Practicing the Common Core: What is the Work of Teaching?

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What is so important about the Common Core State Standards?

1. That they are common. This is the first step in one of the most important differences between the U.S. and other systems.
2. That they give such central content role to mathematical practices. This takes another step from the NCTM "process standards"* and the strands of mathematical proficiency in *Adding It Up*.

*Principles and Standards for School Mathematics, 2000; National Research Council, 2001.

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The mathematical practices (CCSS)

MP.1. Make sense of problems and persevere in solving them.

MP.2. Reason abstractly and quantitatively.

MP.3. Construct viable arguments and critique the reasoning of others.

MP.4. Model with mathematics.

MP.5. Use appropriate tools strategically.

MP.6. Attend to precision.

MP.7. Look for and make use of structure.

MP.8. Look for and express regularity in repeated reasoning.

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What are some resources for implementing this more robust view of mathematics?

1. A history of efforts to incorporate some version of mathematical practices: Process standards of NCTM; Strands of mathematical proficiency in *Adding It Up*.
2. Extensive research literature on certain practices – problem solving; reasoning and proving; modeling.
3. English language arts (at the elementary level), particularly reading and writing, have long been dominated by a focus on practices. This presents a domain of school learning in which teachers have successfully made practices central to instruction.

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What are obstacles to making the Common Core "common"?

1. Textbooks and assessments are not aligned with the CCSS.
2. Misperception that mathematical practices are processes, or the means to learning the "real content."
3. Some people may not think that all students are capable of engaging in mathematical practice.
4. Teachers' lack of opportunity to learn.

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View video clip

Diverse class, over half are ELLs

Mid-year, in a unit on even and odd numbers

Focal question for viewing:

What mathematical practices do you see the students using or learning to use?

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Video available at, http://hdl.handle.net/2027.42/65013

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Viewing focus

What mathematical practices do you see the students using or learning to use?

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Mathematical practices in use

- Understand the approaches of others (3)
- Continually ask oneself, "Does this make sense?" (1)
- Understand and use stated assumptions, definitions, and previously established results in constructing arguments. (5)
- Make conjectures and build a logical progression to explore their validity, (2, 3)
- Consider the available tools (representations) when solving a mathematical problem. (5)
- Abstract a given situation, represent it, manipulate the representation. (2)
- Construct viable arguments and critique the reasoning of others. (3)
- Examine claims and make explicit use of definitions. (3, 6)
- Determine (generalize) domains to which an argument applies. (2, 8)
- Attend to precision (use clear definitions). (6)
- Give carefully formulated explanations to others. (3)
- Look for and make use of structure. (7)
- Evaluate the reasonableness of results. (3)

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"Practice" in school mathematics

1. School mathematics has always been about "practice," but it has been more about *drill and practice*, not *the practice of mathematics*.
2. The Common Core State Standards offer the possibility of re-orienting school mathematics around a more robust conception of mathematical competence.
3. The mathematical practice standards support the advancement of equity in mathematics instruction.

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But . . . any change depends on skillful teaching

1. Understanding mathematical practices for teaching (developing new Mathematical Knowledge for Teaching)
2. Developing an eye for opportunities to worj on mathematical practices
3. Intertwining mathematical practices with mathematical concepts, procedures, and language

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Learning to teach mathematical practice

1. Establishing norms and routines that support engagement in mathematical practice in school
2. Making mathematical practices explicit and learnable
3. Assessing students' learning of mathematical practices

