Critical Issues in Mathematics Education Workshop

Teaching and Learning of Algebra

14 - 16 May 2008

Mathematical Sciences Research Institute • Berkeley, CA

Workshop sponsored by
Mathematical Sciences Research Institute, National Science Foundation,
Math for America, Noyce Foundation

© 2008 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Core Questions for this Workshop

1. What are some organizing principles around which one can create a coherent pre-college algebra curriculum?
2. What is known about effective ways for students to make the transition from arithmetic to algebra?
3. What algebraic understandings are essential for success in beginning collegiate mathematics?

© 2008 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Critical Issues in Mathematics Education Workshop

(Teaching and) Learning of Algebra

14 - 16 May 2008

Mathematical Sciences Research Institute • Berkeley, CA

Workshop sponsored by
Mathematical Sciences Research Institute, National Science Foundation,
Math for America, Noyce Foundation

© 2008 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

In search of teaching . . .

"Don't forget the middle 'man'" (Ed Silver)

© 2008 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Problem:
Teaching often remains invisible

- Limited and misleading language (classroom processes, development, facilitation, no word for the fundamental transaction)
- Some of the work is "inside" the teacher's head
- Lack of frameworks for seeing teaching
- Grain size issues: at what scale to look
- Disciplinary traditions and perspectives that focus closely only on part of the story
- Teaching is undervalued and misunderstood

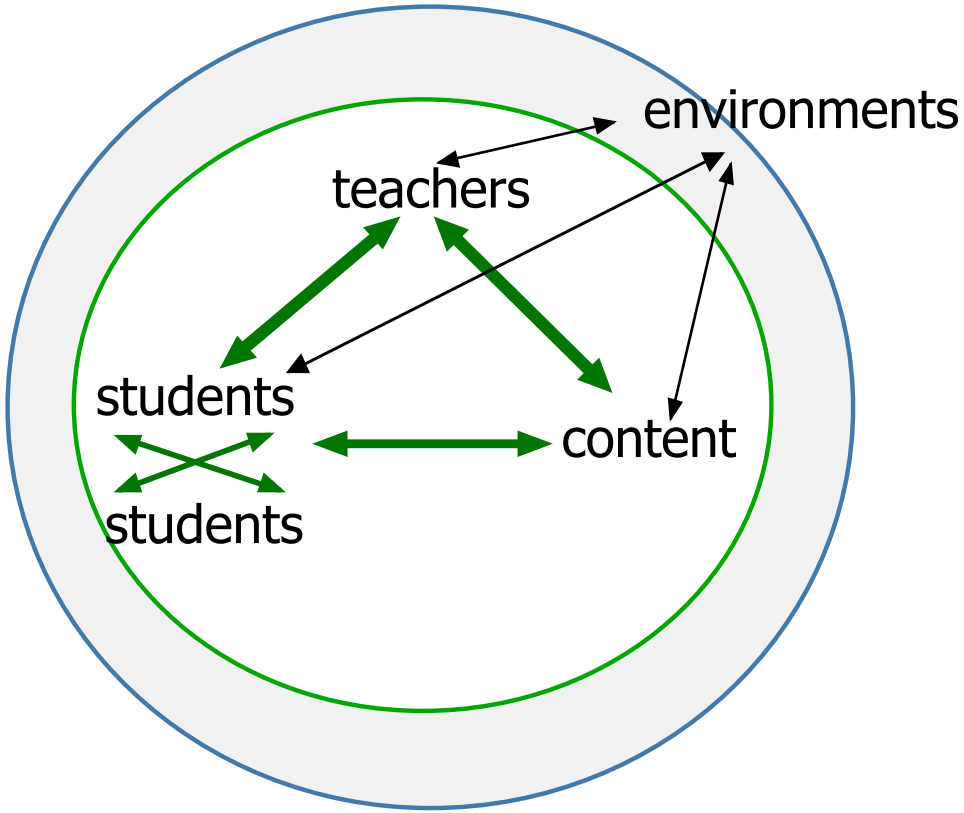
© 2008 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

