




# Using Mathematics Homework with an Eye on Equity and on Mathematical Integrity


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**National Council of Teachers of Mathematics**

**San Diego, CA • April 23, 2010**

SCHOOL OF EDUCATION UNIVERSITY OF MICHIGAN


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# Homework problems

1. Students don't do it.
2. Students have trouble with it and their efforts only reinforce misunderstandings.
3. Students' out-of-school schedules are overflowing.
4. Students have widely varying access to resources to support homework.
5. Difficult to create homework that is mathematically valuable and still doable.



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# Overview of session

1. Solving problems of practice: The Elementary Mathematics Laboratory
2. Teaching students "to be people who study" (Lampert)
3. Homework
4. Studying for a test



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# The Elementary Mathematics Laboratory (EML)

A University of Michigan summer program for students who are struggling in mathematics

- Focused on the development of —
  - Mathematical proficiency: Fractions, basic arithmetic skills, mathematical explanation and reasoning, and the use of language and representations in math
  - Study skills: participating productively in class; homework; preparing for a test

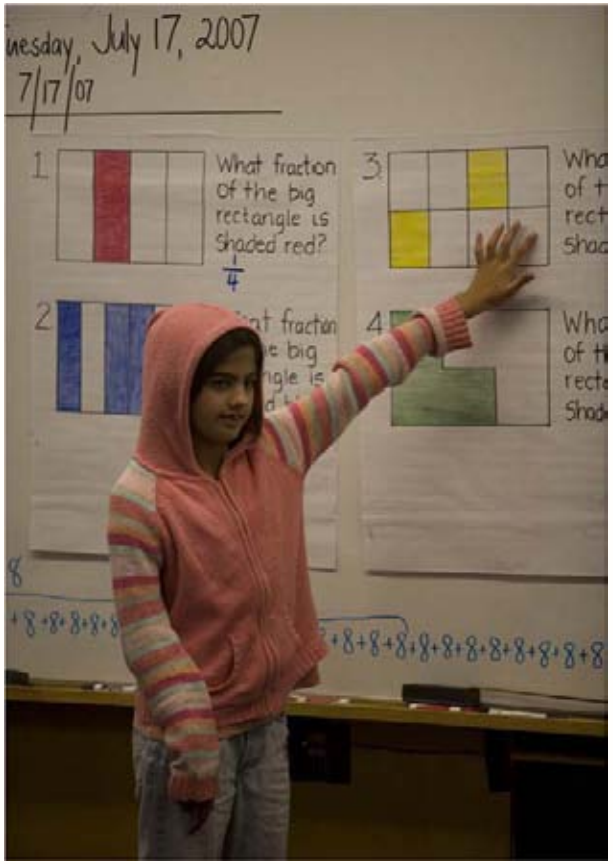




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# The urgency

1. Enormous gaps in learning opportunities and disparities in achievement (within U.S. and in international comparisons)
2. Rapidly changing school population
3. Higher, more complex academic goals
4. High expectations for all students






