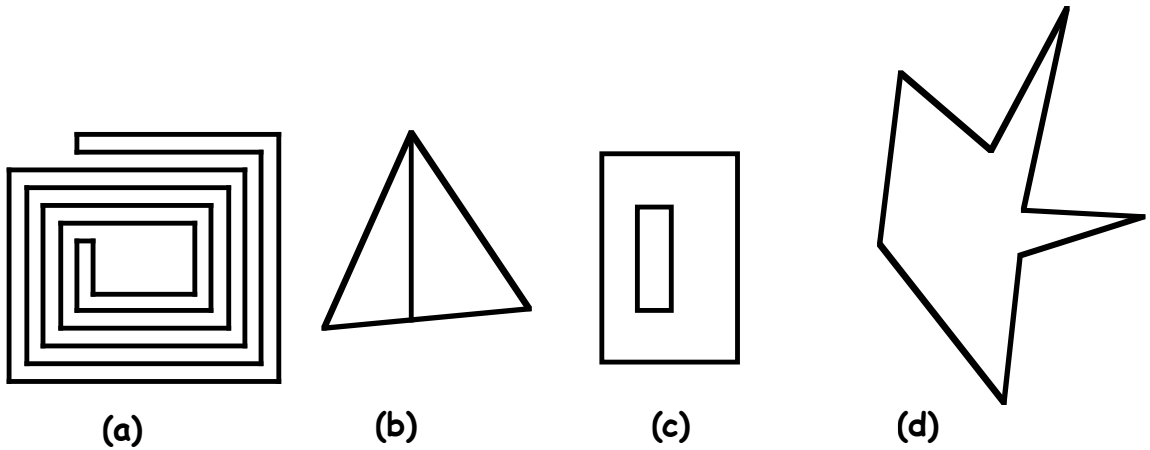
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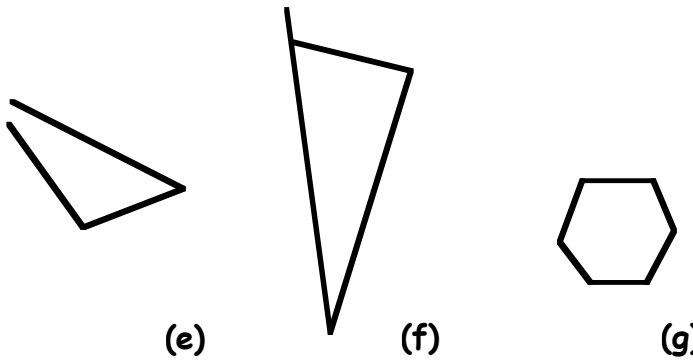
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Possible Examples: What is the Reason for Each?





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Considerations: Which Strategies to Use with Which Terms?

- Will working on the definition be useful for other mathematical goals?
- What challenges might the term create for students?
- Is the term one that warrants detailed definition?

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Conclusions

1. Attending to the acquisition of mathematical language is key to building students' mathematical capacities, and can help support their success on mathematics assessments.
2. Professional education should provide explicit opportunities for teachers to learn about and develop fluency and skill with
   - Mathematical language and what it provides for mathematical work
   - Strategies for teaching mathematical language
   - Examining assessments for their linguistic demands

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