A closer view: Teaching as mathematical work

- Using and analyzing representations, and mapping across different kinds of representations
- Defining terms and attending closely to language
- Using and inventing notation
- Producing and analyzing explanations
- Generating simpler and more complex versions of a problem
- Asking mathematical questions
  - Why does this work? Does this work in all cases? Do we have all the solutions? How are these two representations related?
- Thinking of special cases
  - Boundary cases, or examples that might push an initial idea

© 2008 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Teaching and learning of algebra

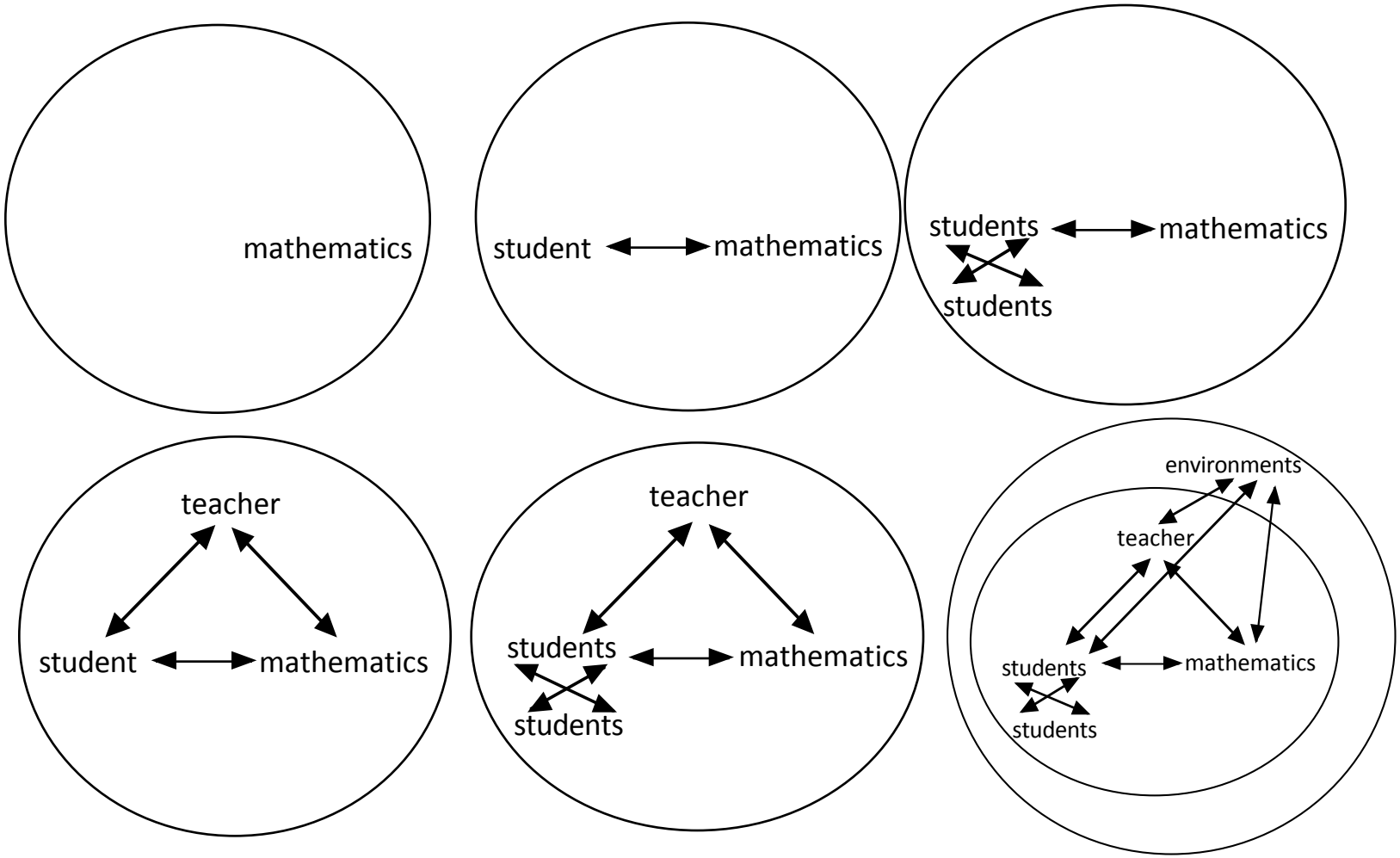


- Instruction as the "black box" of interactions among students, teachers, content
- (Cohen) Teaching is a thoughtful human construction designed to improve learning
- Teaching practice is deliberate and attentive.
- Practicing teachers seek to connect their teaching to students' learning.

