Mathematics and Education: Collaboration in Practice

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Conference in Honor of Ted Eisenberg

Beer-Sheba, Israel • 3 May 2012

SCHOOL OF EDUCATION UNIVERSITY OF MICHIGAN

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Overview

1. Our premise, and the context of our collaboration
2. Our laboratory and our approach to the study of mathematics instructional practice
3. Two claims
4. Conclusions

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Our premise

- Children are not learning mathematics nearly as well as they could (and should).
- Improving children's learning of mathematics depends on expertise both in mathematics as field and expertise in instruction.
- Addressing these issues is our driving purpose; what we have learned about collaboration is a by-product of our actual work together.

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Our collaboration

- Started in 1994, with the question from DB to HB: "What mathematics do you see in this episode of teaching?"
- Reason for the question: Trying to tackle the problem of the mathematical resources needed for the work of teaching

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Situating our work in practice: Our current research laboratory, and our other approaches

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The Elementary Mathematics Laboratory (EML)

A University of Michigan summer program for students who have not been successful in school mathematics

- Focused on the development of —
  - Mathematical proficiency: Fractions, basic arithmetic skills, mathematical explanation and reasoning, and the use of language and representations in math
  - Study skills: participating productively in class; homework; preparing for a test



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Why is it called a "laboratory"?

Laboratory:

A setting for real-time studies of the interplay of instructional design, teaching, and learning



Features of the laboratory:

- Teaching that is public and designed to be open for collective study
- Careful documentation (high quality audio and video, notes, all student and teacher artifacts)
- Deliberate practice of making teaching and learning more visible
- Planned instructional experiments

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Research questions

- What are some of the key sites of using mathematical knowledge in and for teaching (MKT)?
- What constitutes "knowledge at the mathematical horizon" and how is it entailed by and used in practice?
- What mathematical problems are central to beginning teaching, and how might this inform high-leverage initial professional training in mathematics instruction?
- What are some of the challenges and affordances of using the number line as a central mathematical object in the teaching and learning of rational numbers in elementary school?
- What dilemmas are inherent in seeking to help students develop basic skills in active mathematical practice?
- What are the mathematically-intensive instructional resources that are highest leverage to advancing children's mathematics learning and engagement, and how are they more effectively used?
- What is involved in "turnaround" instruction: Making "remedial" instruction accelerated
- What are the problems and challenges of designing and using homework in ways that advance children's academic development and that are equitable and sensitive to home-school connections?

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Other key research sites

- Records of practice from other settings and classrooms
- Studies of beginning teachers of mathematics (elementary and secondary)

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We will make two claims to unpack what we have learned *through* — and *about* — collaboration.

Claim #1: Mathematics in instruction is importantly different from mathematics considered in the abstract, or in curriculum.

Claim #2: Mathematics instruction is fundamentally a special kind of mathematical practice.

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CLAIM #1:
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What mathematics do you see in this problem?

How many different three-digit numbers can you make using the digits 1, 2, and 3, and using each digit only once.

Show all the three-digit numbers that you found.

How do you know that you found them all?