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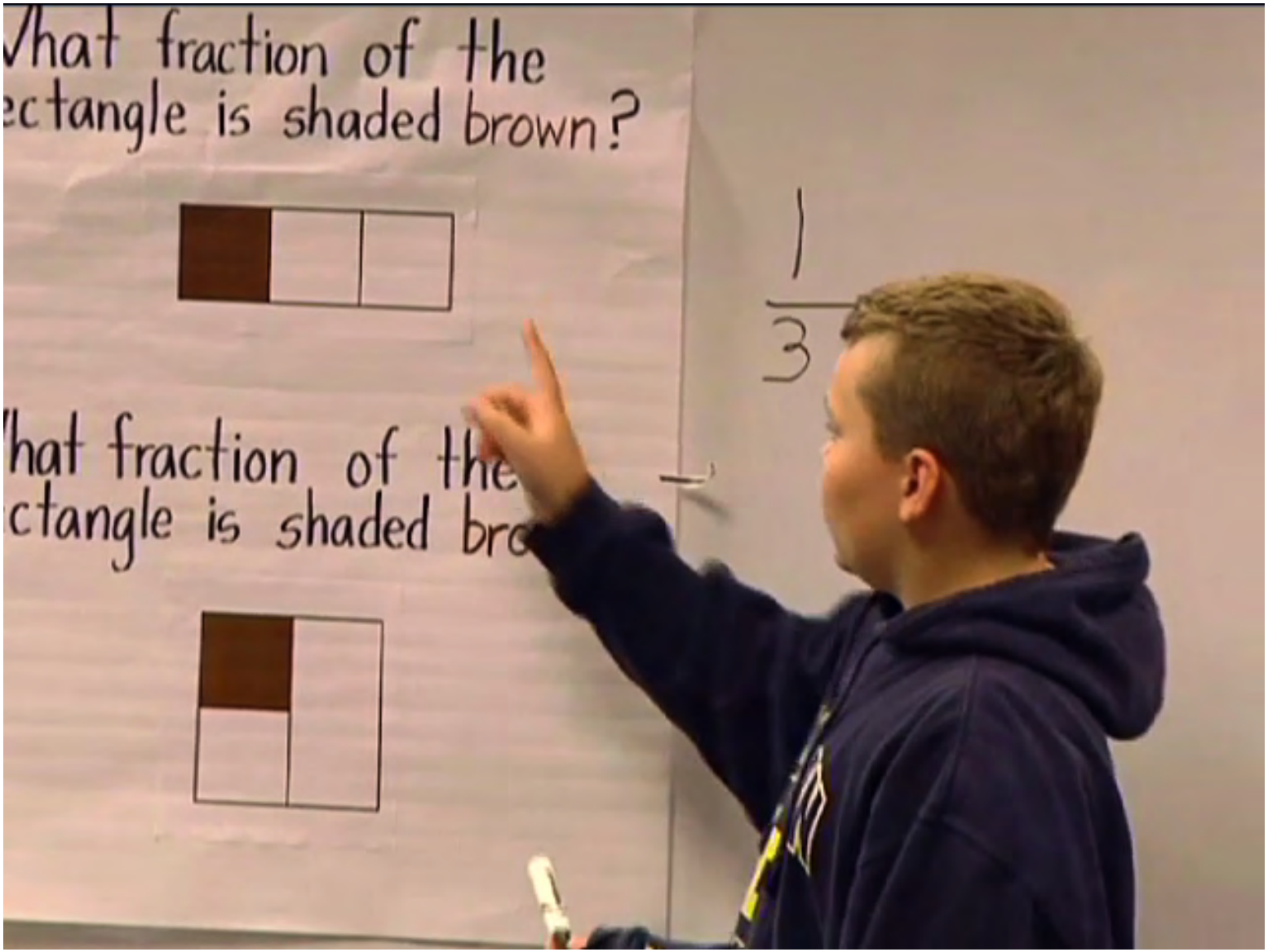
① Establishing norms and routines that support engagement in mathematical practice in school

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Viewing focus

- *What norms are being established that can support work on mathematical practices?*
- *How are these norms being established?*
- *Is there evidence that children are learning these norms?*

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Norms to support work on mathematical practices

- Listening closely and respectfully to others' ideas
- Interpreting and responding to others' ideas
- Expecting explanation, and questioning ideas
- Speaking so others can hear
- Using board space, posters, smartboards and other representational resources in ways that facilitate attention to important mathematical details and substance