Cohen, Raudenbush, & Ball (2003). Resources, instruction, and research. EEPA.

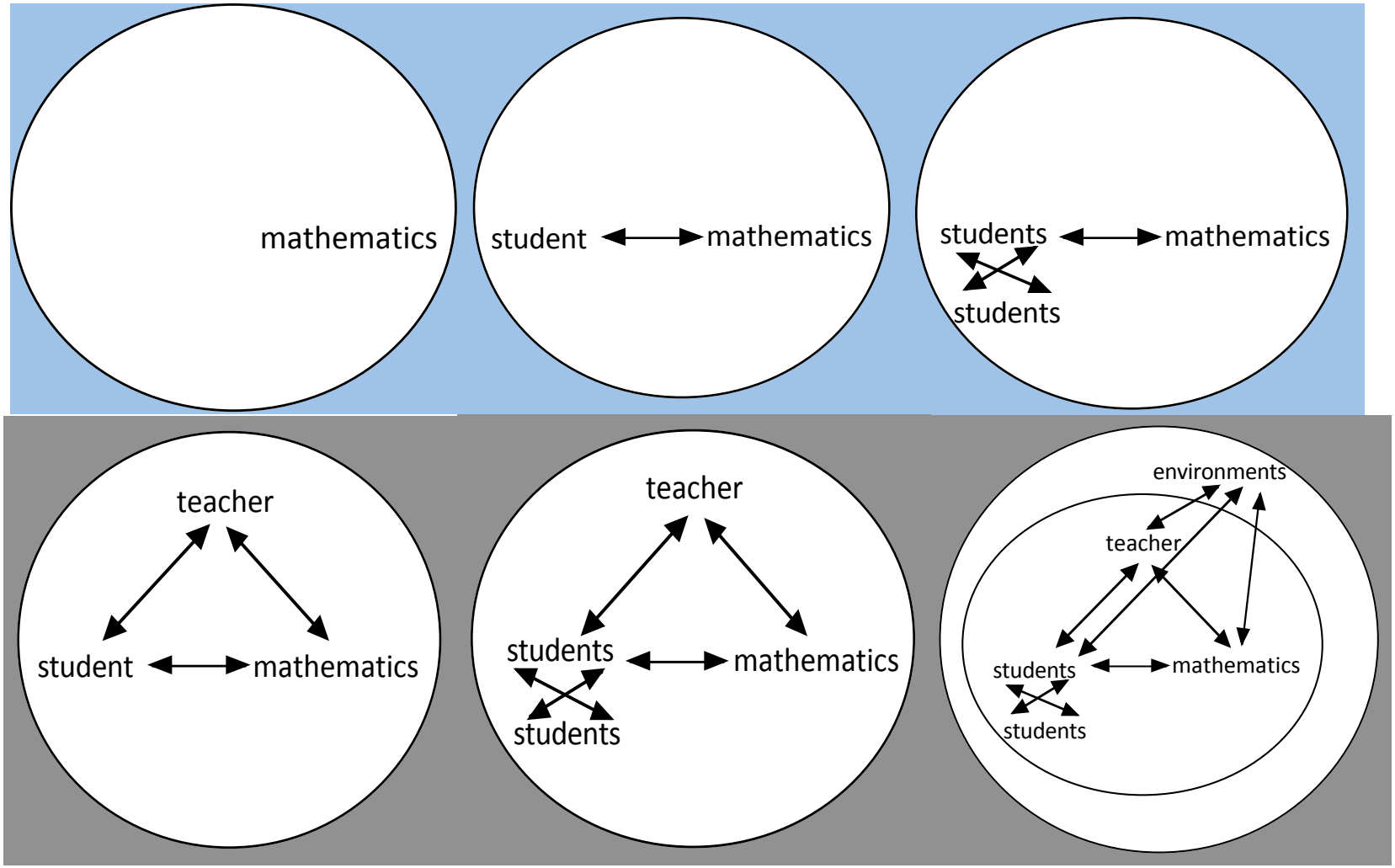
© 2008 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Alternative optics on the learning of algebra