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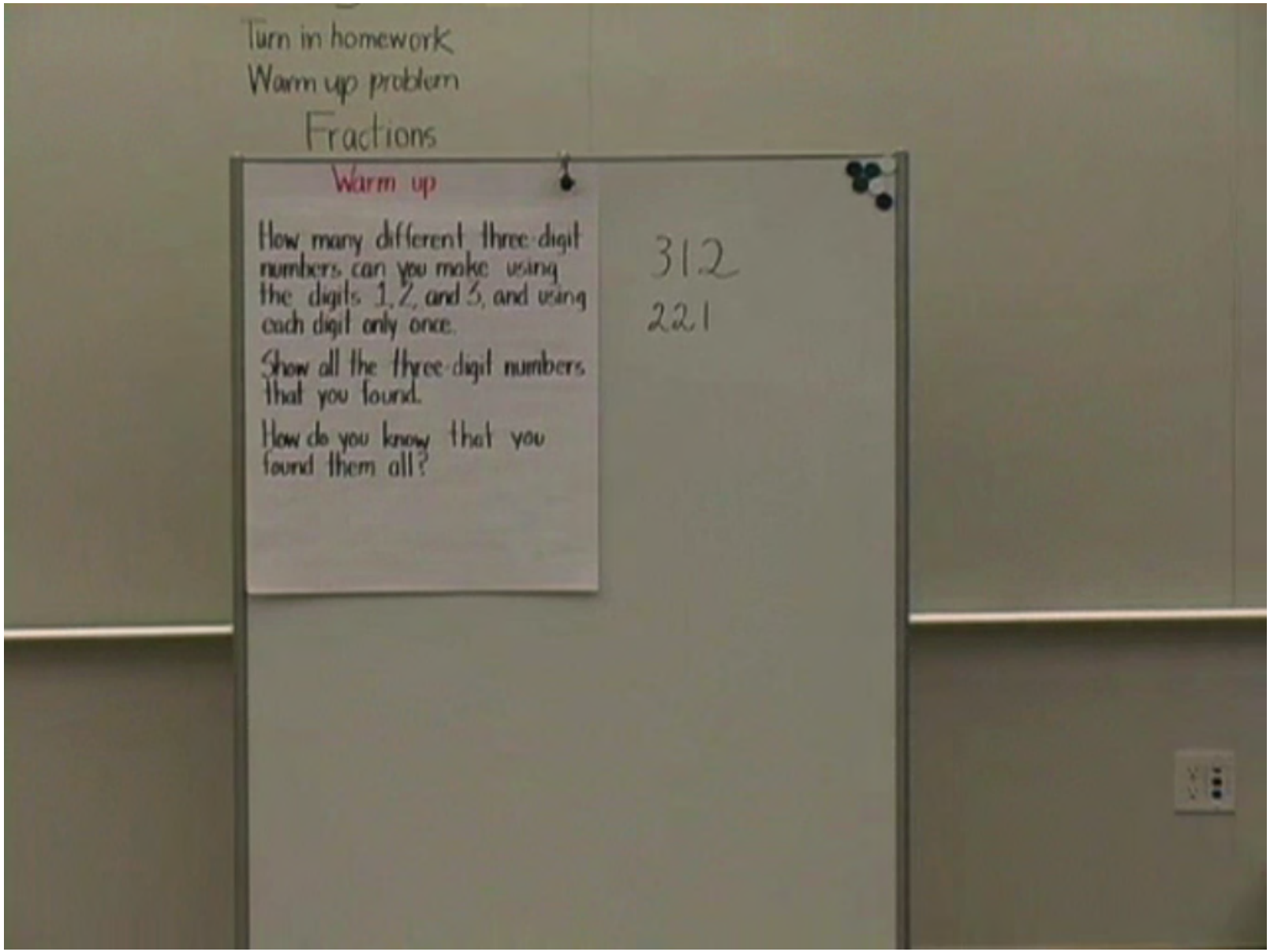
What mathematics do you see in this problem?

- Permutations of 3 objects
- Finding all solutions
- Proving completeness of the solution set
- Other?

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Now: What mathematics do you see in this segment of *instruction* with this problem?

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What mathematics do you see in this segment of instruction?

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What mathematics do you see in this segment of instruction?

- Interpreting problems
- Permutations of 3 objects
- Determining what constitutes an answer or a solution to the problem
- Proving the completeness of the solution set
- Producing complete mathematical explanations
- Mathematical language: conditions, digits
- Concept of zero as a number
- Other?

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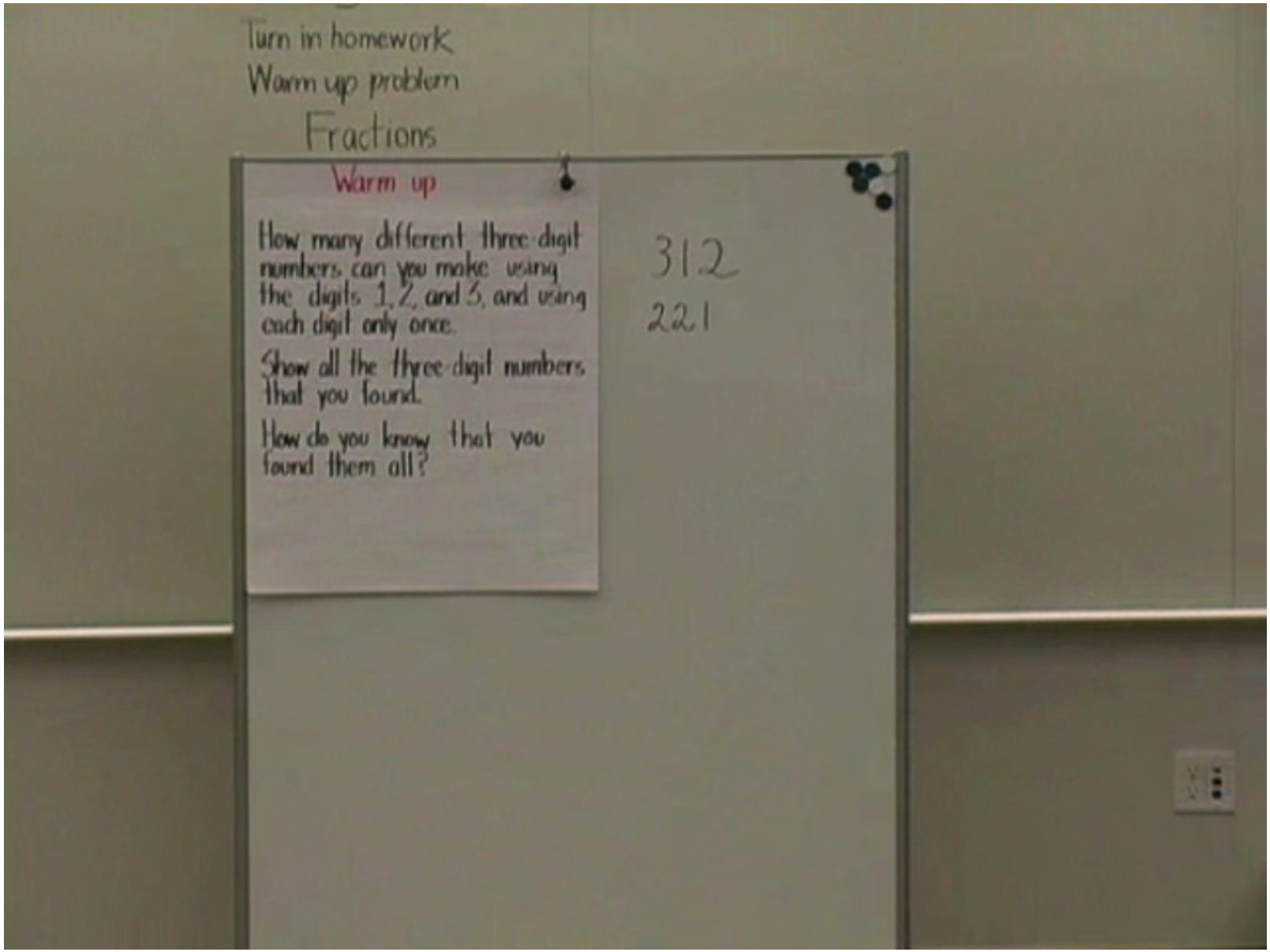
Watch the video again

What *instruction* do you see?

Name specific things the teacher is doing, or did before the lesson.

What is the work of teaching this mathematics?

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What instruction do you see?

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What instruction do you see?

- Displaying the problem in a clear visible place
- Structuring time and norms for children to work on problem for a few minutes before discussing it
- Posing pedagogically and mathematically purposeful and strategic questions in ways that children can understand them, e.g.:
  - Asking child to read the problem
  - Asking child to interpret "what the problem is asking"
  - Asking what would *not* be an answer to the problem, and why
  - Asking what *would be* an answer to the problem
  - Asking children to explain others' proposed answers
  - Asking children to evaluate and critique proposed solutions
- Developing norms of listening to others' mathematical ideas
- Introducing and scaffolding mathematical language (e.g., conditions, digit)
- Distributing turns of talk purposefully
- Writing so children can see
- Others?

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What does this experience show about claim #1?

1. There is far more mathematics visible in watching it in action than one can see from the curriculum alone.
2. The students' opportunities to learn mathematics depend on the mathematical transactions that occur during instruction.
3. The more we examined the mathematics instruction, the more we uncovered about the content, the learning, the work of building bridges between the mathematics and the children.

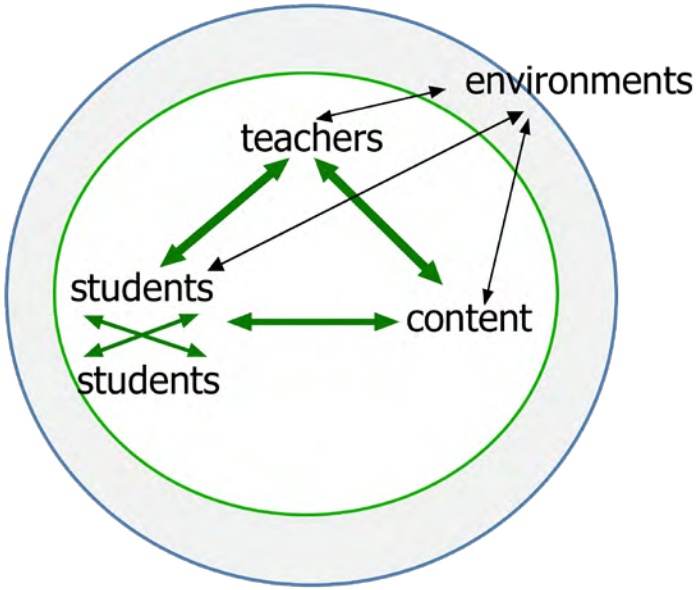
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What is "mathematics instruction"?

It is not:

- What teachers do
- The cause of students' learning
- Equivalent to curriculum

Teaching is what is *co-produced* by students and teachers in contexts, around specific mathematics and curriculum.



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CLAIM #2:
Mathematics instruction is fundamentally a special kind of mathematical practice.

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View video clip

Diverse class, over half are ELLs

Mid-year, in a unit on even and odd numbers

Focal question for viewing:

What mathematical practices do you see the teacher or the students using or learning to use?

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Video available at, http://hdl.handle.net/2027.42/65013

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What mathematical practices do you see the teacher and the students using or learning to use?

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Mathematical practices in use

- Understand the approaches of others
- Continually ask oneself, "Does this make sense?"
- Understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- Make conjectures and build a logical progression to explore their validity,
- Consider the available tools (representations) when solving a mathematical problem.
- Abstract a given situation, represent it, manipulate the representation.
- Construct viable arguments and critique the reasoning of others.
- Examine claims and make explicit use of definitions.
- Determine (generalize) domains to which an argument applies.
- Attend to precision (use clear definitions).
- Give carefully formulated explanations to others.
- Look for and make use of structure.
- Evaluate the reasonableness of results.

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The two-fold basis of "practice" in school mathematics

1. Teaching involves the practice of mathematics (asking mathematical questions, using representations and models, explaining and proving, evaluating arguments, using mathematical structure).
2. The mathematical goals for children's learning are also that they become proficient in the practice of mathematics (reasoning, making and critiquing arguments, interpreting problems, generalizing, using models and representations, using mathematical language).

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Some mathematical practices that stood out

- Explaining and justifying mathematics
- The role of definitions even in early mathematics learning
- Seeing, and generalizing mathematical structure
- Close listening to others' mathematical ideas and claims, and learning to evaluate them

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CONCLUSIONS

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What have we learned?

1. Improving mathematics learning depends on intertwining deep expertise in the practice of both mathematics and instruction.
2. These connections can be built strongly when collaborations engage *in practice* — through direct engagement in instruction, through artifacts that can be discussed, studied, and re-examined over and over.
3. Cross-cultural work is involved — about what the key questions are, about what counts as a claim, what counts as evidence — and mutual respect.

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THANK YOU

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Slides will be available on Deborah Ball's website Google "Deborah Ball"

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