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② Making mathematical practices explicit and learnable

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Mathematical practice must be taught, not just requested

Examples:

- Explaining one's thinking
- Producing a mathematical argument
- Critiquing a mathematical argument
- Interpreting problems
- Using tools

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Making the practice of mathematical explanation learnable

- Scaffolding
- Modeling
- Explicitly appraising explanations
- Making principles of good mathematical explanations explicit

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Warm up problem

- How many different three-digit numbers can you make using the digits 1, 2, and 3, and using each digit only once.
- Show all the three-digit numbers that you found.
- How do you know that you found them all?

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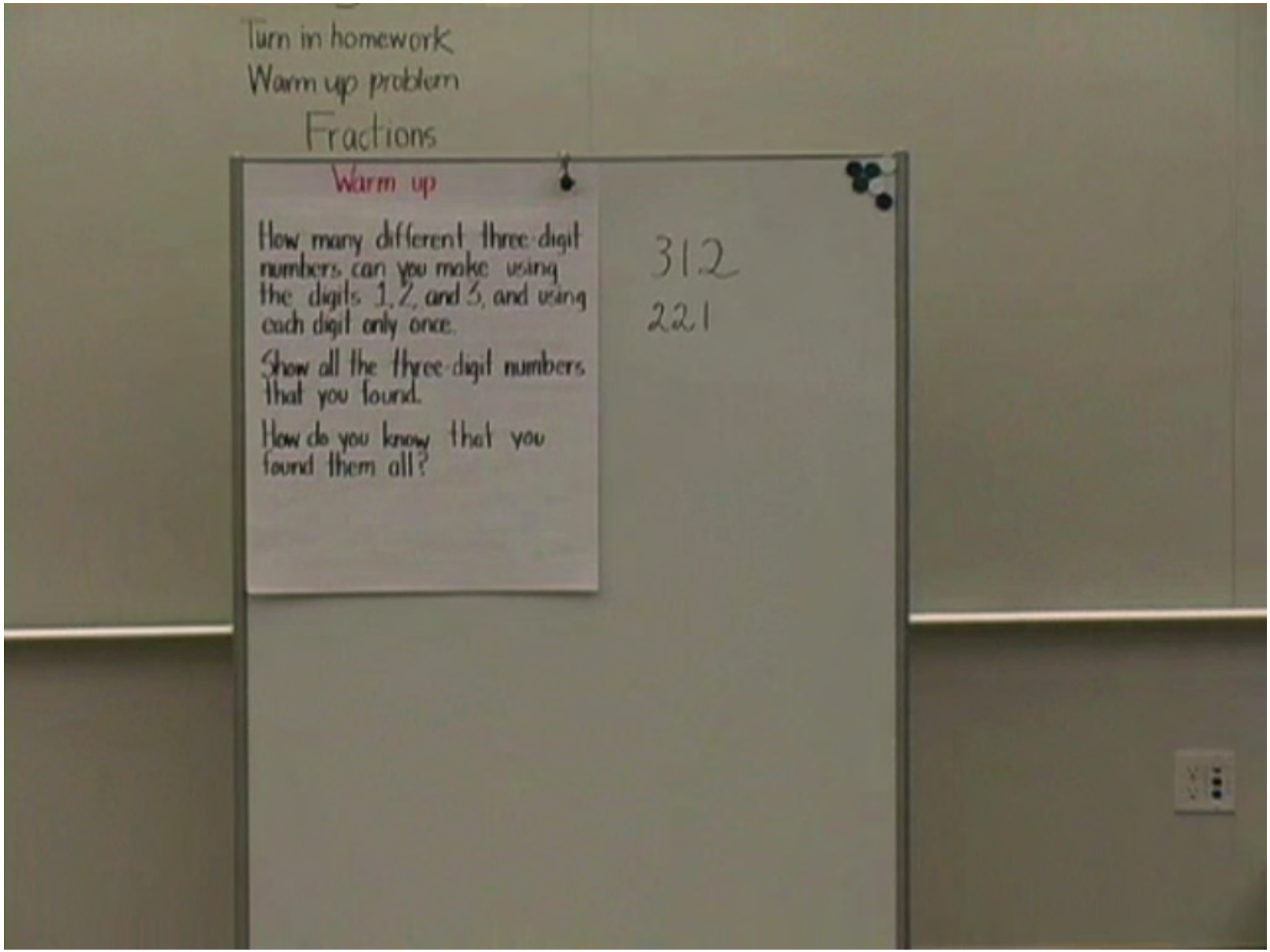
Viewing focus