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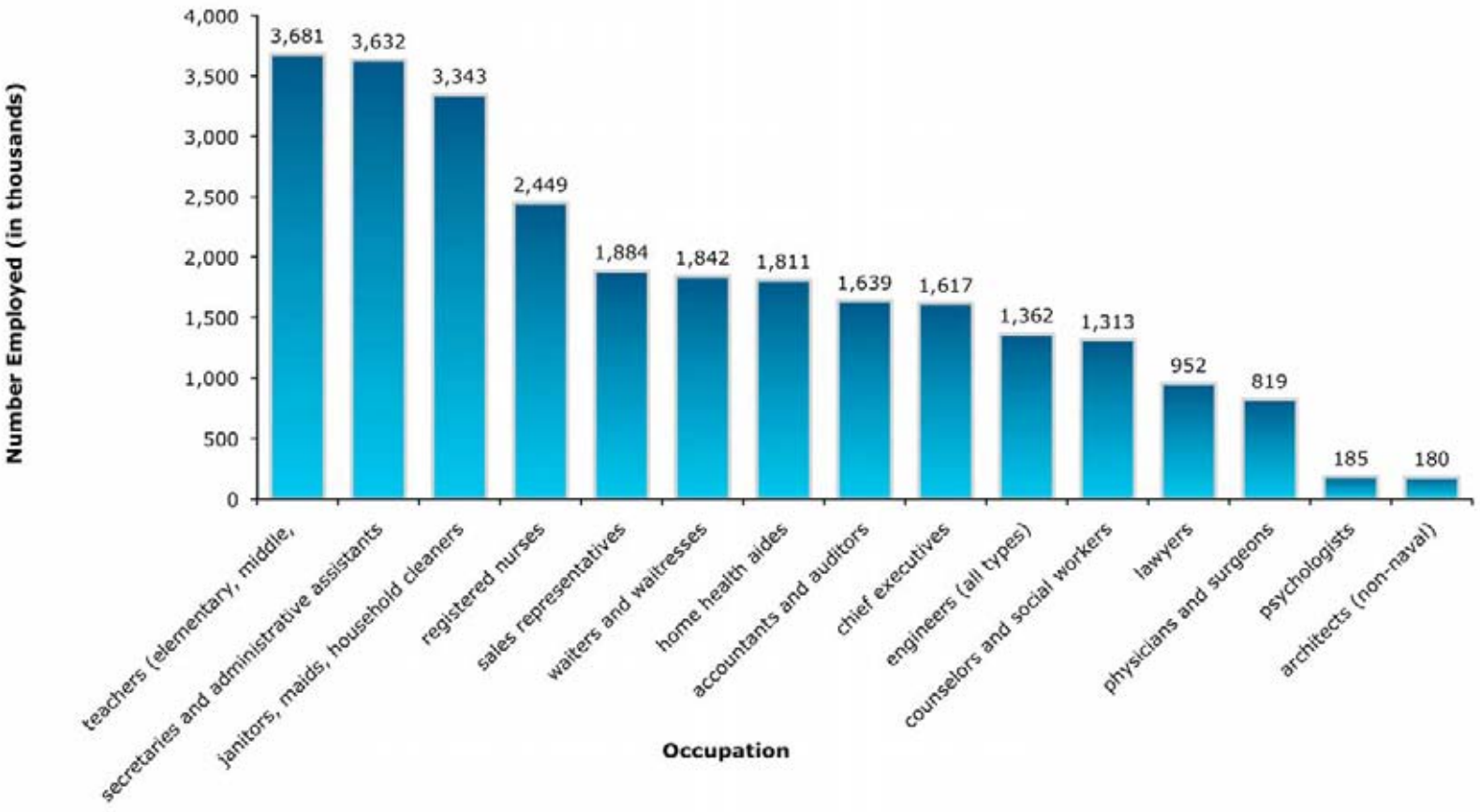
# Teachers matter — a lot

- Persistent evidence that a large proportion of the variability in student achievement gains is due to the teacher
- So one obvious strategy for improving students' opportunities and learning is to ensure that they have teachers who are able to help them learn



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# Teaching is the largest single occupation in the U.S.






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# Why is it called a "laboratory"?

*Laboratory:*

A setting for real-time studies of the interplay of instructional design, teaching, and learning



*Features of the laboratory:*

- Teaching that is public and designed to be open for collective study
- Careful documentation
- Planned instructional experiments



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# Principal research questions: Students

- "Turnaround" instruction: Combining remedial work with enhanced academic extension and challenge
- How can high academic expectations and performance be achieved in a diverse classroom for every student?
- How can homework be better designed and used to enhance students' learning mathematics and learning to learn?
- How can the teaching and learning of fractions, strategic mathematical thinking, and mathematical reasoning be improved?



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# Principal research questions: Professionals

- Mathematical knowledge for teaching: What is it and how can teachers and teacher developers acquire it?
- How can a "live" laboratory setting be designed and used to improve the training of education professionals?
- How can teachers develop skills with high-leverage practices of instruction?
- How can the educators of teachers be better prepared for their work?