© 2008 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Where we have focused



© 2008 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Knowledge and practice

- Dewey (1902): the logical and psychological structures of a subject
- Bruner (1960): intellectually honesty
- Schwab (1971): substantive and syntactical disciplinary structures (nature of knowing)
- Shulman (1986): pedagogical content knowledge

© 2008 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Why this matters: Teaching as unnatural and intricate work

© 2008 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Teaching as mathematically "natural" work, and the limits of this perspective

- Some aspects of teaching depend on mathematical instincts, habits of mind, practices
- So an additive view of learning to teach may make sense — add other knowledge to mathematical knowledge and habits

but —

- Teaching mathematics also involves doing things that are mathematically *unnatural*

© 2008 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Examples of mathematics teaching as mathematically unnatural work

1. Unpacking mathematical ideas
2. Listening to mathematically imprecise language
3. Not automatically affirming correct statements
4. Hearing what others say, not what you think
5. Surfacing “error”

© 2008 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

One core problem of teaching mathematics

- Managing the journey toward compression
- Keeping one eye on the mathematical horizon while avoiding "compression impatience"
- Keeping an eye on the students and on the mathematics and recognizing opportunity moments for compression
- Teaching and learning mathematics as language work whose tools are definitions, representations, symbols, language

- (Hy Bass)

© 2008 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Teaching as intricate work: One look

1. "Off camera": Before this episode
2. During these 2 minutes

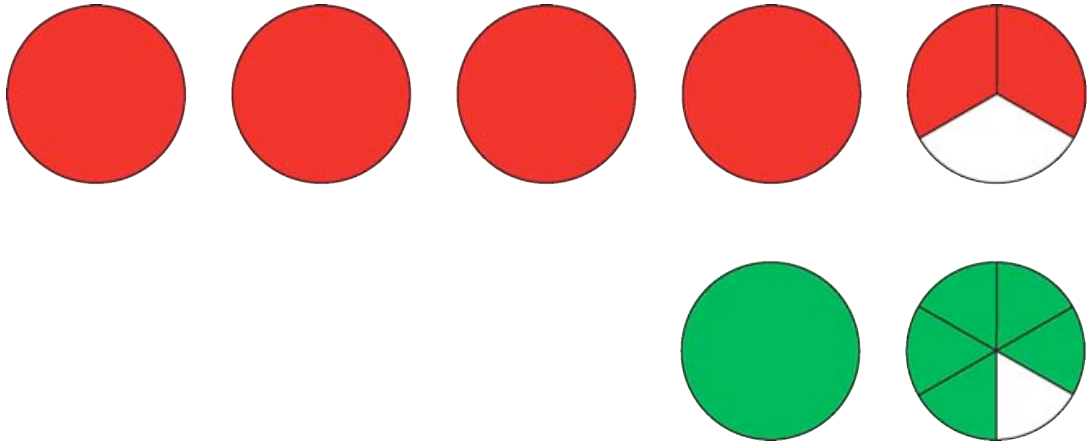
What are some specific tasks required in order to teach this segment?