How is the practice of mathematical explanation being scaffolded?

How is it being supported?

How might it be better supported?

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Unpacking mathematical explanation

Has a clear purpose

- Makes clear at the outset what is being explained and why you start there; and carefully connects the explanation to the question or idea being explained
- States what is known and what needs to be determined

Has a logical structure

- Summarizes at the end what has been explained, and links that back to the original question, claim, or problem

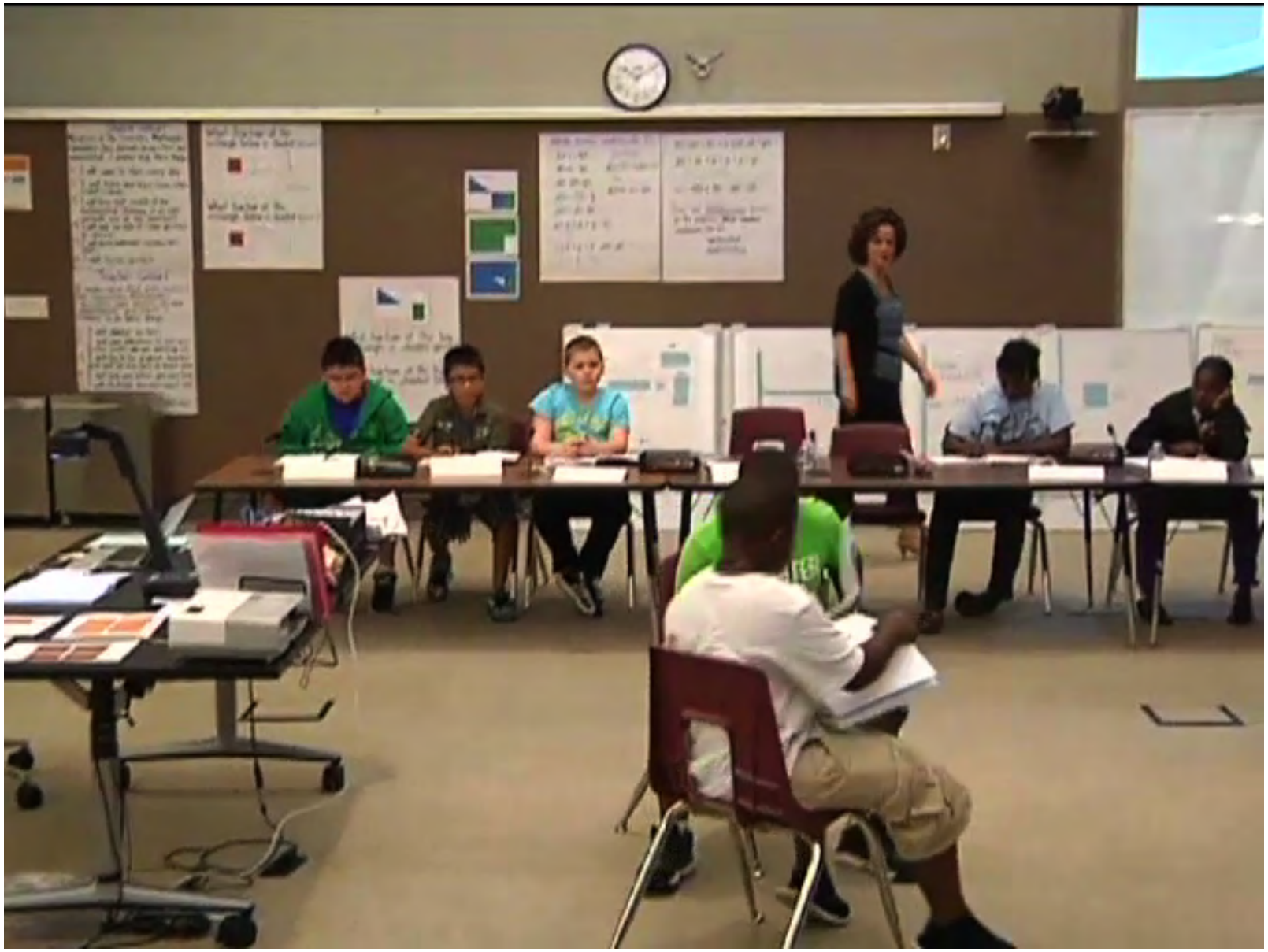
Uses representations and language clearly and carefully