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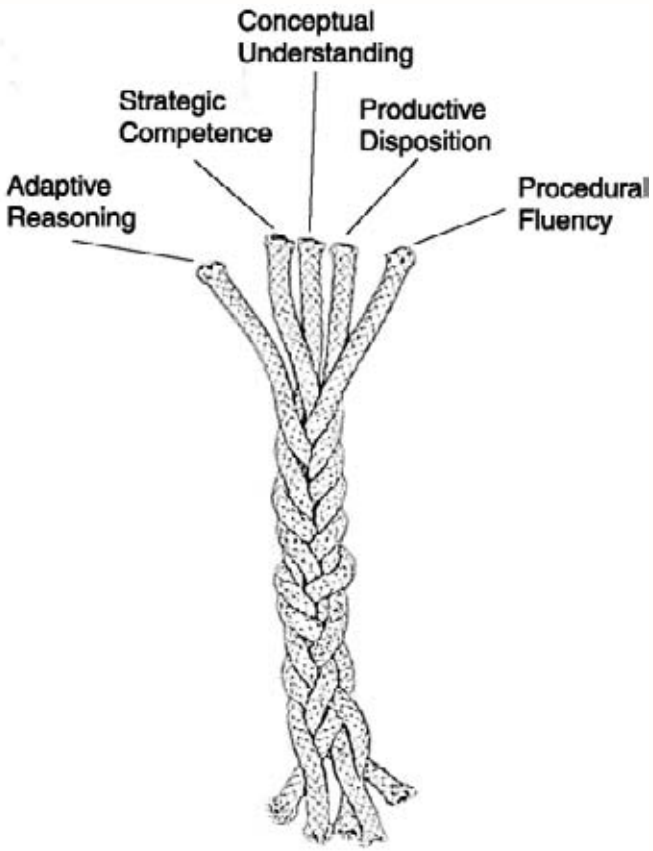
# Purposes for homework

1. As “between class” work to support and extend learning.
2. To offer structured independent work.
3. To develop study skills, learning how to learn and study mathematics, and productive disposition.



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# Strands of mathematical proficiency




- ***Conceptual understanding*** - comprehension of mathematical concepts, operations, and relations
- ***Procedural fluency*** - skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- ***Strategic competence*** - ability to formulate, represent, and solve mathematical problems
- ***Adaptive reasoning*** - capacity for logical thought, reflection, explanation, and justification
- ***Productive disposition*** - habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

Kilpatrick, J., J. Swafford, and B. Findell. (2001). *Adding It Up: How Children Learn Mathematics*. Washington, DC: National Academy Press.



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# Equitable instruction: Twin imperatives of responsibility and responsiveness

- Broaden what it means to be good at mathematics
- Make mathematical practices explicit
- Coordinating among mathematical, school, and students' language
- Using contexts with sensitivity to students' funds of knowledge and to the integrity of the mathematics
- Creating a respectful mathematically-focused learning environment



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# Scaffolding #1:
# How to do homework successfully

1. Explicit expectations and routines
2. Managing the variability in students' out-of-school lives
3. Homework "kits" with supplies
4. Three different kinds of problems, explicitly labeled
5. Differentiating the mathematical demand of out-of-school and in-school tasks



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# Student contract

I understand that my ability to succeed in the Elementary Mathematics Laboratory class depends on my commitments and effort this summer. The following are the commitments asked of me to participate in the special class:

1. I will attend class every day.

2. I will keep neat detailed records of my mathematical thinking in my math notebook.

3. I will complete my homework carefully each night.

4. I will seek help or advice from my teacher or parents/caregivers if I have questions or concerns.

5. I will give my best effort to all the work I complete for this class.

I have read, understand and will follow through with the statements in this contract to the best of my ability.

Print student's full name: _____

Student signature:_____
Date:_____



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# Teacher contract with

## Student's name

I understand that your ability to succeed in the Elementary Mathematics Laboratory class depends on my commitments and effort this summer. The following are the commitments I make to you as your teacher in this special class:

1. I will attend our class every day.
2. I will come to class prepared to teach interesting and appropriate material to you.
3. I will expect you to do your very best work all the time and I will help you learn to do so.
4. I will make sure that you understand the mathematics and problems we work on.
5. I will answer any questions or concerns that you may have about this class or our work.
6. I will keep careful records of my mathematical thinking and ideas.

