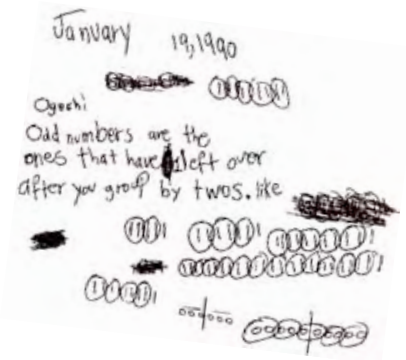
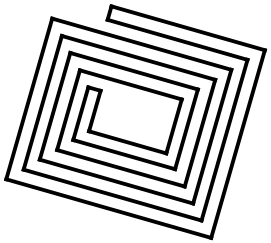
Knowing Mathematics for Teaching

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University of Michigan





Albert Shanker Institute Forum

05/05/05

Length of side of main square

Possible sized squares	1	2	3	4	5	6	7	10	50
1x1	1	4	9	16	25	36	49		
2x2		1	4	9	16	25	36		
3x3			1	4	9	16	25		
4x4				1	4	9	16		
5x5					1	4	9		
6x6						1	4		
7x7							1		
8x8									
9x9									
10x10									
Total	1	5	14	30	55	91	140		



$$\begin{array}{r} 28 \\ \times\ 6 \\ \hline 368 \end{array}$$

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Clarifying the Problem

Teacher quality

Teachers' knowledge of mathematics

and their ability to use it in teaching

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Common Framing of the Problem
→ Failure to Solve the Problem

- Teachers do not know mathematics well enough (evidence and causes vary)
- Concentration of under-qualified teachers in urban and high poverty schools

- Increase number and rigor of mathematics requirements for teacher certification
- Recruit mathematically-trained people into teaching

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A Practice-Based Approach to Answering the Question

1. Study instruction, and identify the mathematical work of teaching
2. Analyze what mathematical knowledge is needed to do that work effectively, and how it must be understood to be useful for the work
3. Develop, test, and refine measures of mathematical knowledge for teaching using multiple methods as a means to improve theory

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Mathematical Knowledge for Teaching

What do we mean when we use this term, "**mathematical knowledge for teaching**"?

- Mathematical knowledge, skill, habits of mind that are entailed by the work of teaching

What do we mean by the "**work of teaching**"?

- The tasks in which teachers engage, and the responsibilities they have, to teach mathematics, both inside and outside of the classroom

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What *is* "Mathematical Knowledge for Teaching"? An Example from Multiplication of Decimals

Multiply:

$$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline \end{array}$$

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Analyzing Incorrect Answers for $\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline \end{array}$

Why do you multiply before you add?

(a)
$$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline 255 \\ 80 \\ \hline 10.55 \end{array}$$

(b)
$$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline 62.5 \end{array}$$

Why do you have as many decimal places in the answer as the total number of decimal places in the problem?

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Analyzing Correct Answers for $\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline \end{array}$

(a)

$$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline .25 \\ 1.5 \\ 1 \\ 6 \\ \hline 8.75 \end{array}$$

(b)

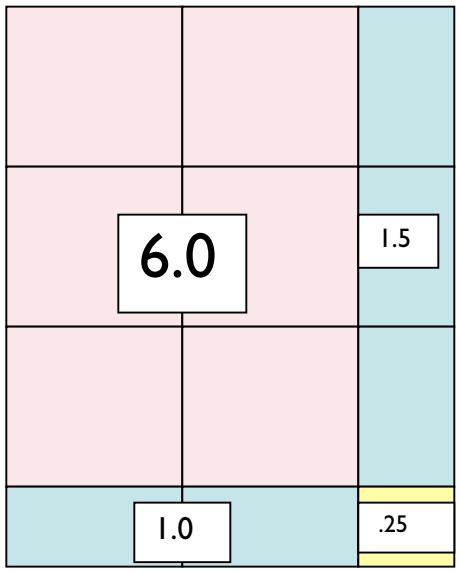
$$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline 1.25 \\ 7.5 \\ \hline 8.75 \end{array}$$

Is there a method?
Does it work for
all decimal numbers?
How do you know?