- Strives to be as simple and clear as possible
- Uses mathematical language accurately and consistently
- Uses representation(s) accurately

Focuses on meaning and is geared to the learner

- Shows what something means or why is true, and is convincing to the person to whom you are explaining
- Takes into account the background knowledge of the listener/reader
- Uses words that will be understood by the listener/reader
- Breaks things down – does not assumes the listener/readers knows what you are thinking

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③ Assessing students' learning of mathematical practices

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Monitoring children's development of mathematical practices

1. End-of-class checks
2. Questioning in class

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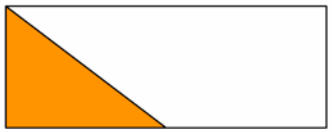
Features of good "end-of-class checks" (aka "exit tickets")

1. Minimizes copying; consider "paste-in" checks.
2. Is easy for every student to begin and complete.
3. Includes content-focused and reflective questions.
4. Is easy for the teacher to "eyeball" quickly for completion and correctness.

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End-of-class check #1

1. Write a number sentence for 10 that has more than two terms to the left of the equals sign.

2. What fraction of the rectangle is shaded orange? 

3. Look at the Student Contract on the wall. Do you think you are doing a good job of doing the things that are on the contract? Give two examples of things you did well today that are on the Student Contract.

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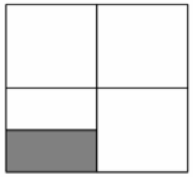
End-of-class check #2

1. Put a number in the blank to make the number sentence <u>true</u>.

   6+ 5 = ____ + 2

   Explain your answer:

2. What fraction of the big square is shaded gray?

   Explain your answer:

3. What is one thing you did today that helped you learn math?

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Question shells (extending and determining "stretch")

1. How would you explain that to a ___ [younger grade] student?
2. Are any other answers possible?
3. Can you think of a situation where that wouldn't be true?
4. Would it still be true if _____?
5. Is that always true?
6. How can we test it?
7. Can we come up with a general rule?
8. What other ways can we represent the solution?
9. Can you use this same method to solve _____?
10. What would happen if the numbers were changed to _____?
11. What if the problem was like this instead: [give slight variation of problem]?
12. If someone said [wrong answer], how would you respond?

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Learning to teach the CCSS mathematical practices

1. Developing appreciation that mathematical practices are **basic to the actual content**, not just part of the means, or something to do once the "basics" are mastered.
2. Developing conviction that **all students can**—and must—develop proficiency with mathematical practices.
3. Developing **mathematical knowledge for teaching (MKT)** with mathematical practices.

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Core instructional capabilities

1. Making the practices explicit and learnable
2. Establishing norms and routines that support mathematical practice
3. Assessing students' capabilities with mathematical practices

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THANK YOU!

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