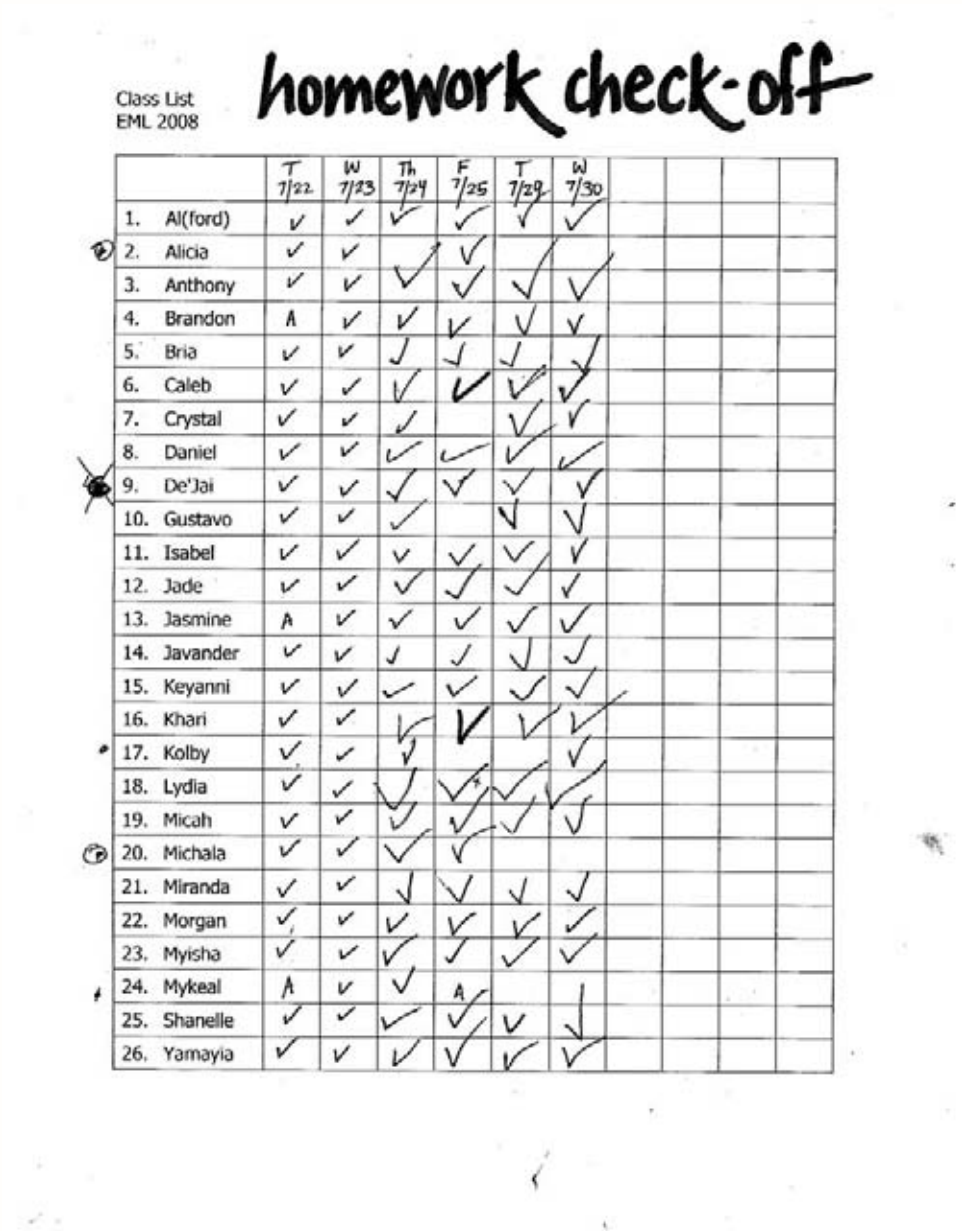
I have read, understand and will follow through with the statements in this contract to the best of my ability.

Print teacher's full name: _____

Teacher's signature:_____ Date:_____



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Class List
EML 2008

# homework check-off

| | T 7/22 | W 7/23 | Th 7/24 | F 7/25 | T 7/29 | W 7/30 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1. Al(ford) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | |
| 2. Alicia | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | |
| 3. Anthony | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | |
| 4. Brandon | A | ✓ | ✓ | ✓ | ✓ | ✓ | | | | |
| 5. Bria | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | |
| 6. Caleb | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | |
| 7. Crystal | ✓ | ✓ | ✓ | | ✓ | ✓ | | | | |
| 8. Daniel | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | |
| 9. De'Jai | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | |
| 10. Gustavo | ✓ | ✓ | ✓ | | ✓ | ✓ | | | | |
| 11. Isabel | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | |
| 12. Jade | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | |
| 13. Jasmine | A | ✓ | ✓ | ✓ | ✓ | ✓ | | | | |
| 14. Javander | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | |
| 15. Keyanni | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | |
| 16. Khari | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | |
| 17. Kolby | ✓ | ✓ | ✓ | | | ✓ | | | | |
| 18. Lydia | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | |
| 19. Micah | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | |
| 20. Michala | ✓ | ✓ | ✓ | ✓ | | | | | | |
| 21. Miranda | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | |
| 22. Morgan | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | |
| 23. Myisha | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | |
| 24. Mykeal | A | ✓ | ✓ | A | | | | | | |
| 25. Shanelle | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | |
| 26. Yamayia | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | | | |



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# Three types of homework tasks

1. **Independent practice**: Skill development and reinforcement, developing students' ability to reflect on their own learning
2. **Preparing for new work**: Anticipating, growing tolerance for difficult work (productive disposition)
3. **Share with others**: Home-school communication, development of students' ability to narrate their work



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# Differentiating the mathematical demand of in-class and homework tasks: How are they similar and different?

## SIMILAR

- Involve the use of mathematical skills as well as concepts
- Routinely require explanation and reasoning
- May have more than one solution



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# Analyzing others' solutions

A fourth grade student did these problems:

$$\begin{array}{r} 48 \\ +\ 27 \\ \hline 615 \end{array}$$

$$29 + 37 + 18 = 84$$

Is each answer correct or incorrect?

How do you know?

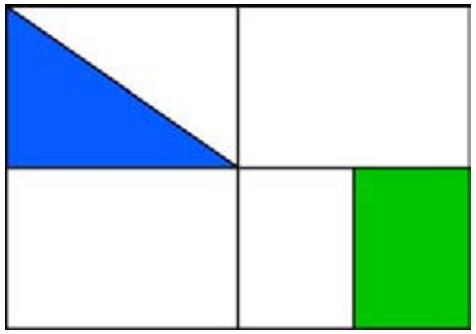
If the answer is incorrect, try to explain what the error is.

If the answer is incorrect, please do it correctly here:



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# Reasoning with representations



What fraction of the large rectangle is shaded blue?

What fraction of the large rectangle is shaded green?

What fraction of the large rectangle is shaded green?

Explain your answers.



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# Reasoning with tools



Which Cuisenaire rod is three times as long as a red one?

Which rod is half as long as an orange one?

One rod is one-fourth as long as another rod. What colors might they be?



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# Differentiating in-class and homework tasks

## HOMEWORK

- Can be done independently
- As unambiguous as possible
- Structures the workspace more explicitly

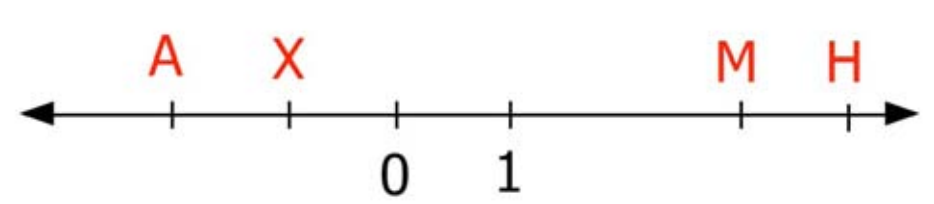
## IN-CLASS

- Profits from exchange with others
- Requires interpretation
- Requires more decisions about representation and recording
- Likely to yield multiple solution methods or solutions



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# Homework




Is A less than or more than H? How do you know?

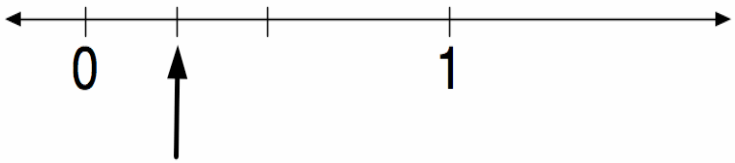
Is A less than or more than X? How do you know?

Label what numbers go at A, X, M, and H. Then, explain how you know what M is.



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# In-class




What number can be written at the mark where the blue arrow is pointing?

Circle your answer:

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{3}$ 1



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# Examining a case of homework

1. How are explicit expectations and routines communicated?
2. How is the variability in students' out-of-school lives managed?
3. What is the nature of the three different kinds of problems, explicitly labeled?
4. What do you notice about the mathematical demand of the homework tasks?
5. What "homework problems" do you see? How might these "homework problems" be further solved?