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Using Representations for $\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline \end{array}$

(a)



(b)



Explain:
Where is the 3.5?
Where is the 2.5?
Where is 8.75?

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Hypotheses About Knowledge of Mathematics for Teaching

PREVALENT HYPOTHESES

1. Teachers need knowledge of the curriculum, plus N levels more knowledge
2. Pedagogical content knowledge, curricular knowledge

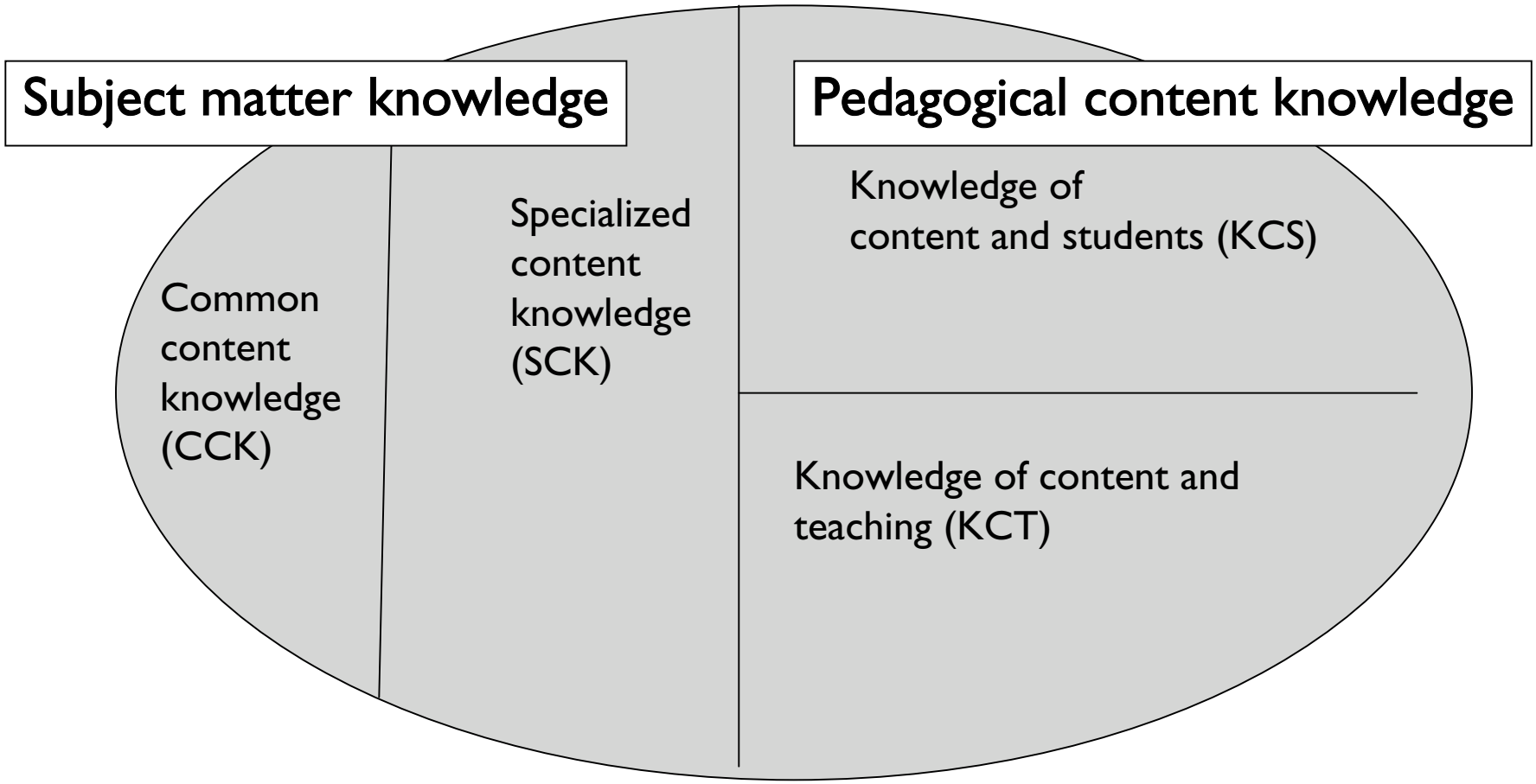
OUR CURRENT HYPOTHESES

- **Common** content knowledge
- **Specialized** content knowledge
- Knowledge of content and **students**
- Knowledge of content and **teaching and curriculum**

What does this have to do with "pedagogical content knowledge"?

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Shulman's Original Category Scheme (1985) Compared with Ours



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Opportunity: Study of Instructional Improvement

- Study of three Comprehensive School Reforms; teacher knowledge a key variable
- Instrument development goals:
  - Usable with 5000 teachers: survey, multiple choice
  - Measure content knowledge teachers *use* in teaching – not just what they *know*
  - Differentiate among teachers
  - Non-partisan

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Sample Item: Operations SCK

Student A	Student B	Student C
35 x25 125 +75 875	35 x25 175 +700 875	35 x25 25 150 100 +600 875

Which of these shows a method that could be used to multiply any two whole numbers?

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Using Data to Test and Improve Theory

- Factor analyses
- Analyses of validity
- Uses of measures
  - To predict student achievement
  - To evaluate professional development

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Factor Analysis

- Enables analyses of the number of underlying factors in a data set
- Can link items to constructs
- Ask similar question of our data:
  - How many factors? What are they named?
  - How do items group?

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Overarching Findings: Factor Analyses

- Multidimensionality of mathematical knowledge for teaching
  - Knowledge of content and students (KCS) different from content knowledge
  - Patterns, functions & algebra content knowledge for teaching
  - Geometry mathematical knowledge for teaching