© 2008 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Modeling addition of negative and positive fractions

$$-4\frac{2}{3}+1\frac{5}{6}$$

- Red "pies" to represent negative numbers
- Green "pies" to represent positive numbers

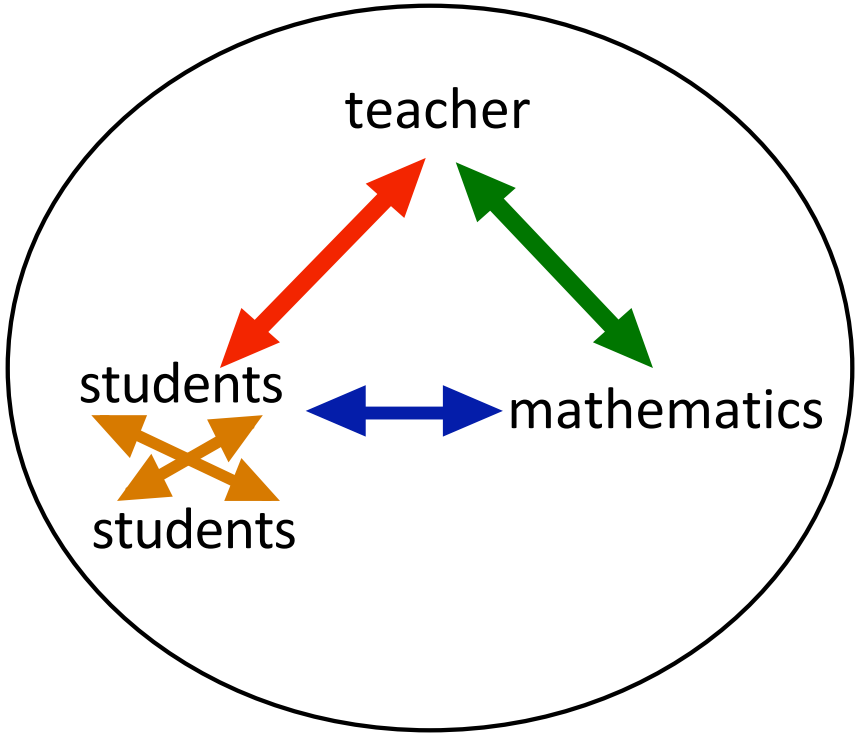


© 2008 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission



© 2008 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

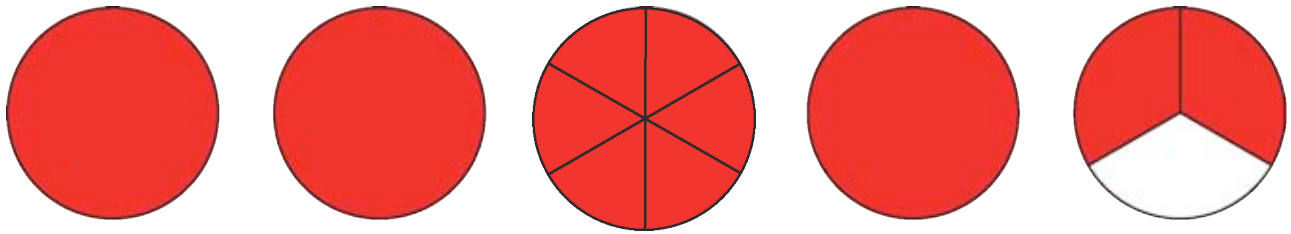
The work of teaching mathematics: What is there to "see"?



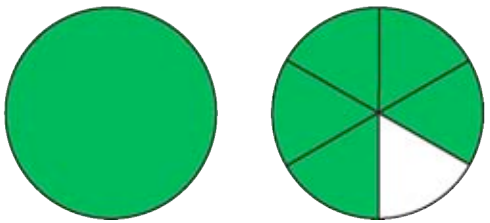
© 2008 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Modeling $-4\frac{2}{3} + 1\frac{5}{6}$

- Red "pies" to represent negative numbers
- Green "pies" to represent positive numbers



Answer: $-2\frac{5}{6}$



© 2008 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Three terrains of teaching practice

1. Knowledge extended to learners
   a) How knowledge is extended
   b) The nature of knowledge
2. Instructional discourse
3. Teachers' acquaintance with students' knowledge and resources

D. Cohen (2008)

© 2008 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission

Toward bringing teaching into our conversations . . .

© 2008 Mathematics Teaching and Learning to Teach
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu
Please do not circulate or cite without permission