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# Scaffolding #2:
# How to study for a math test

1. Explicit support for what to do
2. Access to help from home
3. Problems that are like the ones on the test
4. Hints
5. Answer key with explanations



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**STUDY GUIDE**
**Wednesday, July 30, 2008**

*There will be a test in math class tomorrow, on Thursday, July 31. This is a study guide to help you prepare for the test. The questions on the test will be similar to the questions on the study guide.*

*You should try to answer all of the questions on the study guide. After you have answered the questions, you should check your answers using the answer key at the end of the study guide. If you get stuck, there are hints in the "Are you stuck?" boxes. You may also get help from your family and friends. If you have any questions about a problem or need help, you can also call me (phone number: 734-333-5555) or send an email message (email address: tea@ooo.us). There will also be time for some questions on Thursday morning in class.*



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****How to study for a math test**

*When you study for a math test, it is important to study smartly. Here are steps to use when you study for a math test using a study guide:*

a) Use the study guide to see what is important to know. Make a list of the things you know well and things you need to work on so that when you do the practice problems you are paying special attention to the things you need to work on.

b) Read the practice problems carefully. Try each problem. After you solve each problem, make sure you can explain your answer.

c) Check your answers and mark them right or wrong.

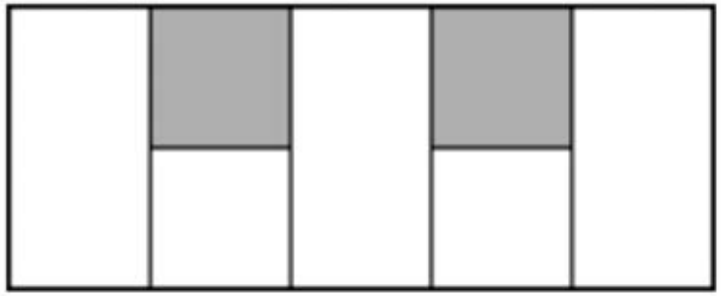
d) If you get an answer wrong, try to figure out what you did wrong:
Did you read the problem incorrectly?
Did you get messed up on the conditions?
Did you just make a careless mistake counting or writing or calculating?

e) Try again to do the problems you got wrong. If you still can't do the problem correctly, ask someone at home or mark it on your study guide so you can ask about it at school.



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2. What fraction of the big rectangle is shaded gray? Explain your answer.



**Are you stuck?**

Try to answer the following questions:

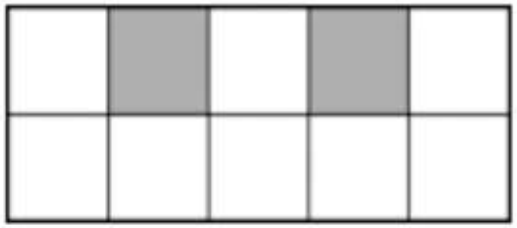
- What is the whole?
- Is the whole divided into equal sized parts? If not, divide the whole into equal sized parts.
- How many equal sized parts are there?
- How many equal sized parts are shaded gray?



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2. Answer: $\frac{2}{10}$ of the rectangle is shaded gray. (You might also say it is $\frac{1}{5}$.)

Explanation: The whole is the biggest rectangle. The parts are not equal. Add lines to make the parts equal:



Now there are ten equal parts. Two of the parts are shaded.



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# Conclusions

1. Homework, understood as work "between class days" is an important part of students' learning opportunities.
2. In order to manage potential problems of inequity and mathematical distortion or misunderstanding, homework can be designed and used in special ways.
3. Explicit expectations, routines, and design, are important.
4. Other strategies include three different types of tasks and clear differentiation of the mathematical demand of in-school and homework tasks.
5. Next steps: Ways to build or adapt new forms of homework; better supports for English language learners; experimenting with other types of tasks.



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# Dates for 2010

Ann Arbor, Michigan: July 26 – August 6, 2010

**Contact**: eml2010@umich.edu



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# Opportunities
## for mathematics teachers and teacher educators

### Dev-TE@M Fall 2010 Pilot Seeking Piloters

*Representing and Comparing Fractions in Elementary Mathematics Teaching*

- Ten 1.5-hour sessions
- Multimedia materials
- Seminar/webinar format (face-to-face or real-time remote)
- Elementary mathematics

http://www.umich.edu/~devteam/

### Professional Development Workshops at the 2010 EML

1) Mathematical knowledge for teaching teachers
2) High leverage practices of teaching mathematics
3) "Public" teaching for teacher leaders, coaches

http://sitemaker.umich.edu/eml2010



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