  - Number and and operations mathematical knowledge for teaching
    - Specialized
    - Common
- Tentative conclusion: existence of professional knowledge of subject matter for teaching

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Validating Our Measures

How do we want to be able to interpret teachers' performance on our questions?

1. Scores capture teachers' mathematical knowledge
   - Cognitive interviews
2. Higher scores are related to higher-quality mathematics instruction
   - Videotape validation study
3. Higher scores are related to improved student learning
   - Study of Instructional Improvement student gains analysis
4. Scores reflect different dimensions of "content knowledge for teaching mathematics"
   - Mathematician and non-teacher interviews
   - Item response theory and factor analysis

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Linking Teacher Knowledge and Student Achievement

Question: Do students learn more when teachers score better on our measures?

- Questionnaire consisting of 30 items (scale reliability .88)
- Model: Student Terra Nova gains predicted by:
  - Student descriptors (family SES, absence rate)
  - Teacher characteristics (math methods/content, content knowledge)
- Teacher content knowledge significant
  - Small effect (< 1/10 standard deviation): 2 - 3 weeks of instruction
  - But student SES is also about the same size effect on achievement

(Hill,Rowan, and Ball, AERJ, in press)

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Conclusion:
Knowing Mathematics for Teaching

- Mathematical knowledge for teaching is the knowledge needed for the work of teaching. That work entails specialized content knowledge distinct from that required by other mathematically-intensive professions.
- Figuring out what teachers need to know requires multiple kinds of work, including research-based evidence.
- It is possible to write questions that reliably and validly measure mathematical knowledge for teaching. The work of producing usable items requires multiple kinds of expertise, a good conceptual frame, and many cycles of revision and improvement.
- Such questions are more likely to seem credible to teachers.
- Work on these questions is central to the improvement of mathematics education.



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Slides will be available at

http://www-personal.umich.edu/~